Contemporary Mathematics

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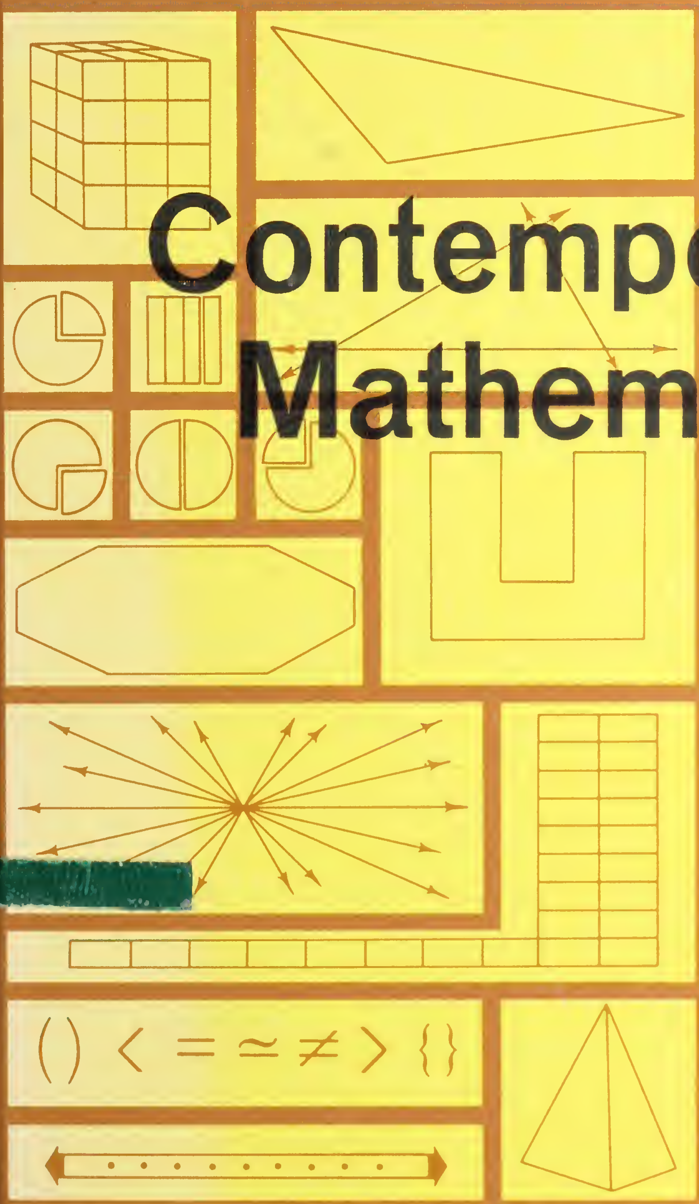

HANWELL MacPHERSON
SMITH CHIVERS

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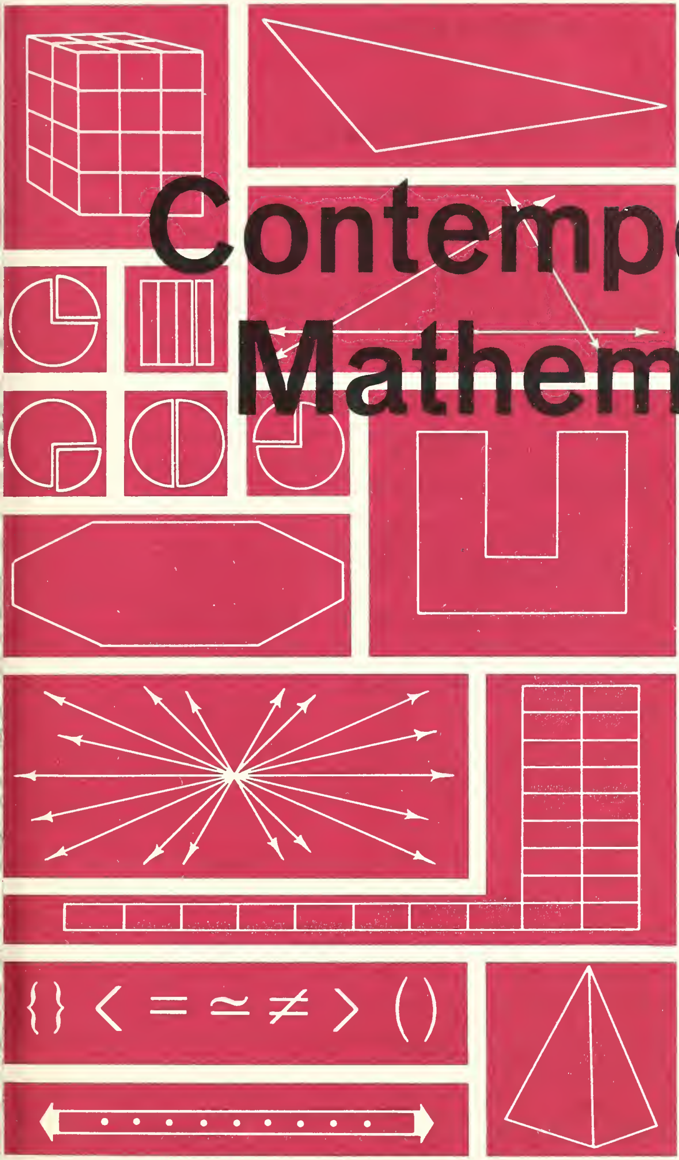

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Contemporary Mathematics 1 is the seventh book in the Holt, Rinehart and Winston Canadian arithmetic programme.

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About This Book

The student who works without understanding is like a production-line worker who, having no idea of the uses of the component he is making, solders wires to terminals all day. He finds little pleasure and less progress in his work and it is difficult for him to apply his knowledge to other things.

This mathematics book has been designed to help you to learn and to *understand* the work in the Grade 7 course. New processes and ideas are presented in several different ways, so that if you find difficulty in understanding one explanation you will probably understand another.

It is most important that you should *think for yourself* when using this book, since thinking produces ideas, and ideas aid understanding. If you work with your book in this way, you will build up an understanding of mathematics so that the practical uses of what you have learned will become apparent to you; you will know what you are doing and why you are doing it.

The
Holt, Rinehart and Winston
Canadian Arithmetic and Mathematics
Programme

Grade 1	Number Patterns a text workbook	Book 1
Grade 2	Number Patterns a text workbook	Book 2
Grade 3	Patterns in Arithmetic	Book 3
Grade 4	Patterns in Arithmetic	Book 4
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1 The Whole Numbers

WHAT IS A SET?

1. We are all used to talking about sets of hockey cards, sets of coins and sets of birds' eggs. Sometimes, instead of the word *set*, we may use the word *collection*. In mathematics we are often interested in *sets* or *collections* of numbers. We may want to discuss the set of numbers which are divisible by three and are less than 10. This set of numbers is 3, 6, 9. In mathematics we would write these numbers within a pair of *curly brackets* or *braces* like this $\{3, 6, 9\}$. We call each number in this set an *element* of the set; so we have:

$$\{3, 6, 9\}$$

elements of the set

In mathematics we call a collection of things a set.

2. Write the following sets within curly brackets:
   a the numbers less than 20 that are divisible by 9
   b the odd numbers that are less than 7
   c the even numbers that are greater than 5 but less than 15
   d the numbers between 20 and 40 that are divisible by 11
   e the numbers between 15 and 25 that are divisible by 4

3. How many elements are there in each of the sets in example Number 2?

NATURAL NUMBERS AND WHOLE NUMBERS

1. When we are counting, we use the numbers 1, 2, 3, 4, 5, 6 . . . Where we have put three dots, these three dots mean that these numbers go on for ever and ever. These numbers make up a set of numbers that we call the *counting* numbers. We also call this set of numbers the set of *natural* numbers. We often represent this set of numbers by the letter N and we write:

$$N = \{1, 2, 3, 4, 5, 6 \ldots\}$$

This means that the elements of set N are 1, 2, 3, 4, 5, 6 . . .

2 If we add 0 to the set of natural numbers we have a new set of numbers that we call the *whole* numbers. We can represent this set by the letter W and we write:

$$W = \{0, 1, 2, 3, 4, 5, 6 \ldots\}$$

This means that the elements of set W are 0, 1, 2, 3, 4, 5, 6 . . .

3. **a** Write the set of the first six whole numbers. We have: $\{0, 1, 2, 3, 4, 5\}$. We see that this set has six elements. The six elements are 0, 1, 2, 3, 4 and 5.
   b Write the set of the first six natural numbers.
   c Write the set of whole numbers that are less than 5.
   d Write the set of whole numbers that are greater than 6 but less than 14.
   e Write the set of natural numbers that are less than 15.

4. How many elements are there in the following sets?
   a the whole numbers less than 20 but greater than 12
   b the natural numbers less than 3
   c the whole numbers less than 15
   d the natural numbers between 11 and 21
   e the natural numbers less than 20 that are divisible by 2

TWO USEFUL SIGNS

1 Which is greater, 9 or 6?
Which is smaller, 9 or 6?
We know that 9 is greater than 6 and that 6 is less than 9. We have a useful, shorthand way in mathematics for writing *is greater than*. We use the symbol $>$. For *9 is greater than 6* we can write $9 > 6$. For *is less than* we use the symbol $<$. For *6 is less than 9* we can write $6 < 9$.

2. Use the appropriate symbol, $>$ or $<$, to make the following into true statements. The first two are done for you.
   a $6+14 \ ? \ 7+9$
   $6+14=20 \quad 7+9=16$;
   so $6+14 > 7+9$.
   b $9 \div 3 \ ? \ 20+5$
   $9 \div 3 = 3 \quad 20+5=25$;
   so $9 \div 3 < 20+5$.
   c $14+7 \ ? \ 2 \times 7$
   d $9 \times 6 \ ? \ \frac{1}{2}$ of 200
   e $16 \div 4 \ ? \ 20 \div 2$
   f $7 \times 3 \ ? \ 3 \times 5$

g $14+27$? $15+16$

h 16×24 ? 18×25

i $305-196$? $196-72$

j $1{,}315+2{,}460$? 27×85

k $4{,}135\div5$? 2×397

l $2{,}025\div25$? 7×216

3. In the exercises below, x has been used to represent a whole number. What set of numbers could be used to replace x in each of the examples? The first one is done for you.

a $x+3<6$

We see that $0+3<6$, $1+3<6$, $2+3<6$;

so for x we could substitute 0, 1 or 2.

The set for x is $\{0, 1, 2.\}$.

b $x+5<2\times5$

c $3+6>x+2$

d $2\times7>x+6$

e $x-2<9$

f $x+14<21$

g $3\times6>x+10$

h $x-5<12$

i $9\div3>x$

j $15+x<20-1$

k $13+x<30$

IDENTICAL SETS

1 Consider the sets below:

$X=\{2, 4, 6\}$ $\qquad$ $Y=\{4, 6, 2\}$

How many elements are there in each set?

Does each element in set X have an element in set Y that is equal to it? Can you match each element in set X with each element in set Y? Are the elements of set X exactly the same as the elements of set Y?

When two sets have exactly the same elements, no matter what the order of the elements is, we say that the sets are identical sets. We can write set X = set Y. In set language the equals sign (=) means *is identical with.* This tells us that set X and set Y contain the same elements.

2 Consider the sets below:

$A=\{0, 1, 2, 3\}$ $\qquad$ $B=\{1, 2, 3\}$

Are these two sets identical? Why not?

Which element of set A is not contained in set B?

Since the two sets are not identical we write set A $\neq$ set B.

In set language $\neq$ means *is not identical with.*

3. Which of the pairs of sets below consist of two identical sets?

a Set $N=\{1, 2, 3\}$; Set M is the set of natural numbers < 4

b Set X = {6, 9, 3, 12}; Set Y = {12, 3, 9, 6}

c Set A is the set of natural numbers < 10; Set B = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

d A = {0, 4, 2, 6, 8}; Set B is the set of even numbers less than 10

e The set of odd numbers on a clock face; {11, 3, 5, 9, 7, 1}

f The set of whole numbers < 6; the set of natural numbers < 6

g Z = {1, 2, 3, 4}; F = {4, 3, 2, 1}

h The set of all even numbers; the set of all whole numbers divisible by 2

i K = {1, 0}; the set of whole numbers less than 3

j {(1+2), (1+3), (1+4)}; {(4−1), (5−1), (6−1)}

FINITE AND INFINITE SETS

1 Set X is the set of the natural numbers that are less than 5. We see that X = {1, 2, 3, 4}.
How many elements are there in this set?
There are four elements in the set. When we can count the number of elements in a set we say that the set is *finite*.

2 Set Y is the set of the natural numbers that are greater than 5. We see that Y = {6, 7, 8, 9 . . . }.
How many elements are there in this set?
We see that the set goes on for ever and ever. When we *cannot* count the number of elements in a set we say that the set is *infinite*. Infinite means *without end*.

3. Which of the sets below are infinite?

a the set of whole numbers

b the set of natural numbers

c the set of natural numbers less than 1,000,000,000

d the set of even numbers

e the set of odd numbers

f the set of natural numbers greater than 1,000,000,000

g the set of whole numbers less than 150,000

h the set of whole numbers between 36 and 63

i the set of natural numbers divisible by 10

j the set of natural numbers greater than 150,000

THE EMPTY SET

1 Write the set of whole numbers that are less than 10 but greater than 8. The only whole number that is less than 10 and greater than 8 is the whole number 9.

The set required is $\{9\}$. Here we see that the set has only one element.

2 What is the set of whole numbers that are greater than 9, but less than 10? We see that there is no whole number that is both greater than 9 and less than 10. There are no elements in this set. The set that contains no elements is called the **empty set**. We can write the empty set as $\{\}$. We write the braces with no elements between them.

3. Select the empty sets from the following:

a the whole numbers less than 8 which are divisible by 9
b the odd numbers divisible by 2
c the natural numbers less than 2
d the whole numbers less than 1
e the natural numbers less than 15 but greater than 16
f the whole numbers less than 12 but greater than 10
g the even numbers divisible by 3
h the whole numbers less than 2×4 but greater than 1×4
i the odd numbers less than 2
j the whole numbers between 0 and 1

NUMBER AND NUMERAL

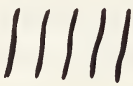
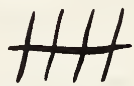
From the earliest time man has wanted to know how to count things. He has used many ways. Although we are not sure of the first way in which he counted, we think that he may have used one of these methods:

a Let us think of a shepherd in days gone by. He may have wanted to count the number of sheep as he sent them out to graze. To do this, he would have a pile of pebbles, and as each sheep went out he would put one pebble into a pile. By the time that all the sheep had left, he would have had a pile of pebbles and each pebble in the pile would have represented one sheep. When the sheep returned, he would take one pebble from the pile for each sheep that returned. If he found that he had no pebbles left in the pile when the sheep had returned, he would know that all the sheep had returned. But if any pebbles remained in the pile he would know that this number of sheep

was missing. In this way of counting, he let one pebble represent one sheep. Each pebble had a sheep and each sheep had a pebble. Sometimes we count this way. If we have a classroom with 30 desks in it and each desk contains a pupil, then we can say, without counting that there are 30 pupils in the room. This method of counting depends on what we call **one-to-one** relationship.

b Another way of counting was to use what was called a tally stick. Suppose a miller wanted to keep count of the number of sacks of flour that he had ground in a day; he would take a stick and cut a mark on it for each of the sacks of flour that was ground. Each mark would represent one sack of flour. The number of marks and the number of sacks of flour would be the same. There would be one mark for each sack of flour; there would be one sack of flour for each mark. This is another example of one-to-one relationship.

c Man has also used other symbols to represent the number of things. At first for 5 separate things he may have made 5 separate marks. Example:

 or 

When the number to be kept count of was very big, this method was unsatisfactory. Can you think why? In time, man came to use symbols or signs to represent numbers. For example, for five things he could have put

 or 

For ten things he could have put

 or  or 

Each of the marks that we see above is *not* a number. It is only a mark or a sign to represent a number. We call marks or signs that represent numbers, **numerals.**

NUMERALS THAT WE USE

1 If we want to represent a number of things we write numerals like 2, 56, 134, 789, 33. To make our numerals we use the figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These figures we call **digits.** Using just these ten symbols

or digits, we can make all the numerals that we use in counting. We have learned how to combine these symbols to name any number that we use in counting. We could use '4' and '8' to name the number 48. We could use just two digits, '4' and '5', to name the number 4,455.

2 Let us have a look at these two sentences:

'Tom' has three letters.

Tom has three letters.

What different meanings do these two sentences have? In the first sentence when we use 'Tom' we are thinking of the word Tom and we think that this word is made up of the three letters, T, O and M. In our second sentence we are thinking of the person Tom. Tom is the name of the person and we are thinking of the person Tom. In this sentence we are thinking of him as having three letters that have come in the mail.

Now let us look at these arithmetical sentences:

'134' is the numeral that we use to represent the number which is made up of one hundred, three tens, and four ones.

In this Grade 7, there are 134 pupils.

In the first of these two sentences, '134' is a numeral. We are thinking of the name of a number. In the second sentence, 134 is being used as a number and it tells us how many pupils there are. From now on when we are thinking of the number, we shall write the digits with no quotation marks around them. When we are thinking of the numeral, or the name of the number, we shall write the digits with quotation marks around them.

3 Study the sentences below:

a John has 56 picture cards.

b '56' is made up of two digits.

c To represent fifty-six we can use the symbol '56'.

d In the numeral '2,158' the digit symbol '1' comes before the digit symbol '8'.

e 2,158 people attended the hockey game.

Can you see the distinction between talking about numerals and talking about numbers in the sentences above? What we have done in **d** and **e** is to represent the number as 2,158 and the numeral as '2,158'. The '2,158' means the numeral we use to represent the number 2,158. In other words, the symbols or figures plus the quotation marks represent the *name* of the number, whereas the symbols or figures without quotation marks represent the actual quantity (or **number** of objects) involved.

SOME WAYS OF WRITING NUMERALS

1 There were 15 girls in the room.

The compound symbol that we have used to represent the number fifteen is '15'. We could represent this number in many ways. Here are some of the numerals (numerical symbols) we could have used:

3×5	$1+4+10$
$7+8$	$9+1+1+4$
$30\div 2$	$17-2$

All these are numerals that we could have used to represent the number 15.

A numeral is the symbol, or set of symbols, that we use to represent a number.

2. Represent each of the following numbers by 5 different numerals:
   a fifty **b** twenty-six **c** fourteen **d** one

BRACKETS ARE USEFUL

1 Find $6+4\times 7$

a If we work this example in the order of the operation signs we will first add the 6 to the 4, which gives us 10, and then find the product of 10 and 7, which is 70. Here, we added first.

b If we do the operation of multiplication first we will have $6+4\times 7=6+28=34$. This is a different answer from what we get when we add first.

2 In mathematics we have a rule: if there are no brackets or parentheses in an exercise with multiplication and addition in it, we do the multiplication first. So $6+4\times 7$ means $6+(4\times 7)$, or $6+28$, which is equal to 34.

3 Sometimes we may wish to do our addition first. When we do, we put brackets, [ ], or parentheses, ( ), around the part of the exercise we want done first. If we write $(6+4)\times 7$, we mean add the 6 and the 4 first to give us 10, and then find the product of 10 and 7, which gives us 70.

4 There are several different kinds of brackets that we use in mathematics. We often give distinctive names to each kind.

{ } are often called **curly brackets** or **braces**.

[ ] are often called **square brackets**.

( ) are often called **parentheses** or **round brackets.**

5 Sometimes in mathematics we have an example such as this: $(7+5)\times(3+8)$. In this example we add the 7 and the 5, and the 3 and the 8, before we multiply. Thus,

$$(7+5)\times(3+8)=12\times11=132$$

6. Work the following: (Remember, when there are parentheses, to work the part inside the parentheses first. When there are no parentheses, do the multiplication first.)

a $(3\times5)\times(4+8)$ **b** $6\times(4+6)\times6$ **c** $3\times(4+15)\times(5+11)$
d $9\times6+4+8$ **e** $4\times6+8\times5$ **f** $3\times(9+8)+3$
g $(11+9)\times(12+8)$ **h** $4+6\times3\times(8+1)$

EXPANDED NUMERALS

1 Let us think about the numeral '487'. '487' represents $400+80+7$. But '400' means 4×100, and '80' means 8×10, and '7' means 7×1. Accordingly, another way that we might write '487' is $4\times100+8\times10+7\times1$. To make sure that we do the multiplication first, although we have a rule which tells us to do the multiplication first when we have multiplication and addition together, we can write this numeral as

$$(4\times100)+(8\times10)+(7\times1)$$

2 Let us think what the numeral '629,537' means. '629,537' means $6\times100{,}000+2\times10{,}000+9\times1{,}000+5\times100+3\times10+7\times1$. If we use parentheses, we write:

$$(6\times100{,}000)+(2\times10{,}000)+(9\times1{,}000)+(5\times100)+(3\times10)+(7\times1)$$

Another way of writing '100' is 10×10; 1,000 is 10×100; so, for 1,000 we can write $10\times10\times10$. 10,000 is $10\times1{,}000$. For 10,000, then, we can write $10\times10\times10\times10$.

Now let us think of the number 487 again. We can indicate this number as $(4\times10\times10)+(8\times10)+(7\times1)$.

100,000 can be written as $10\times10{,}000$, which is the same as $10\times10\times10\times10\times10$. So for '629,537' we can write $(6\times10\times10\times10\times10\times10)+(2\times10\times10\times10\times10)+(9\times10\times10\times10)+(5\times10\times10)+(3\times10)+(7\times1)$.

The numerals we write in this way we call **expanded numerals.**

3 The expanded numeral we could write for the number '4,060' is $(4\times10\times10\times10)+(0\times10\times10)+(6\times10)+(0\times1)$.

4 We have learned in our earlier grades that every digit of a numeral in our way of writing numbers tells us the number of 1's, or the number of 10's, or the number of 100's (which are 10×10's), or the number of 1,000's (which are $10\times10\times10$'s), and so on.

Expanded numerals are just another way of showing us this feature of our numeration system. Because our numeration system depends on the number 10, we call it the **decimal** numeration system. **Decimal** comes from the Latin word **decem,** which means 10. We also say that our numeration system uses **the base ten**; that is, ten is the foundation or basis of our numeration system.

5. Write expanded numerals for each of these numbers:

a 48	**b** 63	**c** 500	**d** 9,000
e 350	**f** 729	**g** 6,133	**h** 9,015
i 10,000	**j** 2,001	**k** 309	**l** 304,906

6. Write decimal numerals for each of the following: (The first one is done for you.)

a $(2\times10\times10)+(8\times10)+(1\times1)=281$

b $(6\times10)+(7\times1)$

c $(3\times10)+(5\times1)$

d $(1\times10)+(0\times1)$

e $(9\times10)+(0\times1)$

f $(8\times10\times10)+(4\times10)+(2\times1)$

g $(6\times10\times10)+(0\times10)+(0\times1)$

h $(6\times10\times10\times10\times10\times10)+(7\times10\times10\times10\times10)+$
$(3\times10\times10\times10)+(0\times10\times10)+(0\times10)+(2\times1)$

i $(3\times10\times10\times10)+(0\times10\times10)+(0\times10)+(0\times1)$

j $(4\times10\times10\times10\times10)+(0\times10\times10\times10)+(8\times10\times10)+$
$(9\times10)+(0\times1)$

7. Write decimal numerals for the following: (The first one is done for you.)

a $(6\times1)+(9\times10)+(8\times10\times10)=896$

b $(0\times1)+(0\times10\times10)+(8\times10\times10\times10)$

c $(6\times1)+(7\times10)+(8\times10\times10)+(9\times10\times10\times10)$

NUMBER SENTENCES

1 **a** When we are speaking or writing we use sentences to express our thoughts. In mathematics we often wish to make statements about numbers. We may want to say "Eight plus ten equals twenty minus 2." In mathematics we could write a **number sentence** for this:

$$8+10=20-2$$

In this number sentence we can think of the symbol for 'equals' as being the verb of the sentence. A number sentence that uses the symbol for 'equals' is called an **equality**. Number sentences involving equality are called **equations.** In the number sentence $8+10=20-2$ we call $8+10$ the left hand side of the equation. For *left hand side* we can write L.H.S. We call $20-2$ the right hand side of the equation. For *right hand side* we can write R.H.S.

b Write number sentences that are equations, using the following as the L.H.S.:

(i) $6+9$ (ii) 7×6 (iii) $20\div4$ (iv) $16-5$ (v) $14+12$

2 In mathematics we may write number sentences that are false. Consider the number sentence $6+4=5+3$. $6+4=10$, $5+3=8$, and we know that 10 does *not* equal 8 so the number sentence $6+4=5+3$ is false. We use a symbol in mathematics that means 'is not equal to'. The symbol is $\neq$. We can now write

$$6+4\neq5+3.$$

We have written a number sentence that is now true. We have written $6+4$ does *not* equal $5+3$.
A number sentence such as $6+4\neq5+3$ is called an **inequality**.

3. Copy the number sentences below into your workbooks. Beside each number sentence write 'True' or 'False'.

a $4+5=3\times3$ **b** $7\times9\neq7+9$ **c** $2\times3=12\div4$
d $7\times8\neq8\times7$ **e** $9\times6\neq2\times27$ **f** $16+8=30-6$

4 We already know two other symbols that we can use as 'verbs' in number sentences. They are $<$, meaning 'is less than' and $>$, meaning 'is greater than'. Here are two examples of number sentences using these symbols:

$$7\times3<7\times4; \qquad 6\times5>2\times3$$

5. Copy the following number sentences into your workbook and by each one write whether it is true or false:

a $7+4>14-10$ **b** $6\times5<8\times9$
c $36\div9<27\div9$ **d** $8-2>5+4$
e $17+8<4+2$ **f** $42\div7>15-7$

6. Use one of the symbols $=$, $<$, or $>$ instead of the question mark in the following examples so that you have number sentences which are true:

a 5 ? 10 **b** 3×4 ? 12 **c** $9\div3$? $2\div2$
d 4×9 ? 2×18 **e** $6-4$? $8-4$ **f** 6×4 ? 3×5
g $56\div8$? $30-15$ **h** $63\div9$? $14\div2$ **i** 9×5 ? 5×9
j 16×2 ? 7×3 **k** $19-19$? $14-12$ **l** 6×7 ? 7×8

7. In which of the examples above could you have replaced the question mark with the symbol $\neq$ and still have made number sentences that were true?

THE NUMBER LINE

1 We have seen that the set of whole numbers is $\{0, 1, 2, 3, 4, \ldots\}$. It is often very useful for us to imagine these numbers being arranged in order on a line.

2. Use a ruler to draw a line across your paper. The correct name for what you have drawn is a line segment. The distinction between a line and a line segment will be dealt with later.

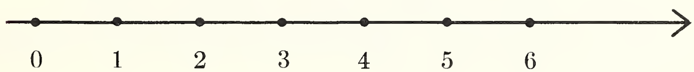
Mark a point near the left end of the line and label it '0'.

3 Later, when we study geometry, we shall learn that points have no size. They indicate positions. We shall also learn that lines are sets of points and that lines have no width. What we are really drawing are *models* of lines and models of points. Until we study points and lines more thoroughly we shall call the *models of points*, **points** and the *models of lines*, **lines.**

4. Measure $\frac{1}{2}$ inch to the right of this point and indicate another point. Label it '1'.

5. Measure $\frac{1}{2}$ inch to the right of the point which is labelled '1' and make another point. Label it '2'.

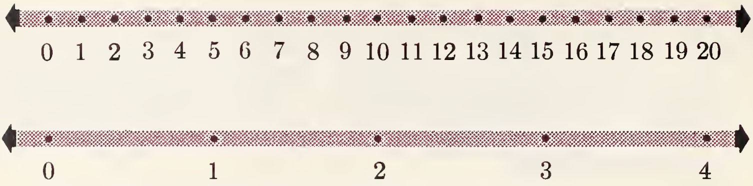
6. Carry on measuring and marking points this way until you come to the end of the line you have drawn.



7 The points that you have drawn are used to represent whole numbers. We have drawn a number line. It shows that 0 is the first whole number and 1 is the next. After 1 comes 2, and so on. The line can go on for ever. To show this we put an arrow on the right of the line.

8 We could extend the line to the left. Later on we shall do this and label points on this part of the line to represent a different kind of number. To show that the line could be extended to the left we put an arrow on the left end of the line.

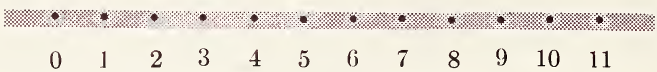
9 We have drawn the points $\frac{1}{2}$ inch apart on our number line. We could use any unit of length to make the distances between the points that we use to represent whole numbers. We must remember, though, to keep the points the same distance apart.



10. What do we notice about the numbers as we move to the right on the line? What can you say about the sizes of the numbers represented by the points labelled A and B on the number line below?

Did you have to count along the line to see which point represented the larger number? Which point represents the smaller number?

11

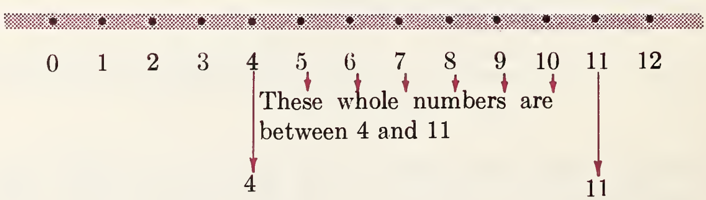


Note: arrows will appear on number lines only when we are thinking of the numbers going on for ever.

a Which number is *larger*, 9 or 5? We see that 9 is to the *right* of 5. We can write $9 > 5$.

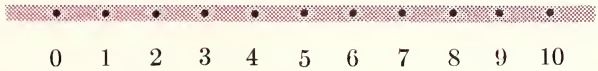
b Which number is *smaller*, 8 or 4? We see that 4 is to the left of 8. We can write $4 < 8$.

12. We can use number lines to tell which numbers are between others. For example, 6 is between 4 and 11.

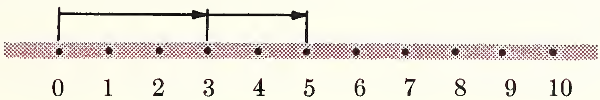


Is 3 between 4 and 11? Why?
Is 12 between 4 and 11? Why?

13 Use a number line to find the solution to $3 + 2 = x$.

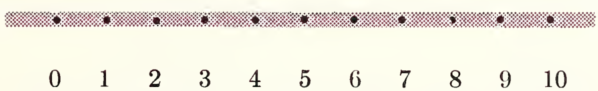


Start at 0 and move to the point which represents 3. $3 + 2$ is 2 more than 3. So the number represented by $3 + 2$ will be represented by a point to the right of the point representing the number 3. We must move two units past the 3 point. This brings us to the point representing the number 5.

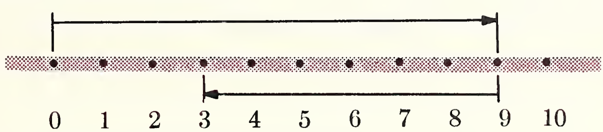


$3 + 2 = 5$, so $x = 5$.

14 Use a number line to find the solution to $9 - 6 = y$.



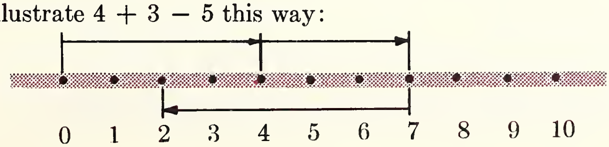
Start at 0 and move to the point representing the number 9. $9 - 6$ is 6 less than 9. The number represented by $9 - 6$ will be represented by a point to the left of the point representing the number 9. We must move 6 units to the left of the 9 point. This brings us to the point representing the number 3.



$9 - 6 = 3$, so $y = 3$:

15 For convenience we shall show addition by lines above the number line. Subtraction will be shown by lines below the number line.

16 We illustrate $4 + 3 - 5$ this way:



17. For the following, draw your own number lines and illustrate the operations:

a $5 + 6 - 4$

b $5 - 3 + 4$

c $2 + 2 + 2 - 5$

d $5 - 4 + 3$

e $8 - 5 + 3$

f $9 - 4 + 3$

g $6 - 6$

h $7 - 6 + 1 - 2$

i $1 + 1 + 1 + 1 - 3$

j $7 + 6 - 10$

18. Use a number line to tell which of these sentences are true:

a $5 < 9$

b $14 > 12$

c $3 + 5 < 4 + 4$

d $12 - 3 + 1 > 4 + 2 - 5$

e $10 - 4 - 3 < 3 + 6 - 5$

f $2 + 7 - 1 < 15 - 7$

g $5 + 8 < 16$

h $2 + 7 - 3 > 8 - 4 - 2$

i $9 + 4 - 7 > 2 + 7 - 1$

j $7 + 5 < 10 - 6$

19. How many whole numbers are between:

a 2 and 5

b 0 and 8

c 10 and 3

d 8 and 1

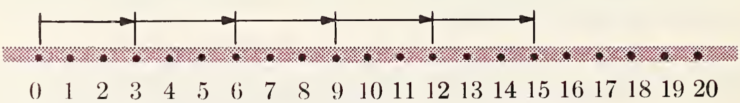
e 14 and 21

f 6 and 19

g 17 and 33

h 206 and 58

20. Use a number line to show that $5 \times 3 = 15$.
5×3 is the same as $3 + 3 + 3 + 3 + 3$



We make 5 moves of 3. Can you explain why?

21. Use a number line to show:

a $2 \times 6 = 12$

b $4 \times 3 = 12$

c $2 \times 7 = 14$

d $8 \times 2 = 16$

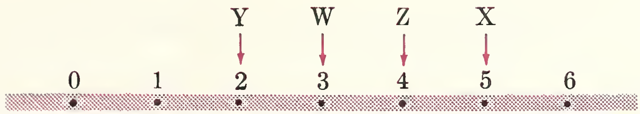
e $4 \times 4 = 16$

f $2 \times 9 = 18$

g $6 \times 3 = 18$

h $2 \times 10 = 20$

1 **a**



What number is represented by the point X?

We see that the point X corresponds with the point which we have labelled as '5'. The whole number 5 is paired with the point X. We say that the *co-ordinate* of the point X is 5. The whole number that we pair off with a particular point is called the co-ordinate of that point.

b What are the co-ordinates of the points W, Y and Z?

2 Write the set of whole numbers that are less than 3; then represent them on a number line.

a The set of whole numbers that are less than 3 is $\{0, 1, 2\}$.

b Consider the number line below:

We can put heavy dots on the points representing 0, 1, and 2; then we have:

This is a picture of the set $\{0, 1, 2\}$.

We call this picture a *graph* of the set $\{0, 1, 2\}$.

3. Draw graphs of the following sets. The first one is done for you.

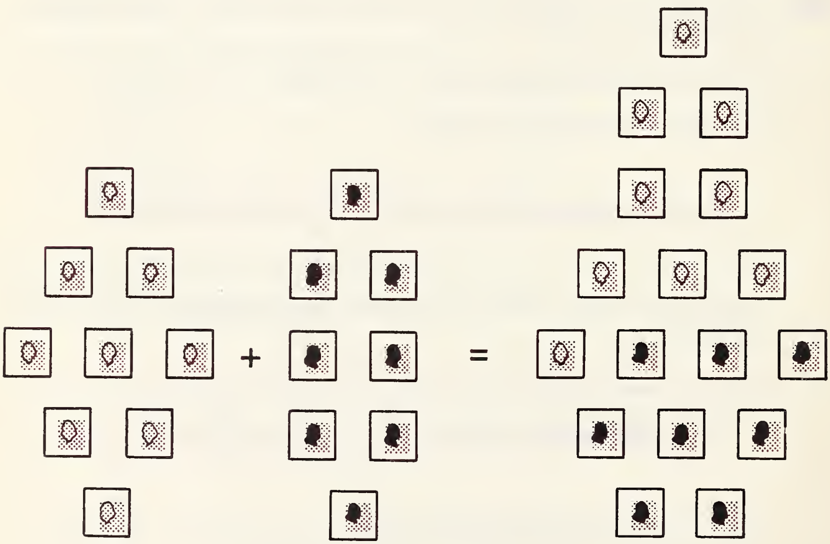
a $A = \{1, 3, 5\}$

b $A = \{2, 4, 6\}$ c $A = \{0, 5\}$
d $A = \{4, 8, 12\}$ e $A = \{7, 9, 16, 25\}$

4. Write the following sets and then graph them on number lines:
   a the set of whole numbers less than 20 that are divisible by 6
   b the set of whole numbers between 20 and 30 that are divisible by 7
   c the natural numbers that come between 10 and 15
   d the natural number which is midway between 7 and 17
   e the whole numbers less than 5 that are also natural numbers

ADDING

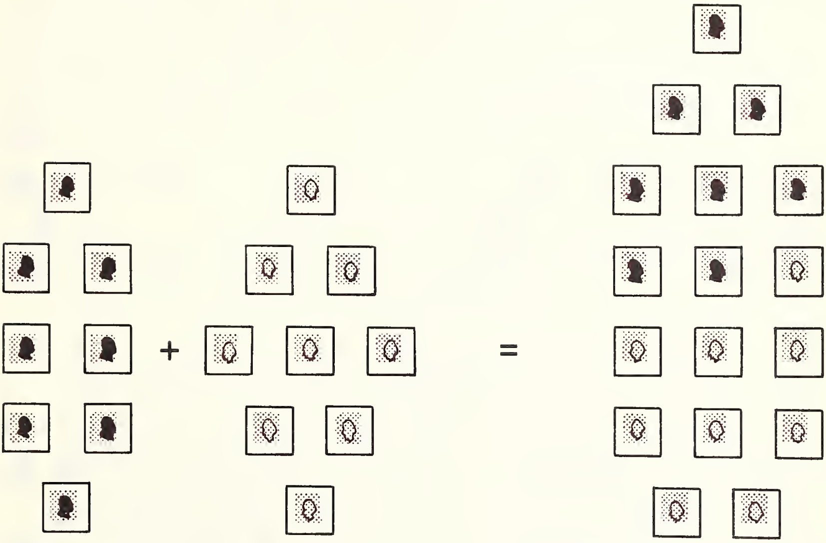
1 A boy had 9 Canadian stamps and 8 British stamps. How many stamps did he have in all?



$9 + 8 = 17$

To find out how many he had in all, we put the two sets of stamps together and then count how many stamps he had in the new set. We see that 9 plus 8 equals 17. When we put sets of things together and find a number to tell how many there are in the new set, we are doing the **operation of addition**.

Now let us look at this addition exercise another way. A boy had 8 British stamps and 9 Canadian stamps. How many stamps did he have in all?



8 + 9 = 17

We see that $8+9=17$.

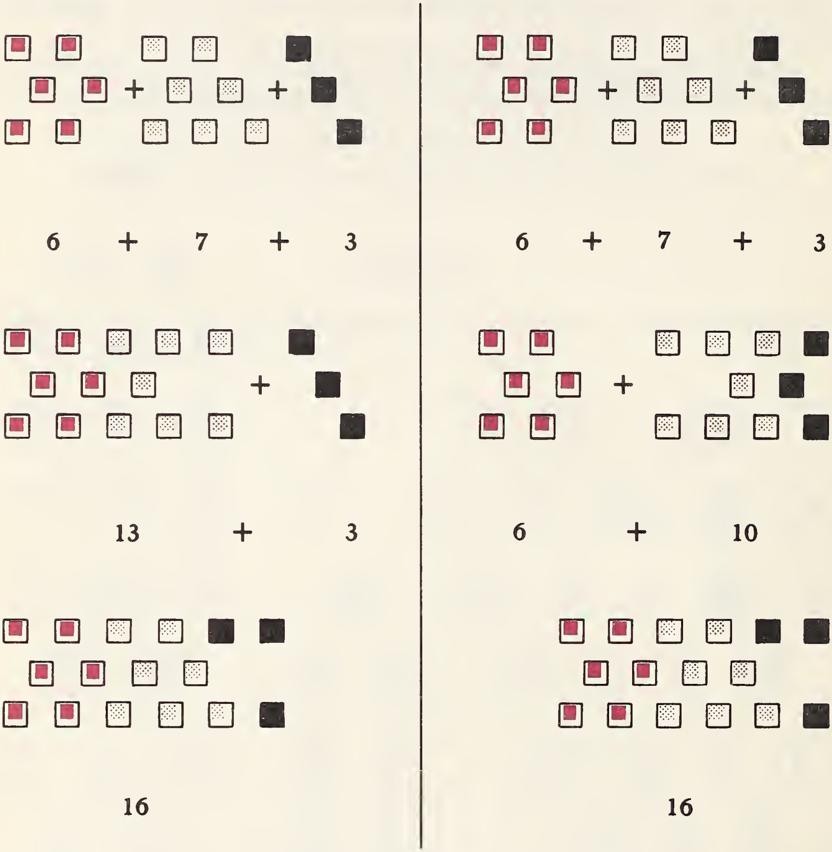
2 Work the following:

a $5+6$; $6+5$ **b** $7+12$; $12+7$

c $329+915$; $915+329$ **d** $654+1{,}086$; $1{,}086+654$

What do you notice about the answers you get in each case? If we let a represent any number and b represent any number, we see that $a+b=b+a$. This is an important rule in addition of numbers.

3 Another boy had 6 Canadian stamps, 7 British stamps, and 3 Italian stamps. How many stamps did he have in all? Now we have to put together a set of 6, a set of 7, and a set of 3.



We see by counting the number of stamps in the set that the total we get by putting the 3 smaller sets together is always 16. We see that $6+7+3=13+3=16$ and we see that $6+7+3=6+10=16$.

4 In the example above when we said that $6+7+3=13+3$ we added the 6 and 7 first. When we put $6+7+3=6+10$ we added the 7 and 3 first. In mathematics we use brackets or parentheses when we want to show which operation we do first. Later in this chapter, we shall learn more about the use of brackets. For $6+7+3=13+3$ we could write $(6+7)+3=13+3$ and for $6+7+3=6+10$ we could write $6+(7+3)=6+10$.

5. Work the following:

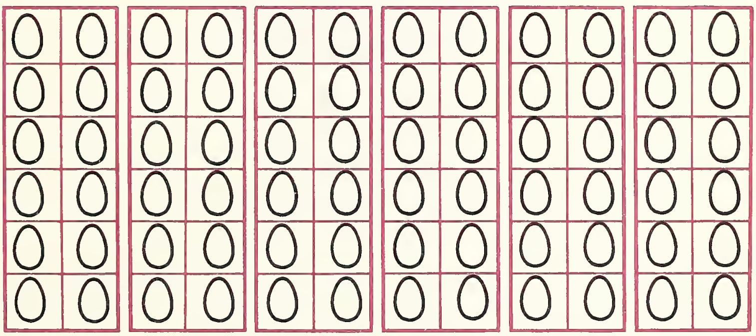
a $(5+9)+4$; $5+(9+4)$
b $(7+8)+5$; $7+(8+5)$
c $(16+22)+13$; $16+(22+13)$
d $(785+19)+286$; $785+(19+286)$
e $(1{,}355+628)+702$; $1{,}355+(628+702)$
f $(537+2{,}069)+7{,}300$; $537+(2{,}069+7{,}300)$
g $(1{,}296+1{,}415)+1{,}962$; $1{,}296+(1{,}415+1{,}962)$
h $(62{,}914+82{,}736)+55$; $62{,}914+(82{,}736+55)$

What do you notice about the sum in each of the examples above?

6 If we let a, b, and c represent any 3 numbers, we see that $(a+b)+c=a+(b+c)$.

MULTIPLYING

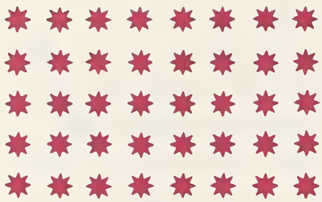
1 a Mother bought 6 cartons of eggs. In each carton there were a dozen eggs. How many eggs did she buy in all?



We see that here we have 12 eggs in each carton and all together we have 6 cartons. To find out how many eggs there are all together we take the 6 sets of 12 eggs and count them. When we find how many things there are in 6 sets of 12, we say we are finding the product of 6 and 12. We are finding the total number of things in 6 sets of 12. We write the product of 6 and 12 as 6×12. Above we arranged our example as 6 rows of things with 12 things in each row. We show that the answer is 72.

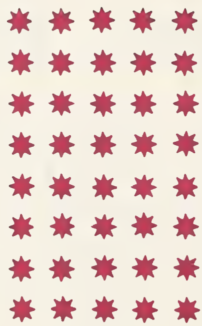
b When we find the product of two numbers we perform **the operation of multiplication.**

2 Look at this arrangement:



This arrangement consists of 5 sets of 8 things. We see that there are 40 things altogether. We say that the product of 5 and 8 is 40.

3 Now let us turn our arrangement the other way.



We now have 8 sets of 5 things. The total number of things is the product of 8 and 5. The product of 8 and 5 is 40. We see that the product of 5 and 8 equals the product of 8 and 5.

We see that $5 \times 8 = 8 \times 5$.

4. Work the following examples:

a 9×6; 6×9

b 15×8; 8×15

c 23×48; 48×23

d 105×295; 295×105

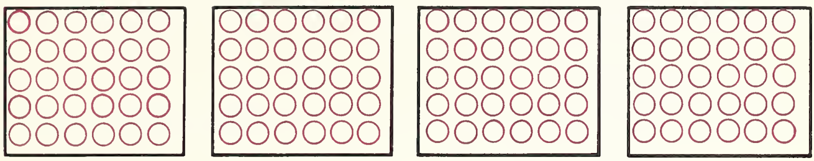
e 18×234; 234×18

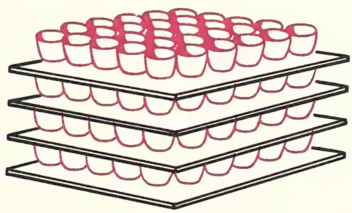
f $62 \times 1{,}015$; $1{,}015 \times 62$

What do you notice about the answers we get in each case?

5 If we let a be any number and b be any number, we see that the product of a and b is the same as the product of b and a. We can write the rule thus: $a \times b = b \times a$.

6 Suppose we have 4 trays. On each tray there are 5 rows of cups with 6 cups in each row. This diagram will show what we have.





We can see that there are 5×6 cups on each tray; that is, we have 30 cups on each tray. But we have 4 trays; so all together we have $4 \times (5 \times 6)$ cups. This equals 4×30 cups, or 120 cups.

But we can look at this another way. If we think of the trays as being stacked on top of one another, we can see that there are 4 levels of cups with 5 in each level; that is, $4 \times 5 = 20$ cups. But this vertical arrangement is repeated 6 times; so we have $6 \times (4 \times 5) = 6 \times 20 = 120$ cups. We see that $4 \times (5 \times 6) = 6 \times (4 \times 5)$.

7. Work the following:

a $7 \times (8 \times 9)$; $(7 \times 8) \times 9$

b $2 \times (6 \times 3)$; $(2 \times 6) \times 3$

c $10 \times (5 \times 4)$; $(10 \times 5) \times 4$

d $6 \times (8 \times 2)$; $(6 \times 8) \times 2$

e $23 \times (8 \times 9)$; $(23 \times 8) \times 9$

What do we notice about the answers in each case?

8 If we let a, b, and c be any 3 numbers, then $a \times (b \times c) = (a \times b) \times c$.

ADDING AND MULTIPLYING

If we let a, b, and c be any 3 numbers, we have learned these important rules about adding and multiplying:

I. Adding:

$$a+b=b+a$$

$$(a+b)+c=a+(b+c)$$

II. Multiplying:

$$a\times b=b\times a$$

$$(a\times b)\times c=a\times(b\times c)$$

We shall learn more about these rules in Chapter 2.

SUBTRACTION

1

Above we have a picture of 9 ★'s. A line is drawn separating the 9 ★'s, into 2 sets, a set of 5 and a set of 4. From this picture we can see that $5+4=9$.

2 Let us think of the 9 ★'s all together. If we subtract 4 of these ★'s we see that 5 will remain. This process of taking part of a set from a whole set is called subtraction. For 'nine take away four' we can write $9-4$. We know that the answer to this is 5. We thus write, $9-4=5$. If we put the 4 back together with the 5, we shall have the

9 that we started with. In this way we can see that $9-4=5$ is the same in arithmetic as $9=4+5$. When we subtract we take one set from another set. When we add we put one set together with another set; thus we can see that subtraction is the opposite of addition.

If we put 4 with 5 and add, we get 9.

If we subtract 4 from 9 we get the 5 we started with. We see that:

$5+4=9$ operation of addition

$9-4=5$ operation of subtraction

We can think of addition as being the *doing* operation and of subtraction as being the *undoing* operation.

We say that subtraction is the **inverse** of addition. Subtraction is the inverse operation to addition.

3. $17-8=9$. This means that $17=8+9$. Now write the following subtraction facts as addition facts: (The first is done for you.)

a $7-3=4$ means $7=3+4$ **b** $11-6=5$

c $32-17=15$ **d** $84-29=55$

e $178-70=108$ **f** $487-119=368$

g $1{,}015-964=51$ **h** $9{,}000-806=8{,}194$

i $75{,}113-6{,}284=68{,}829$ **j** $10{,}000-337=9{,}663$

k $25{,}016-428=24{,}588$ **l** $60-21=39$

m $325-136=189$ **n** $6{,}000{,}025-26=5{,}999{,}999$

4. Work the following in the way that the first example is worked for you:

a $16-7=x$ means $16=x+7$
$9+7=16$; so $x=9$

b $15-4=x$ **c** $21-14=x$

d $33-x=14$ means $33=14+x$
But $14+?=33$, so $x=?$

e $14-x=6$ **f** $34-x=19$

g $713-x=428$ **h** $539-x=479$

i $643-x=124$ **j** $1{,}216-x=800$

k $1{,}500-x=4$ **l** $1{,}001-x=900$

Note: $16-7=x$ and $16=x+7$ tell us the same thing mathematically. We shall write $16-7=x$ *means* $16=x+7$, but we shall remember that when we use *means* in examples like this, we are using *means* as a short way of writing *tells us the same thing as*.

5 $19-5-4=x$

If we subtract in the order in which we see the minus signs we shall have $19-5=14$. $14-4=10$.

If we subtract the 4 from the 5 first, we shall have $19-(5-4)=19-1=18$.

We see that the order in which we do the subtraction makes a difference to the size of our answer; so when we have a series of subtractions to perform we must be careful to put brackets or parentheses around the subtraction we want done first.

6. Work the examples below: (The first one is done for you.)

a $36-(14-8)=36-6=30$
b $(49-8)-3$
c $(67-26)-(14-9)$
d $301-(89-72)$
e $65-(42-19)$
f $(1{,}000-635)-(635-507)$
g $10{,}692-(18{,}623-17{,}841)$
h $(7{,}298-6{,}049)-(12{,}315-11{,}987)$

DIVISION

1 We saw when we were talking about multiplying that when we found how many things there were in 6 sets of 12, namely, 72, we were performing the operation of multiplication. We could write this operation $6\times12=72$. We see that 6 sets of 12 make 72; so if we ask how many sets of 12 there are in 72, we see that the answer is 6. In arithmetic we can write the answer to the question, "How many sets of 12 are there in 72?" as $72\div12$. Another way of writing this is $\frac{72}{12}$. Here, we are asking for the operation of multiplication to be undone. Here, we are asking for the operation of division to be performed.

We can see that:

$6\times12=72$ operation of multiplication
$72\div12=6$ operation of division

We can think of multiplication as being the *doing* operation and of division as being the *undoing* operation. Division is the inverse of multiplication. We see that since $6\times12=72$, therefore $72\div12=6$. We also see that since $6\times12=72$, therefore $72\div6=12$.

2 What is $35\div5$? Since division is the inverse operation to multiplication, we can think of this question as being, how many 5's make 35, or what times five equals 35? We know that $7\times5=35$; so the answer to $35\div5$ is 7. Thus, $35\div5=x$ means $35=x\times5$. For this to be true $x=7$.

3 Find x if $54 \div 6 = x$. $54 \div 6 = x$ means in arithmetic the same as $54 = x \times 6$. How many 6's make 54? For 54 we can write $54 = 9 \times 6$; so $x = 9$.

4. Work the following in the same way that **a** is worked for you:

a $63 \div 7 = x$ means $63 = x \times 7$. $9 \times 7 = 63$; so $x = 9$.

b $32 \div 8 = x$ **c** $49 \div 7 = x$ **d** $36 \div 4 = x$

e $81 \div 9 = x$ **f** $52 \div 13 = x$ **g** $42 \div 7 = x$

h $42 \div 6 = x$ **i** $25 \div 5 = x$ **j** $56 \div 28 = x$

5. Study examples **a** and **b** below and then work the examples that follow:

a $x \div 3 = 7$ means $x = 7 \times 3$; so $x = 21$.

b $18 \div x = 3$ means $18 = 3 \times x$. $3 \times 6 = 18$; so $x = 6$.

c $81 \div x = 9$ **d** $48 \div x = 8$ **e** $x \div 5 = 5$ **f** $x \div 9 = 4$

g $36 \div x = 4$ **h** $32 \div x = 4$ **i** $18 \div x = 9$ **j** $x \div 5 = 30$

ZERO

1. Work the following additions:

a $7+0$ **b** $9+0$ **c** $0+8$ **d** $0+0$ **e** $832+0$

f $0+915$ **g** $0+35$ **h** $362+0$ **i** $x+0$ **j** $0+x$

What do you notice about the answers in each case?

2 We see that when we add 0 to a number, or a number to 0, the answer is always the number itself. If a is any whole number, then $0+a=a$ and $a+0=a$. We say that 0 is the **identity element** for addition in the set of whole numbers. All this means is that when we add 0 to any whole number we get the whole number that we started with.

3. Work the following:

a 5×0 **b** 0×9 **c** 16×0 **d** 37×0

e 0×58 **f** 113×0 **g** 0×922 **h** 0×53

i $x \times 0$ **j** $0 \times x$ **k** 86×0 **l** 0×374

What do you notice about the answers in each case?

4 We see that when we multiply any number by 0, or 0 by any number, the answer is always 0. If we let 'a' represent any whole number then $0 \times a = 0$, and $a \times 0 = 0$.

5. Work the following:

a $16-0$ **b** $72-0$

6 Work the following: $0 \div 8 = \frac{0}{8}$. Let the answer to this be x. Then $0 \div 8 = x$ means that $0 = x \times 8$. What number times 8 will give us 0 as the product? We see that 0×8 will give us 0 as the product; so $0 \div 8 = 0$.

7 Work the following in the same way that the example in 6 above is worked:

a $0 \div 7$ **b** $0 \div 18$ **c** $0 \div n$

What do you notice about the answer in each case? We see that 0 divided by any number gives us 0 as the answer. If a represents any number then $0 \div a = 0$.

8 What is $6 \div 0$? Let the answer to this be x. Then $6 \div 0 = x$. This equation means that $6 = x \times 0$. But what do we always have when we multiply a number and 0? What is $x \times 0$? $6 = x \times 0$ means that $6 = 0$. This is false. There is no number such that when it is multiplied with 0 we get 6 as the answer.

9 Work the following in the way that number 8 was worked:

a $7 \div 0$ **b** $15 \div 0$ **c** $11 \div 0$ **d** $n \div 0$

We see that there is no number which will give us the answer to these division examples.

10 If $\frac{0}{0} = n$ and, from this, $0 = n \times 0$, what can n be? There is no single answer to this, so $0 \div 0$ has no meaning in our number system.

UNITY

1. Work the following multiplication problems:

a 1×9	**b** 7×1
c 15×1	**d** 1×18
e 1×105	**f** 1×1
g 45×1	**h** 1×609
i 1×100	**j** 1000×1

What do you notice about the product in each case? What are you led to believe about multiplication when one of the numbers is 1?

2 We see that when we multiply a number by 1, or 1 by a number, the product is always the number itself. If we let n represent any number, then

$$1 \times n = n \text{ and } n \times 1 = n$$

We say that 1 is the **identity element** for multiplication in the set of whole numbers.

3 Add 1 to any whole number. What number do we get as the sum? We get the next larger number on the number line. Suppose we start with zero and add 1, what is the answer? Now add another 1, what is the result? Can you reach any required whole number by adding 1 at a time?

We see:

$$\begin{array}{lr} 0 & \\ 0+1=1 & \\ 1+1=2 & \\ 2+1=3 & 1+1+1=3 \\ 3+1=4 & 1+1+1+1=4 \\ 4+1=5 & 1+1+1+1+1=5 \end{array}$$

All our whole numbers, with the exception of zero, can be thought of as being the sum of a certain number of 1's.

4 What is the value of $4 \div 4$?

Let the answer be x; then $4 \div 4 = x$.

For $4 \div 4 = x$ we can write $\frac{4}{4} = x$.

$\frac{4}{4} = x$ means $4 = x \times 4$.

By what number must we multiply 4 to get 4 as the product?

$$4 = 1 \times 4; \text{ so } x = 1 \text{ and } \frac{4}{4} = 1$$

5. Show that each of the following quotients is 1:

a $7 \div 7$ **b** $16 \div 16$ **c** $195 \div 195$ **d** $1000 \div 1000$

e $56 \div 56$ **f** $x \div x$ **g** $\square \div \square$ **h** $759 \div 759$

6. What is the answer to $0 \div 0$? What did we discover in the last section when we tried to divide zero by zero?

7 If we divide a natural number by itself the quotient is 1. Let n represent any natural number; then $n \div n = 1$ or $\frac{n}{n} = 1$.

8. Why do we make the rule above for all natural numbers and not for all whole numbers?

9 Try to subtract 1 from any whole number. What do you notice? Can you subtract 1 from zero? Why not? Now study the rule below and explain why we use natural numbers in one part of the rule and whole numbers in the other part of the rule.

If we subtract 1 from any natural number the answer is the number which comes immediately before it on the number line.

CLOSURE

1. We have seen that the set of whole numbers can be written as {0, 1, 2, 3, 4, 5 . . . }

a Add 5 and 3. Is your answer in the set?

b Add 14 and 9. Is your answer in the set?

c Add 6 and 0. Is your answer in the set?

d Add 169 and 213. Is your answer in the set?

e Take any two numbers from the set. Add them. Is your answer a whole number?

f Find two whole numbers whose sum is in the set.

g Add 2 and 2. Is your answer in the set?

h Take any whole number. Add this number to itself. Is the answer a whole number?

2 You have probably discovered by now that when you add two whole numbers together the sum of the two whole numbers is always a whole number. We can say this in several ways: "A whole number + a whole number = a whole number." "The sum of every two whole numbers is in the set of whole numbers."

Another way to say this is:

"The set of whole numbers is **closed** under the operation of addition."

This is an important property of whole numbers and is called **The Closure Property of Addition.**

3 If we let a and b be any two whole numbers, then $a+b$ is always a whole number.

4 $\{3, 6, 9, 12, 15, 18, \ldots\}$

Is this set of numbers closed under the operation of addition?

a Is the sum of 3 and 6 in the set?

b Is the sum of 12 and 12 in the set? How do you know?

c Is the sum of 15 and 48 in the set?

d Can you find any two numbers from this set which added together give a sum which is not in the set?

e Take any two numbers of this set. Add them. Is the sum in the set?

We are led to believe by experimenting that this set of numbers is closed under the operation of addition.

5 $\{0, 1, 2\}$

Is this set of numbers closed under the operation of addition?

a Find the sum of 0 and 1. Is your answer in the set?

b Add 1 and 1. Is your answer in the set?

c Add 2 and 0. Is your answer in the set?

d Add 1 and 2. Is your answer in the set?

We see that $1+2=3$ and that 3 is *not* in this set. Therefore, the set $\{0, 1, 2\}$ is *not* closed under the operation of addition.

6 For a set to be closed under the operation of addition *every* sum of two of its members must be in the set. We need only find one sum that is *not* in the set to be able to say, "The set is *not* closed under the operation of addition."

7 What is the product of 7 and 6? Is 7 a whole number? Is 6 a whole number? Is the product a whole number? Take any two whole numbers. Multiply them. Is the answer a whole number? Can you find two whole numbers such that when you find their product the answer is not a whole number. We see that when we multiply together two whole numbers the answer is always a whole number. We say that the set of whole numbers is closed under the operation of multiplication. **The whole numbers have the closure property of multiplication.**

8 If we let 'a' and 'b' be any 2 whole numbers, then $a \times b$ is always a whole number.

9. $\{4, 8, 12, 16, 20, 24, 28, 32, 36 \ldots\}$
Is this set closed under the operation of multiplication?
a Is the product of 4 and 8 in the set?
b Is the product of 4 and 12 in the set? How do you know?
c Find the product of any two members of the set. Is this product in the set?

Now can you say whether this set is closed under the operation of multiplication?

10 $\{2, 4, 8\}$
Is this set closed under the operation of multiplication?
a Is the product of 2 and 4 in the set?
b Is the product of 2 and 8 in the set? This product is *not* in the set.

The set $\{2, 4, 8\}$ is not closed under the operation of multiplication.

11 From 16 subtract 8. We write $16-8=8$. When we subtract 8 from 16 the answer is a whole number. But let us have a look at $8-16$. Let the answer to this be x. Then $8-16=x$. We have seen that this means that $8=x+16$. What whole number can we add to 16 to make 8? We have learned of no whole number that we can add to 16 to get 8 as the answer. There is no whole number such that when it is added to 16 our answer is 8. So x cannot be a whole number and $8-16$ cannot be a whole number; so the set of whole numbers is not closed under the operation of subtraction.
Because we can find two numbers which have a difference that is *not* in the set of whole numbers, we can say that the set of whole numbers is not closed under the operation of subtraction. There are countless pairs of whole numbers which have a difference that is in the set. $12-6$, $3-1$, $9-5$, $117-98$, are four examples where the pairs of whole numbers have a difference that is a whole number. No matter how many pairs we find that have a difference that is in the set, we need to find only one pair of whole numbers that does not follow the rule for us to say that the set of whole numbers is not closed under the operation of subtraction. A point to notice is that although $12-6$ is a whole number, $6-12$ is *not* a whole number.

12 What is $24 \div 3$? We see that the answer is 8. 24, 3 and 8 are all whole numbers. What is $3 \div 24$? Let the answer be x. Then $3 \div 24 = x$. This means that $3 = x \times 24$. What whole number times 24 will give

3 as the product? There is no whole number such that when it is multiplied by 24 we get 3 as the answer. So we say that the set of whole numbers is not closed under the operation of division.

13. Which of the following sets of numbers are closed under the operation of (a) addition? (b) multiplication?

a $\{5, 10, 15, 20, 25, \ldots\}$ **b** $\{0, 1, 2, 3, 6\}$

c $\{0, 1\}$ **d** $\{10, 100, 1{,}000, 10{,}000, \ldots\}$

e $\{10, 20, 30, 40, 50, \ldots\}$ **f** $\{1, 3, 5, 7, 9, 11, \ldots\}$

g $\{2, 4, 6, 8, 10, 12, \ldots\}$ **h** $\{1\}$

i $\{0\}$ **j** $\{7, 9, 16\}$

CHAPTER TEST

1. Which of the following are numerals that represent the number 50?

a $275-225$ **b** 2×25 **c** $100\div 4$

d $14+75-49$ **e** $(73+2)\div 5$ **f** $2\times 20+10$

2. Represent each of the following numbers by five different numerals:

a ten **b** thirty-five **c** one thousand

d one hundred twenty-five **e** zero **f** one

3. Find the missing numbers.

a $698+x=732+698$ **b** $\square+115=115+27$

c $173+69=y+173$ **d** $201+1143=1143+\triangle$

4. What number is represented by n in the following?

a $n+(14+6)=(8+14)+6$ **b** $20+(110+n)=(20+110)+40$

c $5+(14+6)=20+n$ **d** $3+(15+5)=(n+15)+5$

e $18+(26+n)=44+10$ **f** $7+(19+9)=n+9$

5. Find x.

a $7\times 9=9\times x$

b $105\times x=14\times 105$

c $7\times(3\times 4)=(x\times 3)\times 4$

d $20\times(115\times 10)=(20\times x)\times 10$

e $x\times(9\times 17)=(216\times 9)\times 17$

f $327\times(x\times 68)=(327\times 205)\times 68$

6. Below are pairs of numerals. Which numeral, in each pair, names the larger number?

a $4+(6\times 5)$; $(4+6)\times 5$

b $12-(3\times 4)$; $(12-3)\times 4$

c $(32+4)\div 2$; $32+(4\div 2)$

d $168-(90-70)$; $(168-90)-70$

e $(46\times 2)+6$; $46\times(2+6)$

f $23-(10+4)+6$; $(23-10)+4+6$

g $(72\div 4)\div 2$; $72\div(4\div 2)$

h $114+(6\times 2)$; $(114+6)\times 2$

i $24\div(4\div 2)$; $(24\div 4)\div 2$

j $36+(18\times 2)+3$; $(36+18)\times(2+3)$

7. Write expanded numerals for the following:

a 736

b 1,000

c 86

d 9,010

e 17,065

f 1,000,000

g 4,004

h 2,222

i 1,401

j 1

k 46,808

l 5,500

8. Write the following as decimal numerals:

a $(4\times 10\times 10)+(0\times 10)+(0\times 1)$

b $(3\times 10\times 10\times 10)+(3\times 1)$

c $(8\times 10\times 10\times 10)+(4\times 10)$

d $(1\times 10\times 10\times 10\times 10)+(0\times 10\times 10\times 10)+(0\times 10\times 10)+(0\times 10)+(0\times 1)$

e $(7\times 10\times 10\times 10\times 10\times 10)+(0\times 10\times 10\times 10\times 10)+(3\times 10\times 10\times 10)+(2\times 10\times 10)+(0\times 10)+(1\times 1)$

9. Write the following as expanded numerals.

a 294

b 483

c 470

d 2,000

e 1,000,100

f 9,975

g 1,487

h 12,345

10. Write the following as expanded numerals. The first is done for you.

a $64=4\times 4\times 4$

b 81

c 3125

d 144

e 32

f 343

g 6561

h 10,000

i 169

11. Work the following in the way the first one is done:

a $9-5=4$ means $9=4+5$

b $18-7=11$

c $368-195=173$

d $1{,}000-500=500$

e $701-298=403$

f $16{,}895-183=16{,}712$

g $58{,}064-29{,}175=28{,}889$

12. Study the examples **a** and **b** below and then work the examples that follow:

a $n \div 4=6$ means $n=6\times 4$, so $n=24$

b $32 \div n=8$ means $32=8\times n$. $8\times 4=32$, so $n=4$

c $56 \div n=7$

d $108 \div n=12$

e $n \div 10=5$

f $144 \div n=12$

g $n \div 7=9$

h $n \div 84=4$

13. The set of even numbers is $\{2, 4, 6, 8, 10, \ldots\}$

a Is the set closed under addition?

b Is the product of any two even numbers always an even number?

c Is the set closed under multiplication?

14. Which of the following sets are closed under (a) addition? (b) multiplication?

a $\{1, 3, 5, 7, 9, 11, \ldots\}$

b $\{5, 10, 15, 20, 25, 30, \ldots\}$

c $\{3, 6, 9, 12, 15, 18, \ldots\}$

d $\{7, 14, 21, 28, 35, 42, \ldots\}$

e $\{1, 2, 3, 4, 5, 6, \ldots\}$

f $\{2, 4, 6, 2, 4, 6, 2, 4, 6, \ldots\}$

g $\{1, 1, 1, 1, 1, 1, \ldots\}$

15. Write the following sets:

a the whole numbers between 33 and 65 that are divisible by 7

b the whole numbers that are less than 15 and that leave a remainder when divided by 3

c the whole numbers which, when multiplied by 4, give a product which is less than 31

d the whole numbers less than 100 whose digits add up to 9

2 Open Sentences

NUMBER EXPRESSIONS

1 A grade 7 class was asked to solve the following problems:

a A boy had 50¢ and was given an additional 5¢. His friend had three times as many cents. What number of cents did his friend have?

b When a boy collected on his newspaper route, three persons paid him 50¢ each while one customer gave him a tip of 5¢. What number of cents did he have then?

For problem **a** the answer to the number of cents his friend had is stated as: $3\times50+5$.

For problem **b** the answer to the number of cents he then had is stated as: $3\times50+5$.

The numeral given to represent the number of cents is the same for each problem: $3\times50+5$.

In problem **a** we should think:

The boy had 50 cents; he was given 5¢. The number of cents he had now was $50+5$. The fifty and the five go together: $50+5$. His friend had three times this number of cents; the number of cents the friend had was $3\times50+5$.

We have circled the operation to be performed first.

$3\times50+5=3\times55=165$

In problem **b** we should think:

The boy collected 3×50 cents. The number of cents he collected was 3×50. In addition he received a tip of 5 cents. The number of cents he had then was $3\times50+5$. We have circled the operation to be performed first.

$3\times50+5=150+5=155$

2 We have seen that the numeral $3\times50+5$ could have two meanings, depending on what operation we wish to be performed first.

We circled the operation we wish to be performed first:

$3\times50+5$ or $3\times50+5$.

In mathematics we usually have brackets to show the operation we want to perform first. Brackets have many shapes:

( )	[ ]	{ }
round brackets or parentheses	square brackets	curly brackets or braces

We most often use parentheses to indicate the operation to be performed first.

For $3 \times 50+5$ put $3 \times (50+5)$

For $3 \times 50+5$ put $(3 \times 50)+5$

3 We should be careful in writing expressions with numbers to see that what we write is clear. Often we see number expressions that do not contain parentheses. In cases like this mathematicians have made certain agreements.

a **If addition and subtraction occur in the same expression, perform the operations in the order in which they occur.**

Example:
$$\begin{aligned} & \underbrace{6+4}-2+3 \\ =\ & \underbrace{10\ -2}+3 \\ =\ & \underbrace{8\ +3} \\ =\ & 11 \end{aligned}$$

b **If multiplication and addition or subtraction occur together, perform the operation of multiplication first.**

Example:
$$\begin{aligned} & 14+\underbrace{2 \times 3}-\underbrace{4 \times 5}+2 \\ =\ & \underbrace{14+\ 6}\ -\ 20\ +2 \\ =\ & \underbrace{20\ -\ 20}\ +2 \\ =\ & \underbrace{0\ +2} \\ =\ & 2 \end{aligned}$$

c **If division and addition or subtraction occur together, perform the operation of division first.**

Example:
$$\begin{aligned} & 6+\underbrace{8 \div 2}-3 \\ =\ & \underbrace{6+\ 4}\ -3 \\ =\ & \underbrace{10\ -3} \\ =\ & 7 \end{aligned}$$

d **If multiplication, division, addition or subtraction occur together, first perform the multiplication and division in the order in which they occur and then perform the addition and subtraction.**

Example:
$$\begin{aligned} & \underbrace{5 \times 4}\div 10+6 \\ =\ & \underbrace{20\ \div 10}+6 \\ =\ & \underbrace{2\ +6} \\ =\ & 8 \end{aligned}$$

e If parentheses occur in a number expression, perform whatever operation is enclosed by the parentheses first; then follow the rules above.

$$\begin{aligned} &6 \div \underbrace{(4-2)} \times 3 \\ =&\underbrace{6 \div 2} \times 3 \\ =&\underbrace{3 \times 3} \\ =&9 \end{aligned}$$

Note: Wherever there is likely to be doubt use parentheses.

f If a bar such as we use in fractions occurs in a number expression, evaluate the numerator and the denominator before dividing.

Example: $\dfrac{6\times4+2}{3+40\div4}$

$=\dfrac{24+2}{3+10}$

$=\dfrac{26}{13}$

$=2$

PRACTICE IN EVALUATING NUMBER EXPRESSIONS

Evaluate (find the value of) the following:

1. **a** $19+7-6+10$ **b** $25+37-8-9-6$ **c** $10+3\times14-6$
   d $15+21\div3-3$ **e** $5\times4\div2$ **f** $28\div4+3\times8-6$
   g $36\div(3\times3)+4$ **h** $7\times(9-5)+6$ **i** $64\div(16\div2)$
   j $19+36-15\times3$ **k** $48\div(6-2)-8$ **l** $7+96\div3$
   m $19\times(6-6)$ **n** $3+0\times5+14$ **o** $8\times9-15\div5$

2. **a** $23+15-9\times3$ **b** $36\div6\times2+4$
   c $100\div4-25$ **d** $(9\times7)-(7\times9)$
   e $46\times2-15\times5$ **f** $49+8\div8+49$
   g $144\div12\times3$ **h** $30\div15\times8+10$
   i $93-3\times30-3$ **j** $39\div(10+3)-3$
   k $6\times5\div2\div5$ **l** $7-2+18-6\times2\div4$
   m $(7+16-3)\div(39-9-10)$ **n** $(35+2+13)\times(4+6-8)$
   o $6\times(3+2)\div15$ **p** $(45+5-11)\times(9\times3-3)$

3. a $\frac{7+9+8}{5+2+5}$ b $\frac{20+4-8}{12-3}$ c $\frac{37-15+8}{16+9-15}$

d $\frac{5\times4+4}{3\times3-3}$ e $\frac{50\div2+15}{4+32\div2}$ f $\frac{(6+4)\times(3+2)}{(3+2)\times(4+1)}$

g $\frac{17-9+4}{(20+8)\div4}$ h $\frac{9+24}{17-2\times3}$ i $\frac{4\times(6-3)\div6}{(19-5)\div7}$

j $\frac{15-2\times3}{3\times3}$ k $\frac{58-18-40}{7\times6+4}$ l $\frac{48+5-19}{2\times8+1}$

m $\frac{3\times(15-3)}{6\times(11-8)}$ n $\frac{12\times(7+5)}{64\div(7-5)}$ o $\frac{43\times(3-1)+6}{6+43\times(3-1)}$

4. a $\frac{24-(6+3)}{3\times(10\div2)}$ b $\frac{59-(2+26)-10}{14+2\times7-21}$ c $\frac{5\times9\div3+17}{64-(18+14)}$

d $\frac{100-10-10-10}{40+10+10+10}$ e $\frac{(153-100)\times2}{7\times7+4}$ f $\frac{11+2\div(5-4)}{35\div7-4}$

g $\frac{18+9\times3+15}{27-28\div4}$ h $\frac{(21-9)\times(15-13)}{(8-5)\times(27-23)}$ i $\frac{14+6-9\times2}{(15-6)\div(14-11)}$

j $\frac{2\times9-5+8\div4}{6\times5\div10}$ k $\frac{63\div7\times7}{21\div3\times3}$ l $\frac{16\times(4\div4)}{16\div(4\div4)}$

m $\frac{16\times(4\div4)}{16\div(4\times4)}$ n $\frac{14\times(9\div3)+12}{18\div(26-24)}$ o $\frac{44\div(2\times5+1)}{38\div(16+3)}$

5. a $\frac{4+6}{2\times5}+\frac{3\times7+9}{3\times5}-\frac{6\times4}{4\times3}$ b $\frac{16\times(19-15)}{24\div(7-4)}+\frac{49\div(14\div2)}{15-(2+6)}$

c $\frac{3\times17}{21\div7}-\frac{6+9\div3\times2}{72\div(4+8)}$ d $\frac{129-3+14}{5\times7\times2}-\frac{108-7\times9}{60-3\times5}$

e $\frac{(14+9)-(6+7)}{105\div(15+6)}+\frac{37+(14-11)}{3\times9-7}$ f $\frac{2+6\div3+19+2}{17-(6+9)+3}-\frac{32+8}{5+3}$

g $\frac{2\times19}{2\times8+3}+\frac{14-5}{(3-2)\times3}+\frac{35+19}{3\times9}$ h $\frac{32+(2\times14)}{3\times2\times(24\div2)}+\frac{7\times9-4\times5}{2\times11+3\times7}$

i $\frac{6+14}{2+8}-\frac{7+15}{11+11}+\frac{6\times9}{108\div2}-\frac{7\times7}{98\div2}$

j $\frac{101}{(2\times50)+1}+\frac{(6-2)\times(5-3)}{(9-7)\times(21-19)}+\frac{6\times6+3}{2\times5+3}$

6. Below are pairs of number expressions. Evaluate each of the number expressions and then state whether or not the two expressions name the same number. The first two are done for you.

a $3\times(2+5)-3;\ 4\times6+3$

$$\begin{aligned} 3\times(2+5)-3 &= 3\times7-3 \\ &= 21-3 \\ &= 18 \\ 4\times6+3 &= 24+3 \\ &= 27 \end{aligned}$$

18 is not equal to 27. A symbol for *is not equal to* is $\neq$.

So $3\times(2+5)-3\neq4\times6+3$

b $5\times(16\div2)-5;\ (10-3)\times(8-3)$

$$\begin{aligned} 5\times(16\div2)-5 &= 5\times8-5 \\ &= 40-5 \\ &= 35 \\ (10-3)\times(8-3) &= 7\times5 \\ &= 35 \end{aligned}$$

35 is equal to 35.

So $5\times(16\div2)-5=(10-3)\times(8-3)$

c $4+7-(2\times3);\ (10-5)\times5\div5$

d $(6+8)\times(9\div3);\ (5+7)\times7\div2$

e $(14-6)\times(7-5);\ 12-(7-6)+8$

f $(13+7)\div(15-11);\ 7\times9-(90-32)$

g $5+63\div3;\ 2\times(26-13)$

h $3\times(15\div5)\times3;\ (60-6)\div2$

i $(15\times3)-(8\times4);\ (15\times4)-(8\times3)$

j $\frac{8+27}{2+3};\ \frac{8}{2}+\frac{27}{3}$

k $\frac{8\times5}{2\times2};\ \frac{8}{4}\times\frac{5}{1}$

l $\frac{32-(7-5)}{(8-5)\times(13-8)};\ \frac{15+19-17-(7+8)}{6\times3-12-5}$

m $5+3\times2-4;\ 49\div(11-4)$

n $16-4-4-4-4;\ 12-4-4-4$

OPEN NUMBER EXPRESSIONS

1 We shall see that we can use boxes to represent numbers in number sentences. We call such number sentences **open number sentences.** We shall learn more about open number sentences later.
Consider the expression $3\times\square+4$. This expression can have different values according to the replacement we use for the box. Such expressions are called **open number expressions.** Later you will call these expressions **algebraic expressions.**

2 Evaluate $3\times\square+4$. Why is this not possible? Suppose we are told that $\square$ represents 5. What is the value of the expression now?
We see that $3\times\square+4$ becomes $3\times5+4=15+4=19$.

3. Evaluate the expressions below by replacing the box with the number indicated.

Example: $4\div\square+15$ if $\square=2$

$$\begin{aligned}4\div\square+15&=4\div2+15\\&=2+15\\&=17\end{aligned}$$

a $\square+3+\square$ if $\square=2$

b $5\times\square-4$ if $\square=5$

c $8\times\square-9+3$ if $\square=3$

d $7-\square+15$ if $\square=4$

e $(\square\div3)\times(\square\div4)$ if $\square=12$

f $17+\square-2\times\square$ if $\square=10$

g $2\times\square+3\times\square+5\times\square$ if $\square=8$

h $3\times\square+\square\div4$ if $\square=16$

i $\square+9+\square-\square$ if $\square=9$

j $(3-\square)\times(\square+3)$ if $\square=0$

k $12+(\square-12)$ if $\square=25$

l $\dfrac{3\times\square}{\square-4}$ if $\square=7$

m $\dfrac{\square+9+2\times\square}{6\times\square}$ if $\square=3$

n $\dfrac{2\times\square-(\square+6)}{14+\square}$ if $\square=6$

o $4\times\square+7\times\square+5\times\square$ if $\square=11$

p $3\times\square-2\times\square+3$ if $\square=15$

q $\dfrac{4+2\times\square}{3\times\square+1}$ if $\square=3$

r $\dfrac{\square\times\square\times\square\times\square}{32\div\square}$ if $\square=2$

s $\dfrac{\square-\square+\square}{\square+9}$ if $\square=9$

t $3\times\square-4\times2+\square\times9$ if $\square=4$

u $6\times3\times\square\times\square$ if $\square=3$

v $\dfrac{6+4\times\square}{\square-2}$ if $\square=9$

w $\dfrac{\square\times\square\times\square}{\square-2}$ if $\square=6$

x $\dfrac{\square\times\square\times\square\times\square}{8\times\square}$ if $\square=2$

y $\dfrac{\square\times5+3\times\square}{\square}$ if $\square=4$

z $\square+\square\times7+\square+9$ if $\square=0$

THE REPLACEMENT SET

1 Read this sentence to yourself:

□ is a city in Canada.

We read this diagram as *box*.
We say, "*Box* is a city in Canada."

a Give three different words that you could use to replace the box to make the sentence true.

b Give three different words that you could use to replace the box to make the sentence false.

c How many words do you think there are that would make this sentence true?

d How many words do you think there are that would make this sentence false? Is there any limit?

2 "□ is a city in Canada" is called an **open sentence.** This sentence is neither true nor false. It becomes true or false when we put a name in place of the box.

3 In question **c** above, it does not matter *mathematically* what your answer was so long as you noticed that there must be some limit to how many different words would make the open sentence true. For example, there could not be as many as 30,000,000 such words. (Why?)

Question **d** above is more definite. You could run through the names of other cities in the world, like Tokyo, New York and London, but do these complete the list?

Suppose we wrote:

15 is a city in Canada.

Is this sentence true?

You may think that this is cheating, and in a way you are right. If there is no limit at all to the words we may use to replace the box, certainly there is no limit to how many we can find. We could even write complete nonsense like this:

Ed r is a city in Canada.

Usually we do not allow that sort of thing when we write an open sentence. We tell the reader what sort of word may replace the box.

In this open sentence,

▩ is a city in Canada,

we might say that what replaces the box must be the name of a city in North America, or the name of a city in the world, or the name of a city starting with T or the name of a city with fewer than 6 letters in the name. In each case, we would be stating **the replacement set** for the open sentence. When we talk about **the replacement set** for an open sentence, we mean the full list of things whose names are permitted to replace the box. The names make the sentence true or false.

4. Read the following open sentences. Note carefully the replacement set written beneath each open sentence. Then write as many names as you can which make each open sentence true.

a ▩ is a city larger than Vancouver.

Canadian cities.

b ▩ is a city smaller than Saskatoon.

Saskatchewan cities.

c ▩ is a city in Canada.

Cities whose names start with the letter W.

d ▩ is a man.

Football players.

e ▩ is a woman.

Football players.

f ▩ belongs to the United Nations.

Countries in North America.

g ▩ belongs to the United Nations.

Countries in South America.

h ▩ is a name starting with *a*.

Names of flowers.

i ▩ is a name starting with A.

Names of girls.

5. Read this open sentence.

▩ is larger than 6. Replacement set: *the set of whole numbers from* 0 *to* 10 *inclusive.*

a List the elements in the replacement set that make this open sentence false.

b List the elements in the replacement set that make this open sentence true.

c Change the replacement set for the open sentence to be the numbers from 0 to 50. Now write the numbers that make the open sentence true and write the numbers that make the open sentence false.

d Change the replacement set for the open sentence to be the set of whole numbers less than 7. Now write the numbers that make the open sentence true and write the numbers that make the open sentence false.

6. List the numbers that make each open sentence true. *Replacement set: the set of whole numbers from* 0 *to* 30 *inclusive.*

a □ is less than 6. **b** □ is larger than 25.

c □ is between 9 and 12. **d** □ is greater than 16.

e □ is less than 13. **f** 25 is less than □.

g □ is less than 14 and larger than 9.

h $7+\square=30$. **i** □ is divisible by 9.

j $3\times\square$ is less than 30.

k $4\times\square$ is greater than 20 but less than 25.

l $3\times\square=13$. **m** $12+14=\square$.

n $\square+13=29$. **o** $2+\square+7=19$.

p $11+13+\square=27$. **q** □ is divisible by 5.

r □ is divisible by 3.

s □ is divisible by both 5 and 3.

t □ is a 2-figure numeral.

SEQUENCES

1 Sometimes we like to discuss a list of numbers too long to write out. Even a list like 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30 takes more writing than we care to do. We have agreed to use three dots '. . .' to represent a part of a **sequence** which we have not written down. For example, we could write the above list as 0, 1, 2, 3, . . . , 29, 30. We give enough numerals at the beginning and at the end so that the reader is quite clear as to what we mean. If we wrote 0, . . . , 30, this sequence could mean 0, 2, 4, . . . , 28, 30; or it could mean some other list. We usually read '. . .' as 'and so on'.

We could read

2, 4, 6, . . . , 18

as two, four, six, and so on up to 18.

2. Write down the numerals represented by '. . .' in these sequences:

a 1, 2, 3, . . . , 13
b 4, 6, 8, . . . , 22
c 2, 4, 8, 16, 32, . . . , 256
d 9, 8, 7, . . . , 1
e 2, 3, 5, 8, 12, 17, . . . , 57
f 4, 4, 5, 5, 6, . . . , 9, 9
g 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, . . . , 7
h 1, 2, 1, 3, 1, 4, . . . , 15
i 7, 10, 13, 16, . . . , 43
j 4, 1, 4, 2, 4, 4, 4, 8, 4, 16, 4, . . . , 512
k 1, 1, 2, 3, 5, 8, 13, 21, . . . , 144
l 1, 3, 2, 6, 3, 9, 4, 12, 5, 15, . . . , 45
m 11, 10, 9, . . . , 0
n 13, 26, 39, . . . , 117

3 Sometimes we want to talk about a sequence that has no end. We still use '. . .'

When we write

1, 2, 3, 4, . . .

we mean, one, two, three, four and so on, without end. When we use '. . .' in this way, we must always be very careful to make our meaning clear.

If we write 2, 4, . . . , this can mean 2, 4, 6, 8, 10, 12, . . . or 2, 4, 8, 16, 32 . . . or 2, 4, 2, 6, 2, 8, 2, 10, . . . or almost anything else.

Even if we write 1, 2, 3, 4, 5, . . . , there is *still* the *possibility* that we might mean 1, 2, 3, 4, 5, 1, 2, 3, 4, 5, 1, 2, 3, 4, 5, and so on, or some such sequence. For this reason, it is never *certain* what comes next when we write '. . .', but if we give *enough* numerals in the sequence, we can make our meaning quite clear.

4. Name the three numbers which probably come next in these sequences:

a 7, 8, 9, 10, ...
b 12, 10, 8, . . .
c 14, 17, 20, 23, . . .
d 5, 10, 20, 40, 80, . . .
e 3, 9, 27, 81, . . .
f 1, 2, 3, 1, 2, 3, 1, . . .
g 4, 5, 6, 5, 6, 7, 6, 7, 8, 7, . . .
h 7, 11, 7, 22, 7, 44, 7, 88, . . .
i 3, 3, 6, 9, 15, 24, . . .
j 1, 5, 2, 10, 3, 15, 4, 20, 5, . . .
k 7, 8, 7, 10, 7, 12, 7, 14, . . .
l 10, 1, 9, 2, 8, 3, . . .
m 8, 3, 8, 6, 8, 9, 8, 12, . . .
n 25, 26, 24, 27, 23, 28, 22, 29, . . .

NATURAL NUMBERS AND WHOLE NUMBERS IN OPEN SENTENCES

1 From now on we will use '. . .' to help us talk about lists of numerals. For example, we often want to talk about the **natural numbers.** The natural numbers are $\{1, 2, 3, 4, 5, \ldots\}$. If we wish to include 0, we then say we are dealing with the **whole numbers.**

The whole numbers are $\{0, 1, 2, 3, 4, \ldots\}$.

2 We will now return to open sentences. Study these examples.

a List the numerals that make this open sentence true:

$\square$ is divisible by 3. *Replacement set: the set of natural numbers*

Answer: $\{3, 6, 9, 12, \ldots\}$

b List the numerals that make this open sentence true:

$\square$ is larger than 30. *Replacement set: the set of natural numbers*

Answer: $\{31, 32, 33, 34, \ldots\}$

c List the numerals that make this open sentence true:

$6 \times \square = 20 + 4$. *Replacement set: the set of whole numbers*

The whole numbers are $\{0, 1, 2, 3, \ldots\}$.

Let us replace the box with each whole number in turn.

Try 1: $6 \times \square = 6 \times 1 = 6$
$20 + 4 = 24$
$6 \neq 24$

Try 2: $6 \times \square = 6 \times 2 = 12$ $\quad 12 \neq 24$

Try 3: $6 \times \square = 6 \times 3 = 18$ $\quad 18 \neq 24$

Try 4: $6 \times \square = 6 \times 4 = 24$ $\quad 24 = 24$

Try 5: $6 \times \square = 6 \times 5 = 30$ $\quad 30 \neq 24$

Obviously the only replacement is 4. We might have seen this solution immediately by thinking:

$20 + 4 = 24$ $\quad 6 \times$ what number? $= 24$

$6 \times 4 = 24$ $\quad$ The answer is 4.

3. List the numerals that make each open sentence true.
Replacement set: the set of whole numbers

a $\square$ is divisible by 6

b $\square$ is greater than 9

c $2 \times \square$ is greater than 9

d $9 \times \square$ is greater than 9

e $(4 \times \square)$ is less than 5

f $(2 \times \square) + 7$ is greater than 65

g $5 \times \square$ is greater than 35

h $(4 \times \square) + 3$ is less than 100

i $(6 \times \square) + 4$ is larger than 1,000

j $3 \times \square$ is between 80 and 100

k $\square = 6$ | **l** $7 = \square + 1$
m $3 \times \square = 75$ | **n** $7 \times \square = 18 + 3$
o $11 \times \square = 11$ | **p** $165 = \square + 30$

4 Sometimes we want to write open sentences in which we use the box more than once. When we use the same shape of box more than once in a sentence, the same member of the replacement set has to replace each box.

Example: $\square + \square = 16$. *Replacement set: the set of whole numbers*

Only 8 makes this open sentence true.

$5 + 11 = 16$ is not the correct solution because, even though $5 + 11 = 16$, the same number has to replace *both boxes*.

5. For each question below, find the numeral or numerals making the sentence true. *Replacement set: the set of whole numbers*

a $\square + \square = 36$
b $\square + \square + \square = 51$
c $\square \times \square = 121$
d $(\square \times \square) + 2 = 27$
e $(\square \times \square) - 4 = 12$
f $\square + \square + \square = 261$
g $7 + \square + \square + 9 = 28$
h $9 + \square = 4 + \square + 5$
i $(\square \times \square) \times \square = 8$
j $(\square \times \square) + 2 = 38$
k $(\square \times 3) = \square + 14$
l $\square + \square = 40$
m $\square - \square = 0$
n $\square + \square + \square = (3 \times \square)$
o $(4 \times \square) = (2 \times \square) + (2 \times \square)$
p $\square + \square = 20$
q $2 \times \square \times \square + 5 = 23$
r $\square + 10 = 2 \times \square$
s $2 \times \square + 5 = 20$
t $2 \times \square + 5 = 3 \times \square - 5$

6. Below is a harder exercise. Solve each of the problems.
Replacement set: the set of whole numbers

a $\square + 6 + \square = (3 \times \square) + 3$
b $\square + \square + \square = 48$
c $7 + \square = \square + 6$
d $(2 \times \square) - \square = 0$
e $(2 \times \square) - \square = 10$
f $\square + (3 \times \square) = 72$
g $16 + 4 + \square = 80$
h $(\square \times \square) - \square = 42$
i $(\square \times \square) + 6 = 735$
j $(\square + \square) + (\square \times \square) = 224$
k $(3 \times \square) + (7 \times \square) = 230$
l $(\square \times 6) + (9 \times \square) = 300$
m $(\square \times \square) - (3 \times \square) = 88$
n $(\square \times \square) - (5 \times \square) + 6 = 0$
o $(\square \times \square) + 15 = 8 \times \square$
p $\square + (\square - \square) + \square = 4$
q $12 + \square + \square = 28$
r $\square + \square + 2 \times \square = 4 \times \square$
s $5 \times \square - 10 = 20$
t $(\square \times \square) + 57 = 82$

CUMULATIVE EXERCISE

List the numerals that make each open sentence true.
Replacement set: the set of whole numbers

- **a** $\square$ is divisible by 6
- **b** $\square$ is larger than 9
- **c** $(2\times\square)$ is larger than 36
- **d** $(3\times\square)$ is larger than 14
- **e** $(\square\times\square)$ is larger than 80
- **f** $\square+\square=44$
- **g** $(5\times\square)=\square+\square+\square+\square+\square$
- **h** $\square=\square\div 1$
- **i** $\square=\square\times 1$
- **j** $(\square\times\square)=\square$
- **k** $(\square\times 6)=(15+\square)$
- **l** $(\square\times\square)+(3\times\square)=40$
- **m** $\square+8=(2\times\square)+1$
- **n** $(\square+\square+\square)$ is greater than 40
- **o** $\square$ is divisible by 11
- **p** $\square$ is divisible by both 3 and 4
- **q** $\square\times 3$ is greater than $\square+11$
- **r** $\square\times 5$ is greater than $\square\times 27$
- **s** $\square\times 5$ is less than $\square+27$
- **t** $\square+7=7+\square$
- **u** $(3+\square)+6=3+(6+\square)$
- **v** $(3\times\square)+(12\times\square)=(5\times\square)$
- **w** $(3\times\square)-6=21$
- **x** $\square+\square+19=3+\square+\square+16+\square$
- **y** $(4\times\square)-11$ is greater than 35
- **z** $17+(3\times\square)$ is greater than 15

USING OPEN SENTENCES TO SOLVE PROBLEMS

1. We use open sentences to help us solve problems. Often we can write an open sentence as we read a problem.
Study these examples:

   a If we multiply a number by 3 and then add 6, we get 27. What is the number?
   Solution: $(3\times\square)+6=27$
   The number is 7.

   b If we multiply a number by itself and add 7, we get 128. What is the number?
   Solution: $(\square\times\square)+7=128$
   The number is 11.

2. Write open sentences for these problems, and find the numerals that make each open sentence true. *Replacement set: the set of whole numbers*

   a If we add 7 to a number, the sum is 22. What is the number?

   b If we multiply a number by itself and add 3, we get 28. What may the number be?

c If we multiply a number by 4 and add 6, the sum is greater than 30. What may the number be?

d If we multiply a number by 7 and subtract 10, the difference is less than 25. What may the number be?

e If we take 5 times a number and subtract 6, we get 19. What may the number be?

f If we write down a number 5 times and add, we get 65. What may the number be?

g If a number multiplied by itself is less than 99, what may the number be?

h If a number multiplied by itself is greater than 99, what may the number be?

i If a number multiplied by itself is between 200 and 300, what may the number be?

j If we multiply a number by itself and add 17, we get 21. What may the number be?

k If we subtract a number from 3 times itself, we get double the number. What may the number be?

l If we multiply a number by itself and add 6, we get 5 times 3 What may the number be?

EVALUATING NUMBER EXPRESSIONS

1 Sometimes we need to evaluate number expressions in which two different symbols are used to represent two numbers.

We can use any shape of symbol to represent a number.

Consider $\square + \triangle$

Here we have two symbols. What is the value of this expression?

$\square + \triangle$ has no value until we assign a value to $\square$ and a value to $\triangle$.

Suppose $\square$ represents 2 and $\triangle$ represents 3, we then have:

$\square + \triangle = 2 + 3 = 5$

2 Evaluate: $\triangle + 2 \times \square + 3 \times \triangle + \square$ if $\square = 4$ and $\triangle = 6$.

We must replace each $\square$ with 4 and each $\triangle$ with 6.

We have:

$$\begin{aligned} & \triangle + 2 \times \square + 3 \times \triangle + \square \\ &= 6 + 2 \times 4 + 3 \times 6 + 4 \\ &= 6 + 8 + 18 + 4 \\ &= 36 \end{aligned}$$

3. Evaluate the following expressions by replacing the symbols with the indicated numbers:

a $\square - \triangle$ if $\square = 5$ and $\triangle = 2$

b $\square \times \triangle$ if $\square = 7$ and $\triangle = 12$

c $\triangle + \square - 6$ if $\square = 8$ and $\triangle = 9$

d $(2 \times \triangle) - (3 \times \square)$ if $\square = 4$ and $\triangle = 11$

e $(\square \div 4) \times (9 - \triangle)$ if $\square = 16$ and $\triangle = 5$

f $\square + \triangle + \triangle + \square$ if $\square = 10$ and $\triangle = 3$

g $\square \times \triangle - 16$ if $\square = 4$ and $\triangle = 9$

h $\square \times (\triangle - 2)$ if $\square = 7$ and $\triangle = 6$

i $2 \times \square \times (\triangle - 5)$ if $\square = 4$ and $\triangle = 11$

j $(5 + \triangle) \times \square$ if $\square = 9$ and $\triangle = 5$

k $48 \div (2 \times \triangle) + \square$ if $\square = 6$ and $\triangle = 3$

l $2 \times (\square - \triangle) + \square$ if $\square = 8$ and $\triangle = 2$

m $\dfrac{\square + \triangle}{3 + \triangle}$ if $\square = 9$ and $\triangle = 3$

n $\dfrac{2 \times \square - 3 \times \triangle}{\square - (\triangle + 2)}$ if $\square = 8$ and $\triangle = 5$

o $\dfrac{\square + \triangle}{\square - \triangle}$ if $\square = 12$ and $\triangle = 6$

p $\dfrac{12 \times (\square - \triangle)}{\triangle + 9}$ if $\square = 11$ and $\triangle = 7$

q $\square + 3 \times \triangle - \square$ if $\square = 4$ and $\triangle = 4$

r $\dfrac{5 \times \square - 3 \times \triangle}{4 \times (\square - 1)}$ if $\square = 5$ and $\triangle = 3$

s $\square + \triangle + 2 \div \triangle$ if $\square = 4$ and $\triangle = 2$

t $\dfrac{\square \times \triangle}{\triangle \times \square}$ if $\square = 5$ and $\triangle = 35$

4. a $2 + \square + 9 - \triangle$ if $\square = 4$ and $\triangle = 5$

b $\dfrac{3 \times \square + 2 \times \triangle}{\triangle}$ if $\square = 7$ and $\triangle = 3$

c $\dfrac{\triangle + \triangle + \triangle - \square}{\square}$ if $\triangle = 6$ and $\square = 3$

d $3 \times \square + \triangle + \triangle \times 5 - 16 + 6 \div \square$ if $\square = 2$ and $\triangle = 3$

PAIRS OF NUMBERS IN OPEN SENTENCES

1 Sometimes we want to write open sentences which have more than one box, but we want to allow different numerals in different boxes. Then we write open sentences like this:

▩ + △ = 6 *Replacement set: the set of whole numbers for each place holder*

In this case, any pair of numbers whose sum is 6 will make the sentence true.

$$0+6=6$$

$$1+5=6$$

$$2+4=6$$

$$3+3=6$$

$$4+2=6$$

$$5+1=6$$

We write out the pairs this way:

{(0,6), (1,5), (2,4), (3,3), (4,2), (5,1), (6,0).}

Notice that (3,3) is a correct pair. The numbers *may* be different, but do not *have* to be different.

When we write (2,4), we think of this *pair* of numbers as making the open sentence true. For this reason, from now on we will write

▩ + △ = 6 *Replacement set: the set of whole numbers for each place holder* when a pair of numerals is needed to make the open sentence true.

2 There is one possibility of confusion. Consider this open sentence:

▩ + (2 × △) = 5

Suppose we were told that (1, 3) makes the open sentence true. We would find that if we put '1' in place of the triangle and '3' in the place of the square, the sentence *does* become true, but if we put '1' in place of the square and '3' in place of the triangle, the sentence becomes false. We shall need agreement as to which numeral replaces which box.

For now, we will agree that if an open sentence contains a square (▩) and a triangle (△), the first number of the pair replaces the square and the second number of the pair replaces the triangle.

3. For each open sentence below you are given four number pairs. Copy each open sentence and, beside it, write the four number pairs. Beside each pair, write *true* if the pair makes the open sentence true and *false* if the pair makes the open sentence false. *Replacement set: the set of whole numbers for each place holder*

a $\square + \triangle = 13$ (2,10)
(6, 7)
(7, 6)
(10,2)

b $(2 \times \square) + \triangle = 5$ (1, 3)
(3, 1)
(5, 0)
(0, 5)

c $\square - \triangle = 6$ (7, 1)
(1, 7)
(10,4)
(4,10)

d $(3 \times \square) - (2 \times \triangle) = 5$ (2, 0)
(3, 2)
(2, 3)
(7, 8)

e $\triangle - \square = 6$ (Caution! Read carefully.)
(8, 2)
(2, 8)
(15,9)
(9,15)

f $(2 \times \triangle) + \square = 13$ (7, 3)
(3, 7)
(0, 6)
(1, 6)

g $(5 \times \square) - \triangle = 3$ (0, 2)
(1, 2)
(2, 1)
(3,12)

4 When we are expected to find all of the pairs that make an open sentence true, we try every number that is possible to replace one box and see whether or not some number can go with it to replace the other box.

Study this example: $(2\times\square)+\triangle=9$

Replacement set: the set of whole numbers for each place holder

We try 0 for the box: $(2\times 0)+\triangle=9$

Then 9 must replace the triangle.

(0,9) makes the sentence true.

We try 1 for the box: $(2\times 1)+\triangle=9$

Then 7 must replace the triangle.

(1,7) makes the sentence true.

We try 2 for the box: $(2\times 2)+\triangle=9$

Then 5 must replace the triangle.

(2,5) makes the sentence true.

We try 3 for the box: $(2\times 3)+\triangle=9$

Then 3 must replace the triangle.

(3,3) makes the sentence true.

We try 4 for the box: $(2\times 4)+\triangle=9$

Then 1 must replace the triangle.

(4,1) makes the sentence true.

We try 5 for the box: $(2\times 5)+\triangle=9$

There is no whole number that can replace the triangle to make this open sentence true. And if we place a number larger than 5 in place of the box, we shall have the same trouble. (Why?) Given the open sentence:

$(2\times\square)+\triangle=9$

Replacement set: the set of whole numbers for each place holder

the number pairs (0,9), (1,7), (2,5), (3,3), and (4,1) will make the sentence true.

5. Write the number pairs that make the following open sentences true:

Replacement set: the set of whole numbers for each place holder

a $\square+\triangle=7$

b $(5\times\square)+\triangle=40$

c $(\square+5)+\triangle=13$

d $(2\times\triangle)+\square=10$

e $\square+\triangle=12$

f $16+\square=23-\triangle$

g $(3\times\square)+\triangle=9$

h $17-\triangle=11+\square$

i $\square\times\triangle=60$

j $(4\times\square)+(4\times\triangle)=20$

k $(\square+\triangle)\times 3=12$

l $(3\times\square)+(3\times\triangle)=15$

m $\square+(4\times\triangle)=13$

n $(2\times\square)+(2\times\triangle)=10$

o $(2\times\square)+(3\times\triangle)=18$

p $(3\times\square)+(2\times\triangle)=12$

6 Study this example:

$$\square - \triangle = 7$$

Which of these pairs make the open sentence true?

(7,0), (8,1), (9,2), (10,3), (11,4), (12,5)

Is there any limit to the number pairs making this sentence true? There can be no limit. If we want to describe *all* number pairs making this open sentence true, we must use '. . .' again. As before, we must make it very clear what we are thinking of when we write '. . .'

For this open sentence

$$\square - \triangle = 7$$

we would write

(7,0), (8,1), (9,2), (10,3), . . .

If we were asked what we meant by '. . .' we would say something like, "with each number of the pair increasing by 1 each time", or "so that the numbers of the pair differ by 7". We might even explain it by saying that the three dots mean this: "Continue the series by increasing the number of each pair by 1 and maintaining a difference of 7 between the two members of each pair."

7. Write number pairs to make each of these open sentences true. Use '. . .' shorthand where necessary. *Replacement set: the set of whole numbers for each place holder*

a $\square - \triangle = 3$

b $\square + \triangle = 16$

c $\square + \triangle = 10$

d $\triangle - (2 \times \square) = 6$

e $\square = \triangle$

f $(\triangle \times \square) + 2 = 34$

g $\square = (2 \times \triangle)$

h $\square + 2 = 4 - \triangle$

i $\square + 3 = \triangle$

j $\square + 2 - \triangle = 4$

k $9 - \triangle = \square$

l $\square - 3 = \triangle$

m $\square - 2 = \triangle$

n $(5 \times \square) + (5 \times \triangle) = 50$

o $\square - \triangle = 2$

p $(5 \times \square) - (5 \times \triangle) = 0$

q $\square \times \triangle = 48$

r $\square - \triangle = 0$

s $(2 \times \square) - \triangle = 1$

t $\square + \triangle$ is smaller than 5

u $\square - \triangle = 6$

v $\square + \triangle = 0$

w $\triangle - \square = 7$

x $\triangle - \square = 0$

y $\triangle - \square = 5$

z $\square + \triangle$ is larger than 0

8 We can often write the following kind of open sentence to show what is happening in a problem.

Study this example:

Joe is much older than his sister Mary. Their father agrees that

Joe's allowance should always be twice Mary's. What open sentence describes this situation?

Solution: Joe's allowance is represented by $\square$;
Mary's allowance is represented by $\triangle$;
then $\square = 2 \times \triangle$.

9. Write open sentences to show what is happening in each of these situations:

a Joe and Mary have a total allowance of 75¢.

b We do not know the length or the width of a room, but its area is 300 square feet.

c Mary is 6 inches taller than Susan.

d We do not know the length or width of a field, but the distance around it is 300 yards.

e Both Jack and Bob collect stamps. For every stamp Bob has, Jack has 3 stamps.

f The population of the U.S.A. is nine times that of Canada.

g In our class, there are five boys for every four girls.

h In a certain city, new churches must have 1 parking space for each 10 seats in the church.

i Bill is five years younger than his brother Sam.

j At a school dance, some people came alone and some came in couples. Let the $\square$ represent the number of people who came alone, and let the $\triangle$ represent the number of couples. How many people were at the dance?

THE SOLUTION SET

1 From now on, we shall use a convenient shorthand. If the set of numerals that make an open sentence true is 1, 2, 3, 4, and 5, we will write:

The solution set = {1, 2, 3, 4, 5}

Remember that we read curly brackets as *the set.*

Study these examples:

a $\square + (2 \times \triangle) = 6$ *Replacement set: the set of whole numbers for each place holder*

The solution set is {(6,0), (4,1), (2,2), (0,3)}.

We would read this, "The solution set is the set of pairs: (6,0), (4,1), (2,2), and (0,3)."

b $\square - \triangle = 3$

The solution set is $\{(3,0), (4,1), (5,2), (6,3), \ldots\}$

We would read this, "The solution set is the set of pairs: (3,0), (4,1), (5,2), (6,3), and so on."

c $\square + 9 = \square + 7$

The solution set is $\{\quad\}$.

We would read this "The solution set is empty."

2. Write down the solution sets for these open sentences. Each place holder represents a whole number.

a $\square + 6 = 10$

b $\square - \triangle = 3$

c $\square \times \triangle = 6$

d $\square - 3 = \square$

e $\square + \triangle = 11$

f $(2 \times \square) - \triangle = 2$

g $\square$ is less than 7.

h $\triangle$ is greater than 9.

i $\square + 2$ is less than 6.

j $(2 \times \square) + 1$ is greater than 15.

k $\triangle = (2 \times \square)$

l $(4 \times \square) - (4 \times \triangle) = 0$

m $\square + 4$ is less than 5.

n $\square + \triangle = 12$

o $\square - \triangle = \triangle$

p $(2 \times \square)$ is less than $\triangle$.

q $2 \times \square = 3 \times \triangle$

r $\square + \triangle$ is greater than 6.

s $\square + \square = \square$

t $\square \times \square = \square$

u $\square + 5$ is greater than $2 \times \square$.

v $\square \times \square = 36$

w $2 \times \square = 3 \times \square$

x $\square + \triangle = 6 + \triangle$

3. We will often use open sentences to tell what is going on in a problem. Study these examples:

a Teddy has a stamp collection. If we multiply the number of stamps in his collection by 4 and subtract 31, we get 125. How many stamps has he?

Solution: $(4 \times \square) - 31 = 125$.

We find 39 must replace the box.

Teddy has 39 stamps.

b Both Frank and Ernest have bags of marbles. If we add four times the number of Frank's marbles to the number of Ernest's marbles, we get 26. How many marbles could each have?

Solution: Let $\square$ represent the number of Frank's marbles and $\triangle$ represent the number of Ernest's marbles

Then $(4 \times \square) + \triangle = 26$

The solution set is $\{(0,26), (1,22), (2,18), (3,14), (4,10), (5,6), (6,2)\}$. If we were asked to explain in detail what this means, we would say:

Frank could have no marbles and Ernest 26 marbles.

or

Frank could have 1 marble and Ernest 22 marbles.

or

Frank could have 2 marbles and Ernest 18 marbles.

or

Frank could have 3 marbles and Ernest 14 marbles.

or

Frank could have 4 marbles and Ernest 10 marbles.

or

Frank could have 5 marbles and Ernest 6 marbles.

or

Frank could have 6 marbles and Ernest 2 marbles.

In most cases, we assume that the reader understands what we mean when we write:

Solution set is $\{(0,26), (1,22), (2,18), (3,14), (4,10), (5,6), (6,2)\}$

4. Write an open sentence for each of these problems and find answers:

a Mr. Rousseau has several children. If you multiplied the number of children by 3 and added 5, you would get 29. How many children has he?

b Mr. Harley watched a kangaroo jump 50 feet. He said, "You know, even if I jumped 4 times as far as I can, I would still have to jump 6 more feet to jump that far." How far can Mr. Harley jump?

c Neil said, "Guess how much money I have in my pocket. If you gave me 15¢ and doubled the amount I would then have, I would have \$3.60." How much money did he have?

d John and Les both ran for class president. Four times John's votes plus three times Les's votes add up to 133.

Let ▩ represent the number of John's votes

Let ▲ represent the number of Les's votes

(i) How many votes could each have got?

(ii) If there are 38 students in the class, can you tell how many votes each boy got?

USING NUMBER SENTENCES TO SOLVE PROBLEMS

1 Jim had 24 hockey cards. He bought some more and then found that he had three times as many cards as a friend who had 15 hockey cards. How many hockey cards did Jim buy?

We can use number sentences to help us solve problems such as this. We do not know how many cards Jim bought so we can let n represent the number. Now, Jim had $24+n$ hockey cards. We know that this number of cards is three times the number of cards that his friend had. His friend had 15 cards. Three times this number of cards is 3×15. We know that $24+n$ names the same number as 3×15, so we can write a number sentence to show this:

$$24 + n = 3 \times 15$$

The number of cards Jim had	The number of cards Jim bought	Three times the number of cards his friend had

We now have to do some computation:

We can compute 3×15. $3\times 15=45$

We can now write:

$$24+n=45.$$

How can we compute n? To compute n we subtract 24 from 45. $45-24=21$. We now have $24+21=45$.

The number we use to replace n is 21. Jim must have bought 21 cards.

Here is how we could write our solution to the problem:

Let n represent the number of cards that Jim bought.

$$\begin{aligned} 24+n &= 3\times 15 \\ 24+n &= 45 \\ 24+21 &= 45 \\ n &= 21 \end{aligned}$$

Calculation

$$\begin{array}{r} 15 \\ \times 3 \\ \hline 45 \end{array} \qquad \begin{array}{r} 45 \\ -24 \\ \hline 21 \end{array}$$

Jim bought 21 hockey cards.

2 A boy bought 18 feet of lumber to make a bookcase. He used three pieces, each 4 feet long, to make two shelves and the top. The remainder he used to make the two sides of the bookcase. What length of lumber did he use for each of the sides? If we let n represent the number of feet of lumber for each of the sides, we can write a number sentence to show how the lumber was used.

$$\underset{\substack{\text{number of feet}\\ \text{used in the two}\\ \text{shelves and the top}}}{(3\times4)} \quad + \quad \underset{\substack{\text{number of feet}\\ \text{used in the}\\ \text{two sides}}}{(2\times n)} \quad = \underset{\substack{\text{number of feet}\\ \text{of lumber bought}}}{18}$$

For 3×4 we can write 12. We now have:

$$12+(2\times n)=18$$

Since 12 plus $2\times n$ equals 18, we can find $2\times n$ by subtracting 12 from 18. $18-12=6$. $12+6=18$, so $2\times n$ must represent the same number as 6. $2\times n=6$. What number is represented by n? Here is one way of showing our working:

Let n represent the number of feet of lumber used for each side.

$$\begin{aligned}(3\times4)+(2\times n)&=18\\ 12+(2\times n)&=18\\ 12+\quad 6\quad&=18\\ 12+(2\times3)&=18\\ n&=3\end{aligned}$$

Calculation

$$3\times4=12 \qquad \begin{array}{r}18\\ -12\\ \hline 6\end{array}$$

$$2\times3=6$$

He used 3 feet of lumber for each of the sides.

3. Use number sentences to help you solve the following problems:

a A car, on the average, uses 1 gallon of gasoline to travel 17 miles. At the beginning of a trip the reading on the odometer was 14,298.6. At the end of the trip the reading on the odometer was 14,740.6. How many gallons of gasoline did the car use?

b The population of a city was 67,209. The population was made up 22,914 men, 9,016 boys, 23,785 women and a number of girls. How many girls were there in the city?

c Tom and his sister each drink 1 pint of milk a day. Mother uses 3 pints of milk a day. How many pints less than 20 gallons does the family use in 4 weeks?

d A farmer has 285 acres of land. He grows wheat on 87 acres, oats on 67 acres, and corn on 58 acres. The rest is used for grazing. What number of acres of land is used for grazing?

e The school aquarium contains 68 tropical fish. A girl has 6 more than a half of this number of fish. How many fish does the girl have?

f Pat is in training as a runner, and runs 2 miles every day. His sister runs 200 yards more than one quarter of the distance Pat runs weekly. How far does she run in a week?

g A hotel uses 100 dozen eggs a day. If Mrs. Jones uses 8 eggs a day, on the average, how many days would it take her to use the same number of eggs as the hotel uses in one day?

h Dave Keon scored 69 goals in three years in the N.H.L. How many goals less than Keon would a player score if he averaged 14 goals a year for three years?

i Butter costs 57¢ a pound this week. Last week a housewife bought three pounds which cost her $1.80. How much would she have saved if she had bought the butter this week?

j Joanne had 25¢ when she went to the candy store. She bought 6 licorice sticks which cost 2¢ a piece. How many marshmallow men could she buy if 3 of them cost a penny?

k A recipe for chocolate cake needs 2 eggs, and one for yellow angel-food cake requires 6 eggs. How many eggs would a baker use in making 6 chocolate cakes and 4 yellow angel-food cakes?

l A school wants to buy some baseball bats. At one store they can buy 3 bats for $8.85 and in another store they could buy 5 bats for $15.75. If they want to buy 12 bats, how much would they save by buying them at the first store?

m The distance once around a track is 440 yds. and this distance is called a *lap*. How far would a runner travel in a relay race which was 2 laps long if each member of the relay ran one quarter of the distance?

n Roast beef costs 79¢ a pound, minced steak costs 69¢ a pound, and turkey costs 48¢ a pound. How much would you spend if you bought a 6-lb. roast of beef, 3 lb. of minced steak, and a 12-lb. turkey?

o In an orchard, an average apple tree yields 3 bushels of apples a year. How many bushels of apples would a farmer get if he planted 5 rows of trees to an acre, with 50 trees to a row, and he had 10 acres on his farm?

USING LETTERS IN OPEN SENTENCES

1 If we needed to write an open sentence with more than 3 or 4 different kinds of boxes, we would soon have trouble finding new shapes. For this reason we often use letters of the alphabet. Instead of $6+\boxed{}=15$ we can write

$$6+c=15$$
$$\text{or } 6+d=15$$
$$\text{or } 6+x=15$$

or, in fact, any letter we choose. Exactly the same rule holds as before. If we write $x+x=12$, it is agreed that the same number must replace each box, or as we would now say, the same number must replace each x.

2. Consider the open number expression:

$$x+2\times x+3$$

What is its value if $x=3$?

We have:

$$\begin{aligned} &x+2\times x+3 \\ =&3+2\times 3+3 \\ =&3+6+3 \\ =&12 \end{aligned}$$

Now evaluate the following expressions:

a $x+5+x$ if $x=4$

b $(2\times x)+(5-x)$ if $x=3$

c $(a\times a)-(2\times a)+1$ if $a=5$

d $\dfrac{b+5}{b-5}$ if $b=6$

e $\dfrac{(2\times d)+9-d}{d+1}$ if $d=3$

f $(x\times x)+(x\div 4)$ if $x=8$

g $\dfrac{y-15}{y\div 4}$ if $y=20$

h $2\times(z-9)+7$ if $z=10$

i $m+m+2\times m$ if $m=2$

j $m\times m-2\times m$ if $m=2$

k $\dfrac{a-3+2}{a-5}$ if $a=7$

l $\dfrac{2\times n-3\times 4}{n\div 3}$ if $n=15$

m $p+2\times p-3\times p$ if $p=3$

n $(4\times q)-(q\div 5)$ if $q=30$

o $(2\times q)-(5-q)-4$ if $q=4$

p $(a\times a)-a+(a\times a\times a)$ if $a=1$

q $\dfrac{3\times b}{12+b}$ if $b=6$

r $p-(p\div 3)$ if $p=3$

s $\dfrac{y\times y-2\times y}{y}$ if $y=3$

t $m-(m-4)-m$ if $m=4$

3. Find the solution set for each of these open sentences.

   Replacement set: the set of whole numbers

   a $x+6=10$

   b $b-3=b$

   c $f-4=13$

   d $t+t=22$

   e m is less than 7

   f x is larger than 9

   g $y+z$ is less than 6

   h $(5\times z)+6=21$

   i $(3\times x)-(2\times x)=7$

   j $13-d$ is less than 5 but larger than 1

   k $(2\times x)+1$ is an odd number.

   l $(2\times p)+1$ is an even number

   m $(2\times q)+2$ is an even number.

   n $(2\times h)+1$ is larger than 15

   o $(4\times j)+19=63$

   p $(6\times k)$ is less than 11

   q $s+s+s=33$

   r $(3\times w)+w=36$

   s $(7\times w)+(9\times w)=(16\times w)$

   t $(13\times x)-(10\times x)=(3\times x)$

4. If we write $x+y=6$, this is exactly the same as though we wrote

   $$\square+\triangle=6$$

   The same number need not replace both symbols.

   Write number pairs that make these open sentences true. The first number of each pair should replace x, and the second number should replace y.

   a $x+y=11$

   b $x+(2\times y)=16$

   c $x-y=3$

   d $(2\times x)-y=4$

   e $x\times y=16$

   f $x=y$

   g $x+y=18$

   h $(2\times x)=y$

   i $(x\times x)+y=14$

   j $x+(y\times y)=37$

   k $x-y=0$

   l $x+y=0$

   m $(x\times x)-y=41$

   n $(x\times x)+(y\times y)=100$

   o $(2\times x)+(3\times y)=11$

   p $(3\times x)+(2\times y)=11$

   q $(2\times x)-(3\times y)=0$

   r $x+(3\times y)=10$

   s $(4\times x)+y=19$

   t $(x\times y)-x=2$

5. We have seen how to evaluate open number expressions where we have used two different symbols to represent numbers. We can also evaluate open number expressions where we use letters to represent numbers. Consider $3\times x+y$ if $x=2$ and $y=3$.

   $$\begin{aligned} &3\times x+y \\ =&3\times 2+3 \\ =&\ \ 6\ \ +3 \\ =&9 \end{aligned}$$

Now evaluate the following:

a $x+2\times y$ if $x=1$ and $y=4$

b $x\times x-y+6$ if $x=4$ and $y=9$

c $y+z-6+2\times z$ if $y=4$ and $z=2$

d $\dfrac{m+n}{m-n}$ if $m=6$ and $n=4$

e $(m\times n)-(3\times n)$ if $m=5$ and $n=2$

f $p\times q\times p\times q$ if $p=3$ and $q=7$

g $\dfrac{3\times w-z}{2\times z}$ if $w=8$ and $z=8$

h $\dfrac{5\times a-b\times a}{3\times a}$ if $a=5$ and $b=3$

i $\dfrac{6\div q}{8\div r}$ if $q=3$ and $r=4$

j $2\times p-3\times q+p$ if $p=5$ and $q=2$

ANOTHER LOOK AT SOME NEW SYMBOLS

1 In the next two sections, you will learn nothing new about open sentences. You will learn some new shortcuts in *writing* them. To start with, we shall look at '$<$' and '$>$' again. They are used as follows:

$3<6$: 3 is less than 6

$10>4$: 10 is greater than 4

Find the solution sets for each of these open sentences

Replacement set: the set of whole numbers

a $f<17$

b $(2\times c)<18$

c $(3\times d)>26$

d $(2\times m)>90$

e $(4\times n)>4$

f $(x\times x)-x<45$

g $(x\times x)>x$

h $(x\times x)<(5\times x)$

i $(x\times x)<(2\times x)$

j $(x+x+x)=(3\times x)$

k $(5\times n)<26$

l $2(x\times x)>(3\times x)$

m $(8\times d)<41$

n $(x\times x)-3x>9$

o $(2\times c)>21$

p $(x\times x)+5x<51$

2 The idea behind the use of all these symbols that we have been learning to use is simply an attempt to reduce the amount of writing that we have to do. We have one more such idea to learn right now.

Look at this open sentence:

$$(6\times\square)+(3\times\triangle)=12$$

We could also write the open sentence as

$$(6\times x)+(3\times c)=12$$

The 'x' and the 'c' simply hold places open in the sentence.

We agree that, where there is no danger of misunderstanding, we shall no longer write multiplication signs in open sentences.

For example, we will write the open sentence

$(6\times x)+(3\times c)=12$

as $6x+3c=12$

This saves our having to write multiplication signs and pairs of parentheses.

Suppose we had 3×6. Could we write this as 36? Why not? Suppose we had $(x\times y)=12$. Could we write this as $xy=12$? Why? You saw above that we cannot leave out the multiplication signs between numerals. In most other cases we can leave them out.

For $3\times a$ we can write $3a$. For $m\times n$ we can write mn. What can we write for **a** $5\times p$; **b** $100\times r$; **c** $16\times q$; **d** $5\times b$; **e** $c\times m$; **f** $66\times b$; **g** $100\times s\times s$; **h** $a\times b\times c$; **i** $x\times y\times 2$; **j** $2\times m\times n$?

3. Evaluate the following open number expressions. The first one is done for you.

a $(x\times x)+xy$ if $x=2$ and $y=3$.

$$\begin{aligned}(x\times x)+xy &= (2\times2)+(2\times3)\\ &=4+6\\ &=10\end{aligned}$$

b $x+a+x+a$ if $a=1$ and $x=4$

c $3bc$ if $b=3$ and $c=5$

d $2x-3z+15$ if $x=7$ and $z=1$

e $3(m\times m)+n$ if $m=3$ and $n=4$

f $w+3z-2w$ if $w=5$ and $z=6$

g $2l+3(m\times m)-5$ if $l=10$ and $m=1$

h $5(z\times z)-15p$ if $z=3$ and $p=3$

i $\dfrac{2xy+y}{2xy+4}$ if $x=0$ and $y=4$

j $\dfrac{(x\times x)+b+b}{x+b+b}$ if $x=4$ and $b=1$

k $23x-(2n+4c)-(n+c)$ if $c=2$ and $n=1$

l $3(s\times s)-2r+1$ if $s=3$ and $r=2$

m $(x+2z)-(2x+z)$ if $x=2$ and $z=5$

n $\dfrac{5(p\times p)+2q-3}{2p+q+1}$ if $p=2$ and $q=2$

o $\dfrac{a+b}{a-b}+\dfrac{2a+2b}{2b+2}$ if $a=5$ and $b=3$

p $\dfrac{20m+2mn}{4n+m}$ if $m=2$ and $n=3$

q $\dfrac{p+q-(p-q)}{2q}$ if $p=5$ and $q=4$

r $5(z\times z)+x-6$ if $x=6$ and $z=1$

s $4+2ab+(b\times b)$ if $a=5$ and $b=0$

t $\dfrac{3x+2(y\times y)+1}{2x+1}$ if $x=7$ and $y=2$

u $\dfrac{a-b}{a}+\dfrac{a+b}{b}$ if $a=5$ and $b=5$

4. Find solution sets for these open sentences: *Replacement set: the set of whole numbers for each place holder*

a $xy=6$	b $2x-y=5$
c $x+y=7$	d $x-2y=3$
e $x-y=7$	f $x=3+y$
g $2x+y=2x$	h $2xy=12$
i $2x-2y=0$	j $3x=2x+y$
k $2x+x=3x+y$	l $5x=1+5y$
m $5x=5+5y$	n $5x=8+5y$
o $5x=15+5y$	p $5x+5y=0$
q $3xy=18$	r $9x+3y=69$
s $x=8-y$	t $6y=x+6$
u $2y=4+2x$	v $5x+3y=36$

MORE ABOUT THE PROPERTIES OF NATURAL NUMBERS

1 Write the numerals that make each of these open sentences true:
Replacement set: the set of whole numbers

a $x=x$
b $x+6=6+x$
c $11+x=x+11$
d $(x+3)+4=x+(3+4)$
e $2x+3x=5x$
f $7x-4x=3x$
g $2\times 5x=10x$
h $2x\times 3x=6(x\times x)$
i $2x+3-x=x+3$
j $(5x \quad 2x)-3=10(x\times x)-3$

This is a very important kind of open sentence. What did you find about all of these open sentences?

An open sentence that becomes true no matter what number we place in the box is called an identity or a generalization or a law. In this book we will usually call such a sentence an identity.

Let us look at one of these:

$$5x-2x=3x$$

As values for x, let us try 0, 1, 2, 3, . . .

$(5\times 0)-(2\times 0)=(3\times 0)$. This is true.
$(5\times 1)-(2\times 1)=(3\times 1)$. This is true.
$(5\times 2)-(2\times 2)=(3\times 2)$. This is true.

Are you convinced that this result or statement will *always* be true? If not, try some more numbers. This statement is called an **identity.** After this, we shall often use identities.

2. Which of these open sentences are identities?

a $2x+2x+2x=6x$
b $8x-3x=5x$
c $3x+6=18$
d $x-x=0$
e $x-x=2$
f $4+3x=2x+4+x$
g $5x+4x-2x=7x$
h $6x+7x=x$

3 Some important identities keep coming up so often that we give them special names. One of these is in the identities below.
Convince yourself that each one of these sentences is an identity. Then try to state in your own words the *one big idea* that is behind all of them.

a $7+3=3+7$
b $2+\square=\square+2$
c $\triangle+3=3+\triangle$
d $x+7=7+x$
e $2+y=y+2$
f $x+y=y+x$
g $m+(n+r)=m+(r+n)$
h $\triangle+\square=\square+\triangle$

Did you say something like this? "We can add two numbers in any order we like." One open sentence sums up this idea perfectly:

$a+b=b+a$

Instead of saying, "We can add two numbers in either order as we like," we usually say, "Addition is **commutative.**" This way is shorter and means the same thing. Outside of arithmetic, some things are commutative and some are not. Think of these two acts:

i putting a bullet in a gun   ii pulling the trigger.

Try these the other way around.

i pulling the trigger   ii putting a bullet in the gun.

Is the result the same? Are these two acts commutative? In arithmetic think of subtraction. Is it commutative?

$6-4$ $4-6$

Are these the same?

Subtraction is *not* commutative.

4. Which of these pairs of acts are commutative?

a putting on your socks and putting on your shoes
b buying a milkshake and buying a hamburger
c taking off your clothes and taking a shower
d going 5 miles north, then 4 miles east
e turning around, and then taking 5 steps
f adding 4, then subtracting 4

5 We often make use of the fact that addition is commutative.

When we add

$8+7+2$

We *can* add 8 and 7 to get 15, and then add 2 to 15 to get 17, but we can find this total more readily if we remember that $7+2$ is the same as $2+7$.

$$\begin{aligned}8+7+2 &= 8+2+7\\ &= 10+7\\ &= 17\end{aligned}$$

Now consider: $1+6+9$

We can think as follows:

$$\begin{aligned}1+6+9 &= 1+9+6\\ &= 10+6\\ &= 16\end{aligned}$$

The process is easier because the combinations making 10 can be handled quickly, i.e., the combinations are commutative.

6. Use the fact that addition is commutative to help you add these numbers:

a $6+8+4$
b $8+6+2$
c $7+5+3$
d $6+3+4+7$
e $8+5+2+5$
f $3+4+1+6$
g $8+4+2+1$
h $4+1+6+9+3$
i $3+2+4+7+8$
j $5+1+6+5+4+9$
k $2+3+5+7$
l $8+1+3+2+9+7$
m $9+6+5+4+5$
n $9+3+6+7+4$
o $6+5+3+5$
p $2+7+5+8+5$
q $9+9+5+5+1+1$
r $4+3+9+1$
s $8+2+8+2$
t $5+6+7+4+5$
u $7+3+6+4$
v $8+3+2+7+5$
w $5+2+3+8+7$
x $9+2+4+6+8$

7 We have found in arithmetic that the identity

$$a+b=b+a$$

is an important rule of the game.

We say that addition is commutative.

We can discover more important rules of arithmetic by looking at some more identities.

Examine these identities: In your own words, state the one big idea behind each of them. Can you write the one open sentence that sums up the idea perfectly?

a $\square\times7\times3=\square\times3\times7$
b $7\times\square=\square\times7$
c $\triangle\times6=6\times\triangle$
d $9c=c9$
e $4\times t\times3=4\times3\times t$
f $(\triangle\times3)+6=(3\times\triangle)+6$

The idea behind these open sentences is that we can always change the order in which we multiply two numbers. The open sentence representing this idea perfectly is:

$$ab=ba$$

We say that multiplication is commutative.

8 We use the fact that multiplication is commutative to simplify arithmetic.

When we multiply $5 \times 6 \times 2$

we *can* find 5×6, which is 30, and then find 30×2, which is 60.

But we can also use the fact that multiplication is commutative.

$$\begin{aligned} 5 \times 6 \times 2 &= 5 \times 2 \times 6 \\ &= 10 \times 6 \\ &= 60 \end{aligned}$$

When we multiply $2 \times 7 \times 5$

we may write

$$\begin{aligned} 2 \times 7 \times 5 &= 2 \times 5 \times 7 \\ &= 10 \times 7 \\ &= 70 \end{aligned}$$

This method simplifies our calculation because, again, the 10's are easier to handle.

9. Use the fact that multiplication is commutative to help you multiply these numbers:

a $5 \times 7 \times 2$
b $4 \times 3 \times 25$
c $50 \times 7 \times 2$
d $15 \times 3 \times 2$
e $20 \times 6 \times 5$
f $15 \times 7 \times 2$
g $5 \times 8 \times 2 \times 2 \times 5$
h $5 \times 5 \times 2 \times 7 \times 2$
i $10 \times 7 \times 10$
j $25 \times 11 \times 4$
k $16 \times 3 \times 5$
l $5 \times 5 \times 5 \times 7 \times 2 \times 3 \times 2 \times 2$
m $5 \times 9 \times 2$
n $4 \times 5 \times 6 \times 5$
o $7 \times 6 \times 5$
p $4 \times 5 \times 8 \times 5$
q $9 \times 5 \times 8$
r $25 \times 12 \times 4$
s $70 \times 5 \times 3$
t $75 \times 11 \times 4$
u $16 \times 2 \times 5$
v $4 \times 9 \times 5 \times 2$
w $8 \times 5 \times 9$
x $5 \times 4 \times 3 \times 2 \times 5 \times 9$

10 We have now worked with two important rules of arithmetic. Both of these rules may be stated as open sentences.

$\square + \triangle = \triangle + \square$ **addition is commutative.**

$\square \times \triangle = \triangle \times \square$ **multiplication is commutative.**

$$a + b = b + a$$
$$a \times b = b \times a$$

We shall now find more rules of arithmetic.

11 Study this expression:

$8+3\times6$

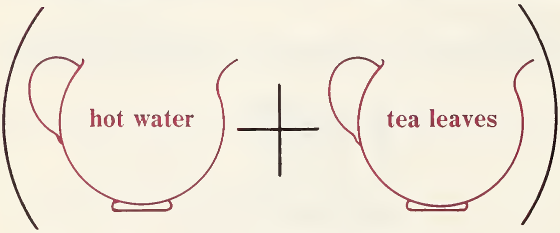
There is more than one way of looking at this expression. We could work this way:

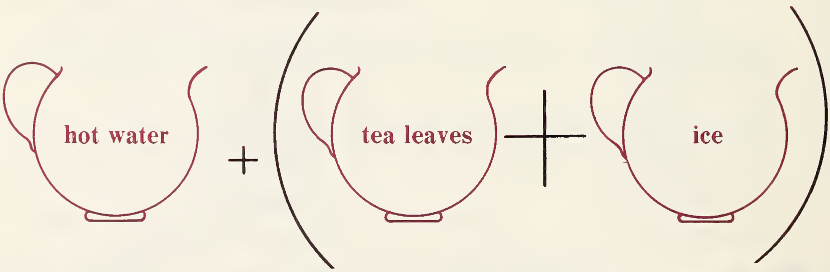
$$(8+3)\times6$$
$$=11\times6$$
$$=66$$

or we could work this way:

$$8+(3\times6)$$
$$=8+18$$
$$=26$$

The answer we get depends upon the way we bracket the question. There are situations like this outside mathematics. Study this example:





The result depends upon where the brackets are. In the example at the top, we end with iced tea; in the bottom example, we end with wet tea leaves.

Chemistry supplies many other examples.

Study these open sentences:

$$(\square+\triangle)+\diamond=?$$
$$\square+(\triangle+\diamond)=?$$

Try some numbers.

When we are to add three numbers, does the bracketing matter?

Since it does not matter, we can write

$$(\square+\triangle)+\diamond=\square+(\triangle+\diamond) \text{ or } (x+y)+z=x+(y+z)$$

Using letters we write $(a+b)+c=a+(b+c)$

Try some numbers. Convince yourself that this is always true.

This is one of the rules of arithmetic.

We say that addition is associative.

12 We can use the fact that addition is associative to help us with addition.

When we add

$53+38$,

we can find the total in the following way:

$$
\begin{aligned}
&53+38\\
&=(50+3)+38\\
&=50+(3+38)\\
&=50+41\\
&=91
\end{aligned}
$$

We used the associative property to put the 3 with the 38 to make mental addition easier.

When we add

$48+23$

we can find the total in the following way:

$$
\begin{aligned}
&48+23\\
&=48+(2+21)\\
&=(48+2)+21\\
&=50+21\\
&=71
\end{aligned}
$$

We have just used the associative property to remove 2 from the 23 and put it in with the 48 to make mental addition easier.

13. Use the fact that addition is associative to help you add these numbers:

a $32+68=\square$

b $73+47=\square$

c $29+64=\square$

d $101+29=\square$

e $13+13+17+17=\square$

f $24+109=\square$

g $41+66=\square$

h $79+22=\square$

i $35+125=\square$

j $85+65=\square$

k $72+138=\square$

l $101+98=\square$

14 Multiplication is also associative. Try some numbers in this open sentence:

$$(\square \times \triangle) \times \diamond = \square \times (\triangle \times \diamond)$$

or $(a \times b) \times c = a \times (b \times c)$

It is always true. We can write the following identity to express this:

$$(x \times y) \times z = x \times (y \times z)$$

i	ii
16×5	15×12
$= (8 \times 2) \times 5$	$= 15 \times (2 \times 6)$
$= 8 \times (2 \times 5)$	$= (15 \times 2) \times 6$
$= 8 \times 10$	$= 30 \times 6$
$= 80$	$= 180$

We can often simplify a question for mental arithmetic if we make use of the fact that multiplication is associative.

15. Use the fact that multiplication is associative to help you with these questions:

a $5 \times 24 = \square$ **b** $28 \times 5 = \square$

c $32 \times 5 = \square$ **d** $5 \times 18 = \square$

e $4 \times 18 = \square$ **f** $26 \times 5 = \square$

g $5 \times 14 = \square$ **h** $5 \times 22 = \square$

i $16 \times 4 = \square$ **j** $66 \times 5 = \square$

k $5 \times 42 = \square$ **l** $5 \times 82 = \square$

16 We often take the associative properties for granted when we write down addition questions.

For example, we often write expressions like $8+6+2$ without parentheses.

We do this because it does not matter which we add first, the $8+6$ or the $6+2$; we still get 16 as an answer.

17. Study the following statements: (We have left out parentheses because addition in arithmetic is associative.)

Compute each total several different ways:

a $16+19+13 = \square$ **b** $18+3+2+6 = \square$

c $11+13+65+64 = \square$ **d** $18+65+64 = \square$

e $45+4+15 = \square$ **f** $11+12+16+17 = \square$

g $29+31+61 = \square$ **h** $44+21+17+36 = \square$

i $44+16+21 = \square$ **j** $56+72+25+16 = \square$

k $11+33+96+18 = \square$ **l** $12+13+75+15 = \square$

18. We are now going to work with the most important of these properties. You should discover this property as you find solutions for these open sentences.

a $(4\times8)+(3\times8)=\square\times8$

b $(6\times11)+(15\times11)=\square\times11$

c $(13\times8)+(6\times8)=\square\times8$

d $\square\times17=(4\times17)+(9\times17)$

e $(2\times26)+(7\times26)=\triangle\times26$

f $(5\times11)+(\square\times11)=9\times11$

g $(\square\times16)+(7\times16)=11\times16$

h $(9\times8)+(6\times8)=\square\times8$

i $(4\times11)+(12\times11)=16\times\square$

j $(4\times17)+(13\times17)=17\times\square$

k $(2\times\square)+(3\times\square)=5\times\square$

l $(6\times7)+(3\times7)=9\times\square$

m $(\square\times6)+(8\times6)=16\times6$

n $(8\times\triangle)=(3\times\triangle)+(5\times\triangle)$

o $(18\times10)+(3\times\triangle)=21\times10$

p $(31\times13)=(15\times13)+(\square\times13)$

q $(4\times63)+(\square\times63)=11\times63$

r $(19\times37)+(17\times37)=\square\times37$

s $(4\times\square)+(9\times\square)=(13\times\square)$

t $(9\times25)+(\square\times25)=25\times25$

Can you state in your own words the one big idea behind all of these open sentences?

19 The rule behind the whole of the last exercise could be called 'the accumulative rule'.

It works like this:

$$(6\times\square) \quad + \quad (7\times\square) \quad = \quad (13\times\square).$$

If we take *6* of a number, and add *7* of the same number, we get *13* of the number. The open sentence above is true for *all numbers*. We accumulate the numbers replaced by the box.

6 of any number plus 7 of that number give us 13 of that number.

If we wrote

$$(3\times17)+(\square\times17)=11\times17$$

we would think in the following way:

We start with *3* 17's, and want to end with *11* 17's. How many 17's must we add?

We must add *8* 17's.

Try 8 in place of the box. Does the sentence become true?

By an historical accident, this is not called the accumulative rule. The first mathematicians who worked with this rule wrote it the other way around.

They wrote

> 7×11, and thought, "That's 7 elevens; so I can *break up* the 7 elevens into two parts if I want to. I can think:
>
> $$7\times11=(7\times3)+(7\times8)$$
>
> 7 elevens is 7 threes plus 7 eights."

This may seem a peculiar way of writing it, but you should see that it is the same idea.

Read these statements: (The first two are done for you.)

a $13\times6 = (13\times4)+(13\times2)$
13 sixes is 13 fours plus 13 twos.

b $29\times13 = (29\times9)+(29\times4)$
29 thirteens is 29 nines plus 29 fours.

c $17\times6 = (17\times5)+(17\times1)$

d $35\times21 = (35\times14)+(35\times7)$

e $42\times17 = (42\times10)+(42\times7)$

f $67\times8 = (67\times5)+(67\times3)$

g $84\times71 = (84\times70)+(84\times1)$

h $35\times26 = (35\times20)+(35\times6)$

i $86\times41 = (86\times20)+(86\times21)$

j $95\times63 = (95\times60)+(95\times3)$

Since the early mathematicians wrote those sentences this way, they saw the first number *spread out* or *distributed* through the sentence. They called this "spreading-out" system the **distributive property**, and we still call it that today.

20 Study these two statements: (Read them both.)

$$(7\times13)+(5\times13) = 12\times13$$
$$13\times12 = (13\times7)+(13\times5)$$

Do you see that they both say exactly the same thing? We say that the one big idea behind them both is the **distributive property**. We use the distributive property all the time in arithmetic.

21. Study these examples:

a
$$\begin{aligned} 7\times26 &= (7\times20)+(7\times6) \\ &= 140+42 \\ &= 182 \end{aligned}$$

This is really the way we multiply all the time. We write:

$$\begin{array}{rl} 26 & \\ \underline{\times7} & \\ 42 & \leftarrow(7\times6) \\ \underline{140} & \leftarrow(7\times20) \\ 182 & \leftarrow(42+140) \end{array}$$

b $9\times 13 = (9\times 10)+(9\times 3)$

$$\begin{array}{rl} 13 & \\ \times 9 & \\ \hline 27 & \leftarrow (9\times 3) \\ 90 & \leftarrow (9\times 10) \\ \hline 117 & \leftarrow (27+90) \end{array}$$

We now could do this question another way:

$9\times 13 = (9\times 8)+(9\times 5)$

$$\begin{array}{rl} 13 & \\ \times 9 & \\ \hline 72 & \leftarrow (9\times 8) \\ 45 & \leftarrow (9\times 5) \\ \hline 117 & \leftarrow (72+45) \end{array}$$

Do you see that we get the same answer? Study these three ways of doing one question. How have we used the distributive property?

$$\begin{array}{rl} 15 & \\ \times 7 & \\ \hline 63 & \leftarrow (7\times \square) \\ 42 & \leftarrow (7\times \square) \\ \hline 105 & \end{array} \qquad \begin{array}{rl} 15 & \\ \times 7 & \\ \hline 56 & \leftarrow (7\times \square) \\ 49 & \leftarrow (7\times \square) \\ \hline 105 & \end{array} \qquad \begin{array}{rl} 15 & \\ \times 7 & \\ \hline 35 & \leftarrow (7\times \square) \\ 70 & \leftarrow (7\times \square) \\ \hline 105 & \end{array}$$

Which way is the easiest?

Does this explain why we multiply the way we do? Do you see how we depend on the distributive property?

22. Work each of these questions three different ways:

a $\begin{array}{r} 12 \\ \times 6 \\ \hline \end{array}$ **b** $\begin{array}{r} 13 \\ \times 5 \\ \hline \end{array}$ **c** $\begin{array}{r} 14 \\ \times 9 \\ \hline \end{array}$ **d** $\begin{array}{r} 15 \\ \times 6 \\ \hline \end{array}$ **e** $\begin{array}{r} 16 \\ \times 8 \\ \hline \end{array}$

23 We use the distributive property in two-figure multiplication. Study this example:

$\begin{array}{r} 16 \\ \times 12 \\ \hline \end{array}\rightarrow$ we can think $12\times 16 = (8\times 16)+(4\times 16)$

$$\begin{array}{rl} 16 & \\ \times 12 & \\ \hline 128 & \leftarrow (8\times 16) \\ 64 & \leftarrow (4\times 16) \\ \hline 192 & \end{array}$$

but it is more *convenient* to think

$$(12\times16)=(10\times16)+(2\times16)$$

$$\begin{array}{r l} 16 & \\ \times12 & \\ \hline 32 & \leftarrow(2\times16) \\ 160 & \leftarrow(10\times16) \\ \hline 192 & \end{array}$$

We *could* find 12×16 all of these different ways:

$$\begin{array}{r l} 16 & \\ \times12 & \\ \hline 48 & \rightarrow(\square\times16) \\ 144 & \rightarrow(\square\times16) \\ \hline 192 & \end{array} \qquad \begin{array}{r l} 16 & \\ \times12 & \\ \hline 96 & \rightarrow(\square\times16) \\ 96 & \rightarrow(\square\times16) \\ \hline 192 & \end{array} \qquad \begin{array}{r l} 16 & \\ \times12 & \\ \hline 112 & \rightarrow(\square\times16) \\ 80 & \rightarrow(\square\times16) \\ \hline 192 & \end{array}$$

24. Work each of these questions three different ways:

a $\begin{array}{r} 15 \\ \times12 \\ \hline \end{array}$ **b** $\begin{array}{r} 32 \\ \times14 \\ \hline \end{array}$ **c** $\begin{array}{r} 25 \\ \times11 \\ \hline \end{array}$ **d** $\begin{array}{r} 29 \\ \times13 \\ \hline \end{array}$ **e** $\begin{array}{r} 36 \\ \times19 \\ \hline \end{array}$

25. One open sentence sums up the distributive property perfectly:

$$(\square+\triangle)\times\diamond = (\square\times\diamond)+(\triangle\times\diamond)$$

Using letters instead of boxes, we find

$$(a+b)x=ax+bx \text{ or } x(a+b)=xa+xb=ax+bx$$

We now have looked at 5 basic properties of natural numbers. Let us look at their open sentences.

a Addition is commutative: $x+y=y+x$.

b Multiplication is commutative: $xy=yx$.

c Addition is associative: $x+(y+z)=(x+y)+z$.

d Multiplication is associative: $x\times(y\times z)=(x\times y)\times z$.

e The distributive property is expressed thus: $(a+b)x=ax+bx$ or $x(a+b)=xa+xb=ax+bx$.

26. In this exercise, first replace the box with a numeral that makes the open sentence correct; then state the property you used to do it.

a $(6+7)+(8+3)=(6+7)+(\square+8)$

b $(100+6)+7=100+(6+\square)$

c $6\times19=(4\times19)+(\square\times19)$

d $(8+7)+(6+3)=[(8+\square)+6]+3$

e $(7\times37)+(3\times37)=\square\times37$

MULTIPLICATION AND DIVISION

1 Find the product: 360×429.

What rule have we learned about multiplication that will help us find this product?

$$360 = 300 + 60$$

so $360 \times 429 = (300 \times 429) + (60 \times 429)$ (What rule have we used?)

$$\begin{array}{rl} 429 & \\ \times\ \ 360 & \\ \hline 128{,}700 & = 300 \times 429 \\ 25{,}740 & = \ \ 60 \times 429 \\ \hline 154{,}440 & = 360 \times 429 \end{array} \qquad\qquad \begin{array}{rl} 429 & \\ \times\ \ 360 & \\ \hline 25{,}740 & = \ \ 60 \times 429 \\ 128{,}700 & = 300 \times 429 \\ \hline 154{,}440 & = 360 \times 429 \end{array}$$

2. Use the distributive property to find the following products:

a $\begin{array}{r} 714 \\ \times 280 \\ \hline \end{array}$ **b** $\begin{array}{r} 9104 \\ \times 470 \\ \hline \end{array}$ **c** $\begin{array}{r} 10823 \\ \times 590 \\ \hline \end{array}$ **d** $\begin{array}{r} 9712 \\ \times 180 \\ \hline \end{array}$

e $\begin{array}{r} 625 \\ \times 309 \\ \hline \end{array}$ **f** $\begin{array}{r} 8293 \\ \times 506 \\ \hline \end{array}$ **g** $\begin{array}{r} 11714 \\ \times 608 \\ \hline \end{array}$ **h** $\begin{array}{r} 10603 \\ \times 207 \\ \hline \end{array}$

i $\begin{array}{r} 716 \\ \times 498 \\ \hline \end{array}$ **j** $\begin{array}{r} 30561 \\ \times 726 \\ \hline \end{array}$ **k** $\begin{array}{r} 29835 \\ \times 153 \\ \hline \end{array}$ **l** $\begin{array}{r} 89614 \\ \times 769 \\ \hline \end{array}$

3 Find the quotient: $33345 \div 247$

$$\begin{array}{r|r} 247\overline{)33345} & 100 \\ 24700 & \\ \hline 8645 & 30 \\ 7410 & \\ \hline 1235 & 5 \\ 1235 & \\ \hline & 135 \end{array}$$

We see that $33345 \div 247 = 135$.

This means that $135 \times 247 = 33345$.

We can check by multiplying.

4 Here is a method we may use to do the computation.

$$\left.\begin{array}{r} 5 \\ 30 \\ 100 \end{array}\right\} 135$$

$$\begin{array}{r|} 247\overline{)33345} \\ 24700 \\ \hline 8645 \\ 7410 \\ \hline 1235 \\ 1235 \\ \hline \end{array}$$

We may shorten the computation this way:

a $\begin{array}{r} 1 \\ 247\overline{)33345} \\ 247 \end{array}$

What does the '1' represent?
What does the '247' represent?

b $\begin{array}{r} 13 \\ 247\overline{)33345} \\ 247 \\ \hline 864 \\ 741 \\ \hline \end{array}$

What does the '3' represent?

What does the '741' represent?

c

```
      135
247)33345
    247
    ---
     864
     741
    ----
    1235
    1235
```

Explain the steps that have been taken here.

5 Study the examples below. Explain each step.

a

```
359)40816 |100
    35900 |
    ----- |
     4916 |
```

```
359)40816 |100
    35900 |
    ----- |
     4916 | 10
     3590 |
     ---- |
     1326 |
```

```
359)40816 |100
    35900 |
    ----- |
     4916 | 10
     3590 |
     ---- |
     1326 |  3
     1077 | ---
     ---- |
      249 |113
```

b

```
      100
359)40816
    35900
    -----
     4916
```

```
       10 }
      100 }
359)40816 |
    35900 |
    ----- |
     4916 |
     3590 |
     ---- |
     1326 |
```

```
        3 }
       10 }
      100 }
359)40816
    35900
    -----
     4916
     3590
     ----
     1326
     1077
     ----
      249
```

c

```
        1
359)40816
    359
    ---
     49
```

```
       11
359)40816
    359
    ---
     491
     359
     ---
     132
```

```
      113
359)40816
    359
    ---
     491
     359
     ----
     1326
     1077
     ----
      249
```

Method **a** is called the **subtractive** method.

Method **c** is the way that was used for many years before the subtractive method became popular. We sometimes call method **c** the **shortened** method.

6. Divide using the shortened method.

Check by using the subtractive method.

a 6650÷28 **b** 8743÷83

c 9505÷37 **d** 8038÷47

e 5025÷31 **f** 48588÷105

g 75112÷520 **h** 35037÷801

i 36248÷176 **j** 47385÷165

k 50509÷155 **l** 96283÷124

7 Compute 4184÷8.

When we have to divide by a single digit, we can use a short method of computing.

We think:

$8\overline{)4184}$

a Will 8 divide 4 thousand and give a number of thousands as the answer?
4 thousands ÷ 8 = how many thousands?
We cannot get a number of thousands as the answer.

$8\overline{)4184}$

b Think of the 4 thousands and the 1 hundred in the dividend as 41 hundreds.
41 hundreds ÷ 8 = how many hundreds?

$8\overline{)4184}$

41 hundreds ÷ 8 = 5 hundreds and 1 hundred over. Put 5 in the hundreds' place in the answer.

18 tens

$\begin{array}{r} {}^{1}\;\; \\ 8\overline{)4184} \\ 5\;\;\;\; \end{array}$

c Think of the 1 hundred we had left over and the 8 tens in the dividend. 100 is 10 tens. 100 and 80 = 18 tens.

$\begin{array}{r} {}^{1}\;\; \\ 8\overline{)4184} \\ 52\;\; \end{array}$

d 18 tens ÷ 8 = 2 tens and 2 extra tens. Put 2 in the tens' place in the answer.

24 ones

$\begin{array}{r} {}^{1\;2} \\ 8\overline{)4184} \\ 5\;2\;\; \end{array}$

e Think of the 2 extra tens and the 4 ones in the dividend as 24 ones.

$\begin{array}{r} {}^{1\;2} \\ 8\overline{)4184} \\ 523 \end{array}$

f 24 ones ÷ 8 = 3 ones.
Put 3 in the ones' place in the answer.

8. Study the examples below and explain the working of each step:

a $7\overline{)83046}$	**b** $6\overline{)35213}$	**c** $9\overline{)10143}$
$7\overline{)8\,{}^{1}3046}$ 1	$6\overline{)35\,{}^{5}213}$ 5	$9\overline{)10\,{}^{1}143}$ 1
$7\overline{)8\;{}^{1}3\;{}^{6}0\;4\;6}$ 1 1	$6\overline{)3\;5\;{}^{5}2\;{}^{4}1\;3}$ 5 8	$9\overline{)1\;0\;{}^{1}1\;{}^{2}4\;3}$ 1 1
$7\overline{)8\;{}^{1}3\;{}^{6}0\;{}^{4}4\;6}$ 1 1 8	$6\overline{)3\;5\;{}^{5}2\;{}^{4}1\;{}^{5}3}$ 5 8 6	$9\overline{)1\;0\;{}^{1}1\;{}^{2}4\;{}^{6}3}$ 1 1 2
$7\overline{)8\;{}^{1}3\;{}^{6}9\;{}^{4}4\;{}^{2}6}$ 1 1 8 6	$6\overline{)3\;5\;{}^{5}2\;{}^{4}1\;{}^{5}3}$ 5 8 6 8 r 5	$9\overline{)1\;0\;{}^{1}1\;{}^{2}4\;{}^{6}3}$ 1 1 2 7
$7\overline{)8\;{}^{1}3\;{}^{6}0\;{}^{4}4\;{}^{2}6}$ 1 1 8 6 3 r 5	$6\overline{)3\;5\;2\;1\;3}$ 5 8 6 8 r 5	$9\overline{)1\;0\;1\;4\;3}$ 1 1 2 7
$7\overline{)8\;3\;0\;4\;6}$ 1 1 8 6 3 r 5	This computation is often called **Short Division.**	

9. Divide, using the short division method:

a $6\overline{)492}$	**b** $5\overline{)375}$	**c** $2\overline{)618}$
d $4\overline{)702}$	**e** $3\overline{)824}$	**f** $8\overline{)152}$
g $9\overline{)171}$	**h** $7\overline{)637}$	**i** $5\overline{)453}$
j $8\overline{)9132}$	**k** $9\overline{)4216}$	**l** $7\overline{)6218}$
m $6\overline{)4135}$	**n** $3\overline{)2167}$	**o** $4\overline{)3218}$
p $8\overline{)53162}$	**q** $9\overline{)81018}$	**r** $7\overline{)60123}$
s $9\overline{)8121}$	**t** $6\overline{)3325}$	**u** $4\overline{)63001}$
v $8\overline{)9077}$	**w** $7\overline{)9543}$	**x** $5\overline{)10001}$

CHAPTER TEST

1. Find names that make these open sentences true:

	Replacement sets:
a '$\square$' is a name of a Canadian Province containing the letter 'o'.	names of Canadian Provinces.
b $\square$ is larger than 16.	whole numbers.
c $2 \times \square$ is less than 19.	whole numbers.

2. Write down the numerals represented by '. . .' in these sequences:

a 4, 6, 8, . . ., 20

b 11, 10, 9, . . ., 2

c 1, 10, 1, 11, 1, 12, . . ., 19

d 4, 4, 8, 12, 20, 32, 52, 84, . . ., 356

3. Evaluate the following:

a $4 \times (9-5)+16 \div 4$

b $(8-2)\times(4+3)\times(10-6)$

c $(49 \div 7)\times(56 \div 8)$

d $\dfrac{13-2\times 6+4}{28 \div 7-3}$

e $\dfrac{(4+9)\times 2}{3\times 3+4}+\dfrac{18 \div 6+3}{12 \div 4}$

f $3x+2(x \times x)+1$ if $x=4$

g $2(x \times x)-x-(x+5)$ if $x=3$

h $(a+3)\div(2a \div 6)$ if $a=9$

i $\dfrac{n+2(n \times n)}{n}+\dfrac{2n-n}{3+2}$ if $n=5$

j $2m+4(m \times m)+5-15$ if $m=2$

k $xy+3x-2y$ if $x=4$ and $y=5$

l $\dfrac{(x \times x)+2y}{4y+1}$ if $x=3$ and $y=4$

m $(3a-b)\times(a+3b)$ if $a=2$ and $b=3$

n $\dfrac{z \times (w-4)}{(w \times w)-5z}$ if $w=6$ and $z=4$

o $\dfrac{mn+(m \times m)+(n \times n)}{m \times (n-m)}$ if $m=1$ and $n=2$

4. Find the numerals that make each open sentence true:
Replacement set: the set of whole numbers

a $(5 \times \square)+6$ is less than 30.

b $(5 \times \square)$ is less than 28.

c $7 \times \square$ is between 80 and 100.

d $\square+\square+\square=57$

e $(\square \times \square)+16=97$

f $(4 \times \square)+\square=95$

g $6+(6 \times \square)$ is more than 45

h $(7 \times \square)+12=40$

i $\square \times \square \times \square=216$

j $\square+\square+(5 \times \square)=56$

5. Write open sentences for these problems, and find the numerals that make each open sentence true:
   a If a number is multiplied by itself, the product is between 400 and 500.
   b If we multiply a number by 8 and subtract 7, we get 97. Find the number.
   c If we subtract a number from 5 times itself, we get 280. What is the number?

6. Write the number pairs that make each of the following open sentences true: *Replacement set: the set of whole numbers for each place holder*
   a $(2\times\square)+\triangle=12$
   b $(3\times\square)+(2\times\triangle)=25$
   c $12-\square=3-\triangle$
   d $\square-(5\times\triangle)=5$

7. Find the solutions of each of these open sentences:
   Replacement set: the set of whole numbers
   a $x+9=13$
   b $3t-7=26$
   c $4f+1=29$
   d $(4\times w)+w=45$
   e $t+t+t=345$
   f $6s-3s=3s$

8. Find the solutions of each of these open sentences:
   Replacement set: the set of whole numbers for each place holder
   a $x+y=9$
   b $2x-y=3$
   c $3x+4y=13$
   d $x-y=6$
   e $x+4y=17$
   f $3x+3y=3$

9. Replace each box with a numeral that makes the open sentence true; then state the property you used to find it:
   a $17\times9=(10\times9)+(\square\times9)$
   b $4+(7+3)=4+(3+\square)$
   c $(8\times6)\times5=(\square\times8)\times5$
   d $(5\times2)\times11=5\times(2\times\square)$

10. Copy these equations. Find a replacement to make each sentence true. Also find a replacement to make each sentence false.
   a $\square-5=7$
   b $2\times\triangle=32$
   c $\square\times\square=25$
   d $3\times\triangle+5=20$
   e $2\times\triangle\times\triangle+5=23$
   f $2\times\triangle+5=3\times\triangle-5$
   g $\square+10=2\times\square$
   h $\square\times\square=\square$

3 Factors

FACTORS AND MULTIPLES

1 Consider the statement $7 \times 3 = 21$.

We say that 7 *divides* 21, and that 3 *divides* 21.

We also say that 7 is a factor of 21 and that 3 is a factor of 21.

Because $8 \times 1 = 8$, we say that 8 divides 8 and 1 divides 8. We see that both 8 and 1 are factors of 8.

Consider the numbers 7 and 23.

We say that 7 does not divide 23; therefore 7 is *not* a factor of 23.

If a number A divides a number B, we say that A is a *factor* of B and that B is a *multiple* of A.

$6 \times 4 = 24$. 6 is a factor of 24. 4 is a factor of 24. 24 is a multiple of 6. 24 is a multiple of 4.

2. **a** Which of these numbers are multiples of 6?
   12, 31, 42, 81, 102, 232, 1,004

   b Which of these numbers are multiples of 7?
   7, 80, 91, 287, 1,001, 6,006, 10,010

   c Which of these numbers are multiples of 25?
   200, 5,000, 6,025, 170, 1,240, 4,075

   d Which of these numbers are multiples of 2?
   2, 16, 31, 97, 1,006, 246, 171, 162, 311, 793, 717

   e Which of these numbers are multiples of 1?
   1, 97, 116, 414, 813, 1,116, 1,021

   f Which of these numbers are multiples of 9?
   36, 45, 379, 416, 2,133, 7,119

   g Which of these numbers are factors of 117?
   1, 13, 53, 76

   h Which of these numbers are factors of 60?
   1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

   i Does 60 have any more factors? What are they?

j Which of these numbers are factors of 80?
1, 2, 3, 4, 5, 10, 15, 20, 40, 60, 80

k Which of these numbers are factors of 23,023?
1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23

l Which of these numbers are factors of 101?
1, 2, 3, 4, 5, 6, 7, 8, 9, 10

3 1 is a factor of all numbers. Another way of saying this is:
$1 \times \square = \square$. $1 \times \square = \square$ is an identity for all integers.
Every number except 1 has at least two factors. The factors of 5 are 1 and 5.

12 has several factors.

1 divides 12 because $1 \times 12 = 12$.
2 divides 12 because $2 \times 6 = 12$.
3 divides 12 because $3 \times 4 = 12$.
4 divides 12 because $4 \times 3 = 12$.
6 divides 12 because $6 \times 2 = 12$.
12 divides 12 because $12 \times 1 = 12$.

1, 2, 3, 4, 6, and 12 are all factors of 12. 12 is a multiple of each of these numbers.

4. Write all the factors of each of these numbers:

a 7	**b** 8	**c** 10	**d** 16	**e** 30	**f** 40
g 71	**h** 90	**i** 100	**j** 101	**k** 91	**l** 88

PRIME NUMBERS

1 Some numbers have only two different factors; others have more than two.

The numbers having precisely two different factors are called prime numbers.

5 is a prime number because it has precisely two different factors: 1 and 5.

11 is a prime number because it has precisely two different factors: 1 and 11.

4 is not prime, because it has 3 different factors: 1, 2, and 4.

1 is not prime because it has only one factor: 1.

The first few primes are these: 2, 3, 5, 7, 11, and 13.

2. **a** Write the next 6 primes after 13.
   b Which of the following numbers are prime?
   41, 43, 47, 49, 51, 57, 59, 67, 77, 87
   c Are there any primes divisible by 5?
   d Are there any primes divisible by 3?
   e Are there any primes divisible by 2?
   f Are there any primes divisible by primes other than themselves? Why?

3 Numbers larger than 1 that are not prime are said to be **composite** numbers. The first few composite numbers are these: 4, 6, 8, 9, 10, 12, 14, 15.

4. **a** Write the next 6 composite numbers that come after 15.
   b Which of these numbers are composite?
   33, 43, 53, 63, 73, 83, 93, 103
   c Are any composite numbers also prime? Why?

5 Any composite number can be written as a product of prime numbers. Study the two examples below.

30 is composite.

$30 = 2 \times 15$

But 15 is composite.

$15 = 3 \times 5$

So $30 = 2 \times (3 \times 5)$

$30 = 2 \times 3 \times 5$ (Why can we leave out the parentheses?)

$45 = 5 \times 9$

But 9 is composite.

$9 = 3 \times 3$

So $45 = 5 \times (3 \times 3)$

$ = 5 \times 3 \times 3$

We can write 45 as $5 \times 3 \times 3$, and these numbers are all prime. When we write a number as the product of prime numbers, we say we are **factoring the number into prime factors.** In this chapter when we say '*factor* a number', we shall understand that we have to write the number as a product of its '*prime factors*'. Example:*factor* 72:

$72 = 6 \times 12$

$ = (2 \times 3) \times (3 \times 4)$

$ = 2 \times 3 \times 3 \times 2 \times 2$

6. **a** Factor each of these numbers:
   12, 16, 18, 25, 27, 29, 36, 42

   b Factor each of these numbers:
   4, 32, 16, 64, 2, 8, 128
   What do these numbers have in common?

   c Factor each of these numbers:
   17, 31, 41, 59, 61, 101
   What do these numbers have in common?

   d Factor each of these numbers:
   4, 25, 1, 36, 64, 9, 49, 16
   What do these numbers have in common?

   e Factor each of these numbers:
   8, 27, 64, 125, 216, 343, 1
   What do these numbers have in common?

7. The following exercise is to help you sharpen your ability to find factors of numbers. Find the numbers that make these open sentences true. There are two ways of finding each number. The first is done for you. Do each question both ways.

   a $105 = 3 \times 5 \times \square$
   First method: Factor 105.
   $105 = 5 \times 3 \times 7$
   Therefore, the missing factor is 7.
   Second method:
   $3 \times 5 = 15$
   $105 \div 15 = 7$
   Therefore, the missing factor is 7.

   b $280 = 2 \times 2 \times \square \times 5 \times 7$

   c $198 = 2 \times 3 \times 3 \times \square$

   d $350 = \square \times \square \times 2 \times 7$

   e $750 = 2 \times 3 \times \square \times \square \times \square$

   f $1{,}368 = 2 \times 2 \times 2 \times 3 \times 3 \times \square$

   g $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times \square$

   h $5{,}005 = 5 \times 11 \times 13 \times \square$

   i $162 = 3 \times 3 \times 3 \times 3 \times \square$

   j $26{,}950 = 2 \times 5 \times 5 \times \square \times \square \times 11$

FACTOR INTO PRIME FACTORS

1 We know that if we are asked to factor 77, we can write:

$77 = 7 \times 11$, or

$77 = 11 \times 7$

Are there any *other* prime numbers which, multiplied together, give 77? How can you tell? The easiest way would simply be to try all possible other numbers. We would find that 77 has no factors but 1, 7, 11, and 77.

With very large numbers, it would be more difficult to tell. For example, it is true that $522{,}291 = 3 \times 7 \times 7 \times 11 \times 17 \times 19$. But suppose we wanted to know whether or not some other list of factors would do? It would take a long time to check all the other numbers that might divide 522,291. Fortunately, we do not have to check any of them! It is not easy to *prove* that this is so, but *it can be proved that every number can be factored in only one way.*

For example:

$12 = 2 \times 2 \times 3$

These are the prime factors of 12. There are no others.

Using the commutative property, we could write this as:

$12 = 2 \times 3 \times 2$

or

$12 = 3 \times 2 \times 2$

but these are just different orders for the same list of factors.

There is no other list of prime numbers having a product of 12.

And this is true for every number.

Every number can be factored in only one way.

This statement is called the

Fundamental Theorem of Arithmetic.

Theorem just means something that can be proved. Mathematicians have found that many further important things in mathematics can be proved once we know this theorem.

2. Answer these questions without multiplying or dividing:

a $2{,}856 = 2 \times 2 \times 2 \times 3 \times 7 \times 17$

(i) Does 13 divide 2,856? Why?

(ii) Does 7 divide 2,856? Why?

(iii) Does 43 divide 2,856? Why?

(iv) Does 19 divide 2,856? Why?

b $1,911 = 3 \times 7 \times 7 \times 13$

(i) Does 7 divide 1,911? Why?

(ii) Does 13 divide 1,911? Why?

(iii) Does 11 divide 1,911? Why?

(iv) Does 17 divide 1,911? Why?

COMPOSITE FACTORS

1 Numbers *do* have factors besides their prime factors. For example, we said that

$12 = 2 \times 2 \times 3$

and that 12 has no other *prime* factors. But *6* is a factor of 12, because $12 = 6 \times 2$.

But 6 is not *prime*. If we factor 6, we get $6 = 3 \times 2$ and then $12 = (3 \times 2) \times 2$

$= 3 \times 2 \times 2$, which are the prime factors of 12 again.

Study 2 (**b**) above. We wrote

$1,911 = 3 \times 7 \times 7 \times 13.$

You know that 17 does not divide 1,911, because 17 is prime, and it is not in the list of prime factors of 1,911. We can also find out what *composite* numbers divide some other number. You will begin to learn how to do so in this section.

2 $231 = 3 \times 7 \times 11$

This means that 3 is a *factor* of 231.

There are two different ways in which we can find how many 3's there are in 231.

First method $\quad 3 \times \boxed{} = 231$

$231 \div 3 = 77$

Second method $\quad 231 = 3 \times 7 \times 11$

$= 3 \times (7 \times 11)$

$= 3 \times 77$

$130 = 2 \times 5 \times 13$

13 is a factor of 130.

We shall use both ways of finding how many 13's there are in 130.

First method $\quad 13 \times \boxed{} = 130$

$130 \div 13 = 10$

Second method $130 = 2 \times 5 \times 13$

$$\begin{aligned} 130 &= 2 \times 5 \times 13 \\ &= (2 \times 5) \times 13 \\ &= 10 \times 13 \end{aligned}$$

Do you see how we can do this kind of question easily once the number is factored?

Study this example:

$126 = 3 \times 7 \times 2 \times 3$

How many 7's are there in 126?

$$\begin{aligned} 126 &= 3 \times 7 \times 2 \times 3 \\ &= 7 \times (3 \times 2 \times 3) \\ &= 7 \times \ \ 18 \end{aligned}$$

There are **18** 7's in 126.

How many 3's are there in 126?

$$\begin{aligned} 126 &= 3 \times 7 \times 2 \times 3 \\ &= 3 \times (7 \times 2 \times 3) \\ &= 3 \times \ \ 42 \end{aligned}$$

There are **42** 3's in 126.

How many 2's are there in 126?

$$\begin{aligned} 126 &= 3 \times 7 \times 2 \times 3 \\ &= 2 \times (3 \times 7 \times 3) \\ &= 2 \times \ \ 63 \end{aligned}$$

There are **63** 2's in 126.

3. **a** $42 = 3 \times 2 \times 7$
   - (i) How many 7's are there in 42?
   - (ii) How many 2's are there in 42?
   - (iii) How many 3's are there in 42?

   b $105 = 3 \times 5 \times 7$
   - (i) How many 3's are there in 105?
   - (ii) How many 5's are there in 105?
   - (iii) How many 7's are there in 105?

   c $429 = 3 \times 11 \times 13$
   - (i) How many 3's are there in 429?
   - (ii) How many 11's are there in 429?
   - (iii) How many 13's are there in 429?

d $153 = 3 \times 3 \times 17$

(i) How many 3's are there in 153?

(ii) How many 17's are there in 153?

e Factor 44.

(i) How many 2's are there in 44?

(ii) How many 11's are there in 44?

f Factor 117.

(i) How many 3's are there in 117?

(ii) How many 13's are there in 117?

g Factor 588.

(i) How many 7's are there in 588?

(ii) How many 3's are there in 588?

(iii) How many 2's are there in 588?

4 $308 = 2 \times 2 \times 7 \times 11$

We could use this list of factors to find how many 2's or 7's or 11's there are in 308.

We can also use this list of factors to find how many 4's there are in 308.

$$\begin{aligned} 308 &= 2 \times 2 \times 7 \times 11 \\ &= (2 \times 2) \times (7 \times 11) \\ &= 4 \times (7 \times 11) \\ &= 4 \times 77 \\ &= 77 \times 4 \end{aligned}$$

There are **77** 4's in 308.

Could we use this list of factors to find how many 14's there are in 308?

Study this worked-out example:

$$\begin{aligned} 308 &= 2 \times 2 \times 7 \times 11 \\ &= (2 \times 7) \times 2 \times 11 \\ &= 14 \times 22 \\ &= 22 \times 14 \end{aligned}$$

There are **22** 14's in 308.

5. **a** Factor 182. How many 14's are there in 182?

   b Factor 441. How many 21's are there in 441?

   c Factor 1,430. How many 55's are there in 1,430?

d Factor 392. How many 8's are there in 392?
e Factor 2,057. How many 121's are there in 2,057?
f Factor 6,578. How many 26's are there in 6,578?
g Factor 216. How many 12's are there in 216?
h Factor 1,260. How many 18's are there in 1,260?
i Factor 1,375. How many 25's are there in 1,375?
j Factor 30,030. How many 130's are there in 30,030?

6 When we have factored a number, we can use this list of factors to find every factor of the number. We simply put the factors together in every possible way.

Study this example:

$28 = 2 \times 2 \times 7$

We build the full list of factors this way:

1—because 1 is a factor of every whole number
2—because $28 = 2 \times (2 \times 7)$
4—because $28 = (2 \times 2) \times 7$
7—because $28 = 7 \times (2 \times 2)$
14—because $28 = (7 \times 2) \times 2$
28—because every number divides itself

Then the only factors of 28 are 1, 2, 4, 7, 14, and 28.

We say, "The set of factors of 28 is {1, 2, 4, 7, 14, 28}."

Study this example:

$72 = 2 \times 2 \times 2 \times 3 \times 3$

Factors of 72:

1—because 1 divides every whole number
2—because $72 = 2 \times (2 \times 2 \times 3 \times 3)$
3—because $72 = 3 \times (2 \times 2 \times 2 \times 3)$
4—because $72 = (2 \times 2) \times 2 \times 3 \times 3$
6—because $72 = (2 \times 3) \times 2 \times 2 \times 3$
8—because $72 = (2 \times 2 \times 2) \times 3 \times 3$
9—because $72 = (3 \times 3) \times 2 \times 2 \times 2$
12—because $72 = (2 \times 2 \times 3) \times 2 \times 3$
18—because $72 = (2 \times 3 \times 3) \times 2 \times 2$
24—because $72 = (2 \times 2 \times 2 \times 3) \times 3$
36—because $72 = (2 \times 2 \times 3 \times 3) \times 2$
72—because every number divides itself

The set of factors of 72 is {1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72}.

Most people would not do all this writing. They would simply factor 72:

$72 = 2 \times 2 \times 2 \times 3 \times 3$

and then write down all the possible combinations, keeping track in their heads.

7. First factor each number given below; then write the set of all its factors: (The first one is done for you.)

a $54 = 2 \times 3 \times 3 \times 3$

The set of factors of 54 is {1, 2, 3, 6, 9, 18, 27 and 54}.

b	63	**c**	28	**d**	200	**e**	88	**f**	48
g	36	**h**	441	**i**	173	**j**	114	**k**	306
l	135	**m**	686	**n**	80	**o**	162	**p**	1,155
q	150	**r**	250	**s**	345	**t**	315	**u**	64
v	65	**w**	937	**x**	17,017	**y**	1,075	**z**	1,584

USING FACTORS TO DIVIDE

1 Sometimes we can use factoring to help with long division. Study this question:

$42\overline{)462}$ This is the same as $\frac{462}{42}$

$$
\begin{aligned}
462 &= 2 \times 231 \\
&= 2 \times 3 \times 77 \\
&= 2 \times 3 \times 7 \times 11 \\
42 &= 2 \times 3 \times 7
\end{aligned}
$$

$\frac{462}{42} = 11$ Check: $42 \times 11 = 462$

We did not need to divide at all!

2. Do these division questions, using the factoring method:

a $10\overline{)110}$	**b** $18\overline{)90}$	**c** $84\overline{)168}$
d $45\overline{)495}$	**e** $33\overline{)891}$	**f** $17\overline{)391}$
g $34\overline{)2346}$	**h** $300\overline{)2700}$	**i** $64\overline{)512}$
j $143\overline{)1001}$	**k** $231\overline{)30030}$	**l** $130\overline{)17290}$
m $17\overline{)3434}$	**n** $120\overline{)1440}$	**o** $234\overline{)18720}$
p $320\overline{)2880}$	**q** $400\overline{)64000}$	**r** $182\overline{)6370}$

COMMON FACTORS

1 We are often interested in *comparing* the factors of two different numbers.

For example:

a $30 = 3 \times 2 \times 5$. The set of factors is $\{\mathbf{1}, 2, \mathbf{3}, 6, 10, 15, 30\}$.
$21 = 3 \times 7$. The set of factors is $\{\mathbf{1}, \mathbf{3}, 7, 21\}$.
3 divides 30 and 3 divides 21.
So does 1, but 1 is a factor of all numbers.
We say that 3 is a **common divisor** of 30 and 21, or that 3 is a **common factor** of 30 and 21. 30 and 21 also have a common divisor of 1.

b $42 = 7 \times 2 \times 3$. The set of factors is $\{\mathbf{1}, \mathbf{2}, 3, 6, 7, 14, 21, 42\}$.
$110 = 11 \times 2 \times 5$. The set of factors is $\{\mathbf{1}, \mathbf{2}, 5, 10, 11, 22, 55, 110\}$
2 is a common factor of 42 and 110. Apart from 1, it is their *only* common factor. If we were finding a simpler name for the fraction $\frac{42}{110}$, we would remove the factor 2.

2. Find the common factors of these pairs of numbers:

a 6 and 21	**b** 15 and 35	**c** 63 and 56
d 55 and 77	**e** 15 and 55	**f** 33 and 15
g 51 and 57	**h** 39 and 65	**i** 34 and 51
j 46 and 23	**k** 63 and 92	**l** 16 and 28
m 20 and 34	**n** 49 and 77	**o** 39 and 52
p 95 and 57	**q** 253 and 161	**r** 35 and 49
s 20 and 24	**t** 20 and 30	**u** 56 and 96

3 Sometimes a pair of numbers has more than one common factor.

a Study this example carefully:
$18 = 2 \times 3 \times 3$. Its set of factors is $\{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{6}, 9, 18\}$.
$30 = 2 \times 3 \times 5$. Its set of factors is $\{\mathbf{1}, \mathbf{2}, \mathbf{3}, 5, \mathbf{6}, 10, 15, 30\}$.
18 and 30 have four common divisors.
1 is a common divisor of any two numbers.
2 is a common divisor of 18 and 30.
3 is a common divisor of 18 and 30.
6 is a common divisor because (2×3) divides 18
and (2×3) divides 30.

b Study this example:

List the common divisors of 42 and 63.

$42 = 2 \times 3 \times 7$. Its set of factors is $\{1, 2, \mathbf{3}, 6, \mathbf{7}, 14, \mathbf{21}, 42\}$.

$63 = 7 \times 3 \times 3$. Its set of factors is $\{1, \mathbf{3}, \mathbf{7}, 9, \mathbf{21}, 63\}$.

7 is a common divisor of 42 and 63.

3 is a common divisor of 42 and 63.

$(7 \times 3) = 21$; therefore, 21 is a common divisor of 42 and 63.

4. List all of the common divisors of each pair of numbers below: (The first question is done for you.)

a 30 and 75

$30 = 3 \times 5 \times 2$. Its set of factors is $\{1, 2, \mathbf{3}, \mathbf{5}, 6, 10, \mathbf{15}, 30\}$.

$75 = 3 \times 5 \times 5$. Its set of factors is $\{1, \mathbf{3}, \mathbf{5}, \mathbf{15}, 25, 75\}$.

1 is a common divisor of 30 and 75.

3 is a common divisor of 30 and 75.

5 is a common divisor of 30 and 75.

15 is a common divisor of 30 and 75.

b 42 and 12 **c** 16 and 28 **d** 27 and 99 **e** 75 and 50

f 70 and 110 **g** 24 and 36 **h** 16 and 88 **i** 26 and 34

GREATEST COMMON FACTOR

1 Often we are interested only in the *greatest* common divisor of two numbers.

For example, we found that 30 and 75 have common divisors of 1, 3, 5, and 15.

15 is the **greatest common divisor** of 30 and 75.

Another name for greatest common divisor is **greatest common factor.** Every pair of numbers has a greatest common divisor. Study these three examples:

a Find the greatest common divisor of 21 and 56.

$21 = 3 \times 7$

$56 = 2 \times 7 \times 2 \times 2$

The greatest common divisor of 21 and 56 is 7.

b Find the greatest common divisor of 35 and 32.

$35 = 7 \times 5$

$32 = 2 \times 2 \times 2 \times 2 \times 2$

The greatest common divisor of 35 and 32 is 1.

c Find the greatest common divisor of 32 and 40.

$32 = 2 \times 2 \times 2 \times 2 \times 2 \qquad = (2 \times 2 \times 2) \times 2 \times 2$

$40 = 2 \times 5 \times 2 \times 2 \qquad = (2 \times 2 \times 2) \times 5$

The greatest common divisor of 32 and 40 is $(2 \times 2 \times 2) = 8$.

2. Find the greatest common factor of these pairs of numbers:

a 17 and 18	**b** 20 and 25	**c** 16 and 32
d 45 and 75	**e** 100 and 250	**f** 21 and 93
g 117 and 65	**h** 19 and 20	**i** 64 and 81
j 52 and 70	**k** 40 and 52	**l** 65 and 75

RELATIVELY PRIME NUMBERS

1 What is the greatest common factor of 18 and 25?

$18 = 3 \times 2 \times 3$

$25 = 5 \times 5$

18 and 25 do not appear to have any common divisors.

Actually 1 divides them both, because 1 is a divisor of all numbers.

Pairs of numbers that have only 1 as a common divisor are said to be **relatively prime.**

2. Which of these pairs of numbers are relatively prime?

a 12 and 15	**b** 15 and 16	**c** 32 and 34
d 72 and 61	**e** 64 and 73	**f** 101 and 606
g 74 and 75	**h** 64 and 66	**i** 91 and 117
j 30 and 72	**k** 85 and 105	**l** 99 and 100
m 1 and 2,000	**n** 99 and 101	**o** 64 and 65

COMMON MULTIPLES

1 3 divides 15.

We say that 3 is a factor of 15.

At the same time, 15 is a multiple of 3.

The set of multiples of 3 is:

$\{(0 \times 3), (1 \times 3), (2 \times 3), (3 \times 3), (4 \times 3), (5 \times 3), \ldots\}$

that is, $\{0, 3, 6, 9, 12, 15, \ldots\}$

0 is a multiple of every number because:

$\square \times 0 = 0$

Usually we do not include 0 in the list of multiples of a number.

The set of multiples of 5 is:

$\{5, 10, 15, 20, 25, 30, 35, \ldots\}$

The set of multiples of 11 is:

$\{11, 22, 33, 44, 55, 66, \ldots\}$

The set of multiples of 1 is:

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \ldots\}$

2 If we look at two sets of multiples at once, we notice something interesting.

The set of multiples of 4: $\{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, \ldots\}$

The set of multiples of 3: $\{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, \ldots\}$

Copy out these two sets. Circle the numbers that appear in both sets.

We call these numbers **common multiples** of 4 and 3.

Every pair of numbers must have a sequence of common multiples.

Consider 7 and 11.

The set of multiples of 7 is $\{(1 \times 7), (2 \times 7), (3 \times 7), \ldots (9 \times 7), (10 \times 7), (11 \times 7), \ldots\}$.

The set of multiples of 11 is $\{(1 \times 11), (2 \times 11), (3 \times 11), (4 \times 11), (5 \times 11), (6 \times 11), (7 \times 11), \ldots\}$.

Note that in the set of multiples of 7 we must eventually come to 11×7, and in the set of multiples of 11 we must eventually come to 7×11. But 7×11 must equal 11×7. Why? We do not know for sure that 7×11 is the *least* common multiple of 7 and 11, but 7×11 must be *a* common multiple of 7 and 11.

In the same way, 13 and 17 must have 13×17 as *a* common multiple. 13 must divide (13×17); so (13×17) is a multiple of 13. 17 must divide (13×17); so (13×17) is a multiple of 17. In general, $(\square \times \triangle)$ is a common multiple of $\square$ and $\triangle$, but it may not be the *least* common multiple.

3. List 4 common multiples of each of these pairs of numbers:

a 5 and 7	**b** 6 and 9	**c** 3 and 8
d 2 and 3	**e** 7 and 10	**f** 12 and 8
g 10 and 20	**h** 6 and 8	**i** 10 and 12
j 5 and 10	**k** 9 and 12	**l** 7 and 11
m 15 and 20	**n** 17 and 34	**o** 16 and 18

LEAST COMMON MULTIPLE

1 There is no limit to the list of common multiples that two numbers can have, but there must be a *least* common multiple in every list.

Every pair of numbers has a *least* common multiple. The least common multiple of 4 and 3 is 12. The least common multiple of 8 and 12 is 24.

2. Find the least common multiple of each of these pairs of numbers:

a	6 and 8	**b**	5 and 6	**c**	7 and 14
d	15 and 10	**e**	12 and 18	**f**	8 and 10
g	9 and 12	**h**	6 and 9	**i**	8 and 9
j	6 and 12	**k**	5 and 15	**l**	15 and 30
m	20 and 30	**n**	11 and 12	**o**	3 and 3

3 Factoring can help us find the least common multiple of a pair of numbers.

$8 = 2 \times 2 \times 2$

$12 = 2 \times 2 \times 3$

Let us build a new number that has 8 as a factor and 12 as a factor.

Examine this number:

$2 \times 2 \times 2 \times 3$

8 is a factor of $2 \times 2 \times 2 \times 3$ because $(2 \times 2 \times 2)$ is a factor of $2 \times 2 \times 2 \times 3$.

12 is a factor of $2 \times 2 \times 2 \times 3$ because $(2 \times 2 \times 3)$ is a factor of $2 \times 2 \times 2 \times 3$.

$2 \times 2 \times 2 \times 3 = 24$

24 is the least common *multiple* of 8 and 12.

Study this question:

Find the least common multiple of 45 and 30.

$45 = 3 \times 3 \times 5$

$30 = 2 \times 3 \times 5$

Our common multiple must contain $3 \times 3 \times 5$. Why?

Our common multiple must also contain $2 \times 3 \times 5$. Why?

Can we build a list containing $3 \times 3 \times 5$ and also $2 \times 3 \times 5$?

Examine $2 \times 3 \times 3 \times 5$. Does this contain $3 \times 3 \times 5$? $2 \times 3 \times 5$?

Does 45 divide $2 \times 3 \times 3 \times 5$? How do you know?

Does 30 divide $2 \times 3 \times 3 \times 5$? How do you know?

$2 \times 3 \times 3 \times 5 = 90$. This is the least common multiple of 30 and 45.

4 To find the least common multiple of A and B, we write the shortest possible list of factors containing the factors of A and B.

Let us find the least common multiple of 30 and 105.

$30 = 2 \times 3 \times 5$

$105 = 3 \times 5 \times 7$

Our list must contain 2, 3 and 5. It must also contain 3, 5 and 7.

Consider $\overbrace{2\times3\times5}^{30}$ with $\underbrace{3\times5\times7}_{105}$, that is, $2\times3\times5\times7$

$2\times3\times5$ is in this list.

$3\times5\times7$ is also in this list.

There are no unnecessary factors.

Therefore, $2\times3\times5\times7$ must be the *least* common multiple of 30 and 105.

5. Find the least common multiple of the pairs of numbers given below: (The first one is done for you.)

a 42 and 70

$42=2\times3\times7$

$70=2\times5\times7$

least common multiple $= 2 \times 3 \times 5 \times 7$

$2\times3\times5\times7 \quad =210$

The least common multiple of 42 and 70 is 210.

b	12 and 20	**c**	16 and 12	**d**	18 and 42
e	99 and 66	**f**	40 and 30	**g**	24 and 20
h	35 and 40	**i**	28 and 30	**j**	49 and 98
k	33 and 55	**l**	60 and 48	**m**	36 and 32
n	45 and 72	**o**	108 and 90	**p**	40 and 50
q	13 and 20	**r**	24 and 36	**s**	15 and 14
t	30 and 77	**u**	77 and 98	**v**	32 and 64

6 If two numbers are relatively prime, their least common multiple will simply be their product.

For example:

$8=2\times2\times2$

$9=3\times3$

The least common multiple of 8 and 9 is:

$$\underbrace{2\times2\times2}_{8}\times\underbrace{3\times3}_{9}$$

Since there is no 'overlap' in factors, we end up multiplying the two numbers by each other.

7 We can find the least common multiple of three or more numbers. The set of multiples of 2 is {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, **30**, 32, 34, 36, 38, 40, 42, . . . }.

The set of multiples of 3 is {3, 6, 9, 12, 15, 18, 21, 24, 27, **30**, 33, 36, 39, . . . }.

The set of multiples of 5 is {5, 10, 15, 20, 25, **30**, 35, 40, 45, 50, . . . }.

30 is the least number common to the three sets.

Any three numbers must have a common multiple. For A, B, and C, $A \times B \times C$ must be *a* common multiple. Why?

We can build least common multiples using factors. Study these examples:

Find the least common multiples of 6, 10, and 15.

$6 = 2 \times 3$

$10 = 2 \times 5$

$15 = 3 \times 5$

The least common multiple $= 2 \times 3 \times 5$ because:

2×3 divides $2 \times 3 \times 5$

and 2×5 divides $2 \times 3 \times 5$

and 3×5 divides $2 \times 3 \times 5$

$2 \times 3 \times 5 = 30$

Therefore, the least common multiple of 6, 10, and 15 is 30.

8. Find the least common multiples of these numbers:

a 4, 6, and 10	**b** 9, 12, and 15	**c** 3, 7, and 5
d 10, 15, and 35	**e** 12, 15, and 18	**f** 15, 20, and 25
g 20, 30, and 50	**h** 6, 9, and 20	**i** 8, 20, and 25
j 18, 24, and 27	**k** 40, 50, and 60	**l** 18, 33, and 45
m 8, 10, and 12	**n** 12, 15, and 20	**o** 24, 30, 36, and 42

9. **a** Write 7 different multiples of 9.
   b Write 7 different factors of 100.
   c Name the first three primes larger than 80.
   d Factor 240.
   e Factor 600.
   f Write $\frac{24}{88}$ in lowest terms.
   g Write $\frac{34}{126}$ in lowest terms.
   h Find the missing factors: $\square \times \triangle \times \diamond \times 5 = 1715$

i Find the missing factors: $234 = \square \times \triangle \times 2 \times 13$

j Name all the factors of 30.

k Name all the factors of 40.

l Name all the factors of 56.

m Name all the factors of 47.

n Name all the factors of 84.

o Name all the factors of 300.

p Name all the factors of 70.

q $25\overline{)475}$

r $21\overline{)147}$

s $21\overline{)77}$

t Find the least common multiple of 36 and 60.

u Find the least common multiple of 12 and 18.

v Find the least common multiple of 18 and 27.

w Find the least common multiple of 24 and 18.

x Find the least common multiple of 12, 18,and 27.

EXPONENTS

1

When we wrote expanded numerals such as '$10 \times 10 \times 10$' we may have found the process a little tiring. There is a short way of writing down $10 \times 10 \times 10$. It is 10^3. We read this as *10 to the exponent 3.* This means that we have to put down ten 3 times and multiply the three 10's together, or it means that we use 10 as a factor 3 times.

A short way of writing $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ is 10^7. For this we say *10 to the exponent 7.* It means use 10 as a factor 7 times.

2 What does 10^4 mean? It means use 10 as a factor 4 times; so 10^4 means $10 \times 10 \times 10 \times 10$, or 10,000.

3 When we write symbols like 10^9, we call 10 the **base** and 9 the **exponent** or **index.** This way of writing numerals is called **exponential notation.**

4 For $10 \times 10 \times 10 \times 10$ we write 10^4 and we say, *10 to the fourth* or *the fourth* **power** *of 10.* For $10 \times 10 \times 10$ we write 10^3. For 10×10 we write 10^2. What can we write for 10? For the number 10, we shall continue to write the numeral 10, and for the number 1 we shall write '1.'

Thus 10^4 is called a **power**, the 10 being the base and the four being the exponent or index.

6. Write the following as expanded numerals employing exponential notation. (The first is done for you.)

a $305 = (3 \times 10^2) + (0 \times 10) + (5 \times 1)$

b 392 **c** 580 **d** 786 **e** 1,358

f 10,084 **g** 720,316 **h** 265,000 **i** 1,000,000

EXPONENTS TO OTHER BASES

1 What can we write for 5×5? Here we see that we use the factor 5 twice. If we think of 5 as the base, we can write $5 \times 5 = 5^2$. For $6 \times 6 \times 6$ we could write 6^3.

2. Write the following using exponential notation:

a $4 \times 4 \times 4$ **b** $5 \times 5 \times 5 \times 5 \times 5$ **c** $7 \times 7 \times 7 \times 7$

d 9×9 **e** $26 \times 26 \times 26$ **f** $19 \times 19 \times 19$

g $10 \times 10 \times 10 \times 10$ **h** $1 \times 1 \times 1 \times 1 \times 1$ **i** $6 \times 6 \times 6 \times 6$

3. Write the following as expanded numerals: (The first one is done for you.)

a $7^3 = 7 \times 7 \times 7$ **b** 5^2 **c** 11^3 **d** 9^4

e 8^2 **f** 7^5 **g** 3^4 **h** 18^3

i 26^2 **j** 19^3 **k** 20^4 **l** 4^4

4. Write the following using decimal numeration: (The first one is done for you.)

a $2^3 = 2 \times 2 \times 2 = 8$ **b** 4^4 **c** 3^3 **d** 7^2

e 2^6 **f** 3^5 **g** 10^4 **h** 11^2

i 12^3 **j** 9^3 **k** 13^2 **l** 6^4

5. Write the following as the products of factors and then write them using exponential notation: (The first is done for you.)

a $16 = 4 \times 4 = 4^2$, or $16 = 2 \times 2 \times 2 \times 2 = 2^4$

b 49 **c** 32 **d** 64 **e** 121

f 36 **g** 27 **h** 125 **i** 81

6 In the last unit we learned ways of naming numerals written in exponential notation (powers). We can use this method for naming powers that have different bases from the base of 10. For 4^6 we can say "4 to the sixth" or "the sixth power of four." The important thing is to know what is meant by numerals like 4^6, 2^5, 10^3 and so on.

SQUARE ROOTS BY FACTORING

1 We use another symbol to express multiplication of several equal factors.

We write $9 \times 9 \times 9$ as 9^3. (This means 9 used as a factor 3 times.)

We write $4 \times 4 \times 4 \times 4 \times 4$ as 4^5. (This means 4 used as a factor 5 times.)

We write 11×11 as 11^2. (This means 11 used as a factor twice.)

In general, $\square^2$ means $\square \times \square$.

When we multiply a number by itself, we sometimes say that we are *squaring* the number.

$$3^2 = 3 \times 3 = 9$$

We say, "3 *squared* equals nine."

2. Oral exercise. Find:

a 2^2	**b** 7^2	**c** 10^2	**d** 6^2	**e** 4^2	**f** 3^2
g 11^2	**h** 12^2	**i** 8^2	**j** 5^2	**k** 1^2	**l** 20^2

3 We can reverse this procedure.

Given a number, we can look for its *square root*.

For example $\square \times \square = 25$

Clearly, *5* makes this open sentence true.

5 is the *square root* of 25.

4. What are the square roots of these numbers?

a 4	**b** 25	**c** 81	**d** 49
e 36	**f** 1	**g** 100	**h** 64
i 144	**j** 121	**k** 0	**l** 900
m 625	**n** 169	**o** 400	**p** 225

5 We can use factors to help us find square roots.

Study these examples:

a Find the square root of 225.

$$\begin{aligned} 225 &= 5 \times 5 \times 3 \times 3 \\ &= 5 \times 3 \times 5 \times 3 \\ &= (5 \times 3) \times (5 \times 3) \\ &= 15 \times 15 \end{aligned}$$

The square root of 225 is 15.

b Find the square root of 144.

$$\begin{aligned}144 &= 2\times2\times2\times2\times3\times3\\ &= 2\times2\times3\times2\times2\times3\\ &= (2\times2\times3)\times(2\times2\times3)\\ &= 12\times12\end{aligned}$$

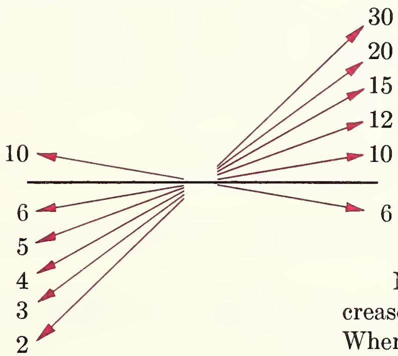
Therefore the square root of 144 is 12.

We rearranged the factors of 144 so as to get two groups the same.

6. Use factors to find the square roots of these numbers:

a 225	**b** 256	**c** 625
d 81	**e** 64	**f** 1,089
g 576	**h** 289	**i** 1,225
j 1,600	**k** 2,500	**l** 1,024
m 1,764	**n** 2,304	**o** 441
p 1,296	**q** 1,521	**r** 4,900

7 Examine this table:



$2\times30=60$
$3\times20=60$
$4\times15=60$
$5\times12=60$
$6\times10=60$

Notice that as one factor increases, the other factor becomes less. Where will the increasing and the decreasing factors meet? At the square root! The square root of 60 is not a whole number, but it must be between 6 and 10. Why? Why will it be a little less than 8?

If we have written a number as the product of two different factors, one of these factors must be less than the square root of the number and the other greater than the square root of the number.

For example, 100 can be written as the product of two factors in several ways.

$2\times50=100 \qquad 5\times20=100$
$4\times25=100 \qquad 10\times10=100$

So long as the two factors are different, one must be smaller than the square root and the other must be larger.

8 This fact is useful in testing whether or not a number is prime. For example, suppose we are asked whether or not 101 is prime. The square root of 101 is about 10.

We do not need to try any factors above 10, because if we were to find a factor greater than 10, there would have to be a factor less than 10 to go with it.

2 does not divide 101
3 does not divide 101
4 does not divide 101
5 does not divide 101
6 does not divide 101
7 does not divide 101
8 does not divide 101
9 does not divide 101
10 does not divide 101

We conclude that 101 is prime.

9. Which of these numbers are prime?

a 83	**b** 49	**c** 91	**d** 67	**e** 97
f 71	**g** 117	**h** 99	**i** 57	**j** 127

10 We have still been doing too much work. We have been trying some unnecessary factors.

For example, there is never any use trying 4. If 4 divides a number, so does 2. This is easy to see. If 4 is a factor of a number, we can write the number as:

$$4 \times \square \times \triangle \times \dots$$

or $$2 \times 2 \times \square \times \triangle \times \dots$$

If we have already tried 2, and it doesn't work, there is no hope for 4. In the same way, there is no hope for 6 or 8 or 10, or any multiple of 2. Why? This leads us to the next step. If we have tried 3 and have found that it is not a factor of a number, there is no hope for 6 or 9 or 12 or any multiple of 3. Why?

In fact, if we try the primes, there is no point in trying any of their multiples. Why? A knowledge of this fact makes our work much easier. For example, if we are asked whether or not 223 is prime, we try:

2: 2 does not divide 223
3: 3 does not divide 223
5: 5 does not divide 223
7: 7 does not divide 223
11: 11 does not divide 223
13: 13 does not divide 223
17 is past the square root of 223 because

$15 \times 15 = 225$

We conclude, therefore, that 223 is prime.

11. Which of these numbers are prime?

a 331	**b** 219	**c** 309	**d** 771
e 61	**f** 411	**g** 191	**h** 121
i 169	**j** 171	**k** 143	**l** 251
m 1,003	**n** 217	**o** 289	**p** 51
q 721	**r** 607	**s** 11,011	**t** 222

12. Find the first 5 primes larger than 200. Check your work carefully. Did you find that this was a lot of work? People have worked out lists of primes that go far past 1,000,000. Fortunately, we now have machines to do that kind of arithmetic.

CHAPTER TEST

1. What are the prime factors of 1,001?

2. How many factors has 36?

3. Name the first 6 primes past 400.

4. Factor these numbers:

   a 72 **b** 45 **c** 250 **d** 720

5. Find the missing factors in the following:

   a $637 = 7 \times 7 \times \square$ **b** $8{,}349 = 3 \times \square \times 11 \times 23$

   c $2{,}401 = \square \times 7 \times 7 \times 7$ **d** $32{,}395 = 5 \times 11 \times 19 \times \square$

6. Given that $47{,}775 = 3 \times 5 \times 5 \times 7 \times 7 \times 13$:

   a Does 13 divide 47,775? Why?

   b Does 17 divide 47,775? Why?

   c Does 21 divide 47,775? Why?

   d Does 105 divide 47,775? Why?

7. Use factoring to find answers to these division questions:

a $35\overline{)665}$ **b** $42\overline{)2646}$

c $28\overline{)1092}$ **d** $40\overline{)5000}$

8. Find the greatest common divisors of these numbers:

a 60 and 72 **b** 105 and 180 **c** 68 and 170

d 175 and 375 **e** 98, 154, and 182 **f** 54, 144, 360, and 216

9. Find the least common multiples of these numbers:

a 10 and 14 **b** 16 and 20 **c** 14 and 21

d 16, 20, and 24 **e** 14, 21, and 15 **f** 30, 42, and 56

10. Use factors to find the square roots of these numbers:

a 36 **b** 400 **c** 1,764 **d** 784

e 3,136 **f** 1,849 **g** 1,296 **h** 3,600

11. Write the sets of all the factors of 1,008 and 792. What are the common factors of these numbers? What is the greatest common factor of these two numbers?

12. Which of the following pairs of numbers are numbers that are prime to each other?

a 35 and 26 **b** 121 and 63

c 372 and 153 **d** 560 and 425

e 111 and 222 **f** 405 and 117

g 143 and 133 **h** 203 and 253

i 1,296 and 1,081 **j** 307 and 169

4 Numeration Systems

ANOTHER LOOK AT THE DECIMAL NUMERATION SYSTEM

1. In chapter 1 we learned about the decimal numeration system. What is the place value of each '3' in the decimal numerals below?

   a 134 **b** 2,013 **c** 3,014,156 **d** 2,316,971 **e** 3

   f 132,459 **g** 63,017 **h** 1,388 **i** 300 **j** 134,006

2. Write expanded numerals for the following decimal numerals. The first is done for you.

   a $4{,}019 = (4\times 1{,}000) + (1\times 10) + (9\times 1)$

   b 2,315 **c** 91,216 **d** 420,000 **e** 1,000,100

   f 4,296,327 **g** 64 **h** 200,153 **i** 53,164,232

3. Write expanded numerals for the following decimal numerals using exponential numeration. The first is done for you.

   a $$\begin{aligned} 15{,}329 &= (1\times 10{,}000) + (5\times 1{,}000) + (3\times 100) + (2\times 10) + (9\times 1) \\ &= (1\times 10^4) + (5\times 10^3) + (3\times 10^2) + (2\times 10) + (9\times 1) \end{aligned}$$

   b 416 **c** 1,095 **d** 17,325 **e** 416,833

   f 6,014,037 **g** 2,098,637 **h** 10,000,000 **i** 2,014,937

4 In the decimal system we group numbers into tens, groups of ten tens, groups of ten ten-tens and so on. We say that 10 is the *base* for the decimal numeration system.

ANOTHER BASE

1 We have seen that the numeration system in most common use is called the *Decimal Numeration System* because we think in terms of tens and groups of ten. Consider the numeral '365'. What number does it represent?

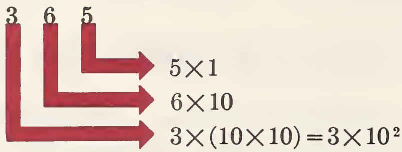
3 6 5

represents 5 ones

represents 6 groups of ten

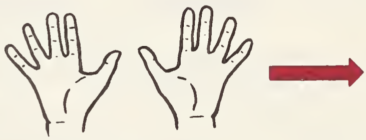
represents 3 groups of ten tens

For 5 we can write 5×1. For 6 groups of ten we can write 6×10. What can we write for 3 groups of ten tens using 10 as a base?

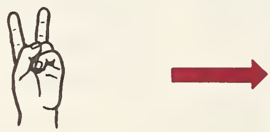


How many digits do we need for our decimal numeration system? What are they?

2 **a** Some people think that 10 was chosen as a base in the decimal numeration system because we have ten fingers. We still use our hands sometimes to indicate a number to someone. We can indicate the number 12 by first holding up two hands to represent 10 and then two fingers to represent 2 more.



represents 10

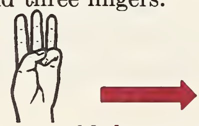


represents 2

For 35, we could show both hands extended 3 times to represent 30 and then one hand extended to represent 5 more.

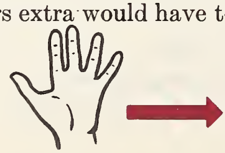
b How could you use your hands to represent: 40 27 36 100

3 **a** Suppose we used only one hand to represent a number. To show 3, we could extend three fingers.



represents 3

b To represent 7 we could show one hand extended to indicate 5. How many fingers extra would have to be extended to show 7?



one hand—represents 5

 two fingers—represent 2

c We could now invent a method of using digits to represent numbers based on the idea of 5.

For 7, we could write:

1 2

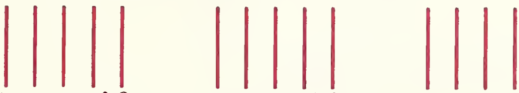
represents 2 ones

represents 1 group of five

In order that this numeral might not be confused with the decimal numeral for 12, we could write it this way: 12_{five} or 12_5.

The small numeral we call a **subscript**. Again, we must have a way of saying 12_5 that will not confuse us. We can agree to say "One-two, base five" for 12_5.

4 Below, each stroke represents a pencil. How could we write a numeral to represent this number of pencils?



We could write 24_5 and say "Two-four, base five".

5 **a** We saw that when we used base 10 numerals, we needed 10 digits to represent numbers. How many digits do you think we shall need when we use base 5 numerals?

b

| | → 2_5. → 2 ones

| | | | → 4_5. → 4 ones

| | | | | → 10_5

0 ones

1 five

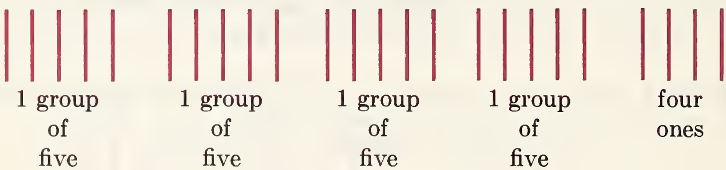
c We would count, using base five numerals:
"one, two, three, four, one-zero, one-one, . . ."
and we would write: 1_5, 2_5, 3_5, 4_5, 10_5, 11_5, . . ."

6. Write the following decimal numerals as base five numerals:
12; 20; 2; 19; 3; 23; 24; 16; 14; 15

7. Write decimal numerals for each of these base five numerals:
12_5; 23_5; 4_5; 30_5; 34_5; 24_5; 14_5; 42_5; 44_5; 10_5

8. Write expanded numerals for each base five numeral in exercise 7.
Here is how we would write an expanded numeral for 12_5:
$12_5=(1\times5)+2$

9 Write an expanded numeral for 44_5.
We have: $(4\times5)+(4\times1)$.
Add 1_5 to 44_5.
We can picture 44 as



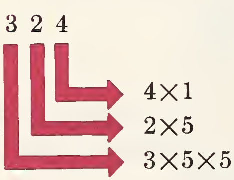
When 1 more is added, we have

We now have five groups of five or 5×5.

Just as we needed an extra column to the left of the 10's column in decimal numeration to show 10 tens, so we need an extra column to the left of the 5's column to show 5 fives.

For 5 fives we can write: 100_5.

10 Write 324_5 as an expanded numeral.



5×5's	5's	1's
3	2	4

$324_5=(3\times5\times5)+(2\times5)+(4\times1)$

What can we write for 5×5 in exponential numeration? $5 \times 5 = 5^2$

For 324_5 we can write: $(3 \times 5^2) + (2 \times 5) + (4 \times 1)$.

11. Now write the following numerals as expanded numerals using base 5 and exponents:

    413_5; 203_5; 100_5; 440_5; 343_5; 402_5

12. Write the base 5 numerals in exercise 11 as decimal numerals.

    Example: $413_5 = (4 \times 5^2) + (1 \times 5\ ) + (3 \times 1)$
    $5^2 = 25$; $1 \times 5 = 5$; $3 \times 1 = 3$

    So $413_5 = (4 \times 25) + (1 \times 5) + (3 \times 1)$
    $= 100 + 5 + 3$
    $= 108$

13. We are now able to show the place value of any digit in a base 5 numeral. Study the place value chart below:

$(5 \times 5 \times 5 \times 5)$'s 5^4	$(5 \times 5 \times 5)$'s 5^3	(5×5)'s 5^2	5's 5	1's 1

14. Write decimal numerals for:

    5^2; 5^3; 5^4; 5^5; 5^6; 5^7

15. Using exponents, write the following as expanded numerals:

    a 1010_5 **b** 3432_5 **c** 4000_5 **d** 1230_5
    e 40134_5 **f** 10000_5 **g** 32103_5 **h** 20202_5

    Example: $1010_5 = (1 \times 5^3) + (0 \times 5^2) + (1 \times 5\ ) + (0 \times 1\ )$

16. Write decimal numerals for each of the base 5 numerals in exercise 15.

    Example: $1010_5 = (1 \times 5^3) + (0 \times 5^2) + (1 \times 5\ ) + (0 \times 1\ )$
    $= (1 \times 5 \times 5 \times 5) + 0 + (1 \times 5) + 0$
    $= 125 + 5$
    $= 130$

17 Write a base 5 numeral for 173.

We have seen that:

$5^2=25;\ 5^3=125;\ 5^4=625;\ \ldots$

a Does 173 contain 625?

b Does 173 contain 125? How many times?
What remainder do we have when we take 125 from 173?
$173=(1\times125)+48$

c Examine the remainder. Does it contain 25? How many times?
What remainder do we have when we take 25 from 48?
$173=(1\times125)+(1\times25)+23$

d Does 23 contain 5? How many times? What remainder do we have when we take 20 from 23?
$173=(1\times125)+(1\times25)+(4\times5)+3$

e What can we write for 125, using exponents and 5 as the base? What can we write for: 25? 5? 4×5? 3?

$$\begin{array}{lll}
1\times125 & =1\times5\times5\times5 & =1\times5^3 \\
1\times25 & =1\times5\times5 & =1\times5^2 \\
4\times5 & & =4\times5 \\
3 & & =3\times1
\end{array}$$

For 173 we can write $(1\times5^3)+(1\times5^2)+(4\times5\ )+(3\times1\ )$.
Write this expanded numeral as a base 5 numeral. We have:

$(1\times5^3)+(1\times5^2)+(4\times5\ )+(3\times1\ )=1143_5$

So $173\ =\ 1\ 1\ 4\ 3_5$

18. Write base 5 numerals for:

a 46	**b** 79	**c** 214	**d** 125
e 35	**f** 696	**g** 720	**h** 1,047
i 1,216	**j** 2,967	**k** 2,149	**l** 3,000

19. Show that $0_5=0;\ 1_5=1;\ 2_5=2;\ 3_5=3;\ 4_5=4$.

In future, we shall write the numerals 0_5, 1_5, 2_5, 3_5, 4_5, as 0, 1, 2, 3, and 4 respectively, unless there is a possibility of misunderstanding what the numerals represent.

OPERATIONS WITH BASE FIVE NUMERALS

1. Add. Write the sums as base 5 numerals.

a	0+0	**f**	1+0	**k**	1+2	**p**	4+1	**u**	3+2
b	0+1	**g**	2+0	**l**	1+3	**q**	2+2	**v**	3+3
c	0+2	**h**	3+0	**m**	1+4	**r**	2+3	**w**	3+4
d	0+3	**i**	4+0	**n**	2+1	**s**	2+4	**x**	4+3
e	0+4	**j**	1+1	**o**	3+1	**t**	4+2	**y**	4+4

2. We can write these *addition facts* of base five numerals as a table. Copy the table below and complete it.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10_5
2					
3			10_5		
4					13_5

The coloured arrows show us how to find the sum 3+2. It shows that $3+2=10_5$.

3. Find the sum of 32_5 and 11_5.
32_5 means 3 fives plus 2 ones.
11_5 means 1 five plus 1 one.
Now add:
How many ones do 2 ones and 1 one make?
How many fives do 3 fives and 1 five make?

32_5 How many ones are there?
$\underline{11_5}$ How many fives are there?
43_5

4. Add the following:

a 20_5	**b** 33_5	**c** 12_5	**d** 23_5	**e** 12_5
14_5	10_5	32_5	22_5	21_5

5 Add: 34_5+13_5.

5 fives 5×5's	Fives 1×5's	1's
	3	4
	1	3
1	0	2

a Add the ones.

Using the table, we have $4_5+3_5=12_5$

one five — two ones

b Put 2 in the *ones'* column and *carry* the one five to the *fives'* column.

c Add the fives:

1 (that has been carried)$+3=4$

$4+1=10$ (using the tables)

d Put 0 in the *fives'* column and *carry* 1 to the 5 *fives'* column. We can show how to work this more clearly by using decimal numerals.

e $4+3=7$ $\quad 7=(1\times5)+2$

Put 2 in the *ones'* place and *carry* 1 five to the *fives'* column.

f $1+3+1=5$. We have 5 fives $=5\times5$.

g Put 1 in the (5×5) column. There are no extra fives; therefore, put a zero in the (1×5) column.

h Here is another way in which we may think of the computation:

$34_5=3$ fives$+4$ ones
$13_5=1$ five $+3$ ones

4 fives$+7$ ones $=4$ fives$+1$ five$+2$ ones
$=5$ fives$+2$ ones
$=1$ five-fives$+2$ ones$=102_5$

6. Add:

23_5	24_5	31_5	33_5	14_5	32_5
13_5	24_5	24_5	14_5	41_5	44_5

7 Add: 44_5+21_5. Check by changing base 5 numerals to decimal numerals.

$$\begin{array}{r} 44_5 \\ \underline{21_5} \\ 120_5 \end{array}$$

a $4_5+1_5=10_5$
Put 0 in the *ones'* column.
Carry 1 to the *fives'* column.

b $1_5+4_5=10_5$, $10_5+2_5=12_5$. Put 2 in the *fives'* place and 1 in the 5 *fives'* place.

Check: $44_5=(4\times5)+(4\times1)=20+4=24$
$21_5=(2\times5)+(1\times1)=10+1=\underline{11}$
35

Add 24 and 11. We have:

$$\begin{aligned} 35=25+10&=(1\times5\times5)+(2\times5)+(0\times1)\\ &=(1\times5^2)+(2\times5\ )+(0\times1\ )\\ &=120_5 \end{aligned}$$

8. Add the following: (Check by changing base 5 numerals to decimal numerals.)

32_5	14_5	44_5	31_5	32_5	11_5
$\underline{34_5}$	$\underline{43_5}$	$\underline{22_5}$	$\underline{32_5}$	$\underline{34_5}$	$\underline{34_5}$

9. Study the examples below:

a $142_5=100_5+40_5+2_5$
$\underline{232_5}=\underline{200_5+30_5+2_5}$
$300_5+120_5+4_5=424_5$

b $142_5=(1\times5^2)+(4\times5\ )+(2\times1\ )=25+20+2=\ 47$
$232_5=(2\times5^2)+(3\times5\ )+(2\times1\ )=50+15+2=\ \underline{67}$
114

$114=(4\times25)+(2\times5)+(4\times1)=(4\times5^2)+(2\times5\ )+(4\times\ 1)=424_5$

10. Work each of the addition examples below in two different ways:

132_5	233_5	422_5	203_5	142_5
$\underline{241_5}$	$\underline{104_5}$	$\underline{343_5}$	$\underline{332_5}$	$\underline{303_5}$
242_5	211_5	123_5	303_5	444_5
$\underline{142_5}$	$\underline{243_5}$	$\underline{213_5}$	$\underline{402_5}$	$\underline{311_5}$

11. Add. Write all the answers for the following, using decimal numerals: (The first is done for you.)

a
$$\begin{array}{r} 1223_5 \\ 2414_5 \\ \hline 4142_5 \end{array}$$
$$\begin{aligned} 4142_5 &= (4\times125)+(1\times25)+(4\times5)+(2\times1) \\ &= 500+25+20+2 \\ &= 547 \end{aligned}$$

b $\begin{array}{r} 4012_5 \\ 1143_5 \\ \hline \end{array}$ c $\begin{array}{r} 3312_5 \\ 2402_5 \\ \hline \end{array}$ d $\begin{array}{r} 3221_5 \\ 4432_5 \\ \hline \end{array}$ e $\begin{array}{r} 4104_5 \\ 2342_5 \\ \hline \end{array}$

f $\begin{array}{r} 4122_5 \\ 3424_5 \\ \hline \end{array}$ g $\begin{array}{r} 2332_5 \\ 4242_5 \\ \hline \end{array}$ h $\begin{array}{r} 1014_5 \\ 1434_5 \\ \hline \end{array}$ i $\begin{array}{r} 2314_5 \\ 4024_5 \\ \hline \end{array}$

j $\begin{array}{r} 2303_5 \\ 1312_5 \\ \hline \end{array}$ k $\begin{array}{r} 2403_5 \\ 1410_5 \\ \hline \end{array}$ l $\begin{array}{r} 3202_5 \\ 2333_5 \\ \hline \end{array}$ m $\begin{array}{r} 1224_5 \\ 3214_5 \\ \hline \end{array}$

12 Addition with base 5 numerals is much like addition with decimal numerals. When we add base 5 numerals, we *carry* fives, twenty-fives, one hundred twenty-fives, etc. instead of tens, hundreds, thousands, etc.

13. Copy and complete:

a $2_5+2_5=4_5$ so $4_5-2_5=\square$ b $1_5+3_5=4_5$ so $4_5-3_5=\square$

c $2_5+3_5=10_5$ so $10_5-2_5=\square$ d $4_5+4_5=13_5$ so $13_5-4_5=\square$

14 We have learned that subtraction is the inverse operation to addition. We can use this fact to help us with subtraction.

a $12_5-3_5=4_5$ because $12_5=4_5+3_5$

Now copy and complete:

b $4_5-1_5=\square$ because $4_5=\square+1_5$

c $11_5-1_5=\square$ because $11_5=\square+1_5$

d $13_5-4_5=\square$ because $13_5=\square+4_5$

e $12_5-3_5=\square$ because $12_5=\square+3_5$

f $11_5-\square=4_5$ because $11_5=4_5+\square$

g $12_5-\square=3_5$ because $12_5=3_5+\square$

h $10_5-\square=2_5$ because $10_5=2_5+\square$

i $\square-3_5=2_5$ because $\square=2_5+3_5$

j $\square-4_5=2_5$ because $\square=2_5+4_5$

k $\square-3_5=4_5$ because $\square=4_5+3_5$

15 Study the examples below:

a $\begin{array}{r} 43_5 \\ -21_5 \\ \hline 22_5 \end{array}$

b $$\begin{array}{rllll} 31_5 & = 30_5+1_5 & = 20_5+10_5+1_5 & = 20_5+11_5 & \\ -12_5 & = 10\ +2_5 = & 10_5+2_5 & = 10_5+\ 2_5 & \\ \hline & & & 10_5+\ 4_5 & = 14_5 \end{array}$$

c $$\begin{array}{rllll} 323_5 & = 300_5+20_5+3_5 & = 300_5+10_5+13_5 & \\ -104_5 & = 100_5 \qquad +4_5 & = 100_5 \qquad +\ 4_5 & \\ \hline & & 200_5+10_5+\ 4_5 & = 214_5 \end{array}$$

d Explain each step in the subtraction examples above. Prove that the answers are correct by doing the inverse operation to subtraction.

e Here is another way in which we may think of computing subtraction examples:

$$\begin{array}{rll} 31_5 & = 3 \text{ fives}+1 \text{ one} & = 2 \text{ fives}+1 \text{ five}+1 \text{ one} = 2 \text{ fives}+6 \text{ ones} \\ 12_5 & = 1 \text{ five} \ +2 \text{ ones} = & 1 \text{ five} \ +2 \text{ ones} \\ \hline & & 1 \text{ five} \ +4 \text{ ones} \end{array}$$

1 five+4 ones $=14_5$.

16. Subtract:

a $\begin{array}{r} 113_5 \\ 104_5 \\ \hline \end{array}$ **b** $\begin{array}{r} 432_5 \\ 141_5 \\ \hline \end{array}$ **c** $\begin{array}{r} 204_5 \\ 122_5 \\ \hline \end{array}$ **d** $\begin{array}{r} 313_5 \\ 204_5 \\ \hline \end{array}$

e $\begin{array}{r} 202_5 \\ 123_5 \\ \hline \end{array}$ **f** $\begin{array}{r} 300_5 \\ 101_5 \\ \hline \end{array}$ **g** $\begin{array}{r} 214_5 \\ 141_5 \\ \hline \end{array}$ **h** $\begin{array}{r} 400_5 \\ 333_5 \\ \hline \end{array}$

i $\begin{array}{r} 103_5 \\ 14_5 \\ \hline \end{array}$ **j** $\begin{array}{r} 213_5 \\ 213_5 \\ \hline \end{array}$ **k** $\begin{array}{r} 403_5 \\ 134_5 \\ \hline \end{array}$ **l** $\begin{array}{r} 203_5 \\ 104_5 \\ \hline \end{array}$

17 Subtract: (Check: (i) by using decimal numerals, (ii) by adding.)

a $\begin{array}{r} 401_5 \\ 132_5 \\ \hline 214_5 \end{array}$ $$\begin{array}{l} 401_5 = (4\times 25)+(0\times 5)+1 = 100+\ 0+1 = 101 \\ 132_5 = (1\times 25)+(3\times 5)+2 = \ 25+15+2 = \ \underline{\ 42} \\ \hspace{8cm} 59 \end{array}$$

$$59 = (2\times 25)+(1\times 5)+4 = (2\times 5^2)+(1\times 5\ )+(4\times 1\ ) = 214_5$$

The answer is correct.

b $401_5 - 132_5 = 214_5$ means $401_5 = 214_5 + 132_5$

$\begin{array}{r} 214_5 \\ 132_5 \\ \hline 401_5 \end{array}$ The answer is correct.

18. Work the following: (Check: (i) by using decimal numbers, (ii) by adding.)

a $\begin{array}{r} 31_5 \\ -4_5 \\ \hline \end{array}$	b $\begin{array}{r} 224_5 \\ -33_5 \\ \hline \end{array}$	c $\begin{array}{r} 4043_5 \\ -2024_5 \\ \hline \end{array}$	d $\begin{array}{r} 3123_5 \\ -2212_5 \\ \hline \end{array}$
e $\begin{array}{r} 4132_5 \\ -2013_5 \\ \hline \end{array}$	f $\begin{array}{r} 2123_5 \\ -224_5 \\ \hline \end{array}$	g $\begin{array}{r} 3000_5 \\ -121_5 \\ \hline \end{array}$	h $\begin{array}{r} 1000_5 \\ -214_5 \\ \hline \end{array}$
i $\begin{array}{r} 1020_5 \\ -123_5 \\ \hline \end{array}$	j $\begin{array}{r} 4000_5 \\ -1111_5 \\ \hline \end{array}$	k $\begin{array}{r} 2344_5 \\ -1344_5 \\ \hline \end{array}$	l $\begin{array}{r} 3413_5 \\ -2424_5 \\ \hline \end{array}$

BASE SEVEN NUMERALS

1 We have seen how we can group numbers of objects into groups of 5, groups of (5×5) and so on. This kind of grouping has enabled us to write numerals using 5 as the base. We learned that we would need five digits in this numeration system, the digits being 0, 1, 2, 3, and 4.

2 Consider the number of X's below:

X X X X X X X X X X

Into how many groups of seven can we arrange this group? How many will be left over? We have:

(X X X X X X X) X X X

1 group of 7   3 left over

How might we write a numeral to represent this number if we use 7 as the base? We could write: 13. To make sure that we understand that this numeral uses seven as the base, we would write: 13_7.

13_7

1 group of 7   3 ones

3. Write the following in groups of seven and use a base seven numeral to represent the number of X's we have:

X X X X X X X X X X X X X X X X X X X

4. How many digits did we use in the decimal numeration system? How many digits did we use in the base five numeration system? How many digits do you think we shall need to form a base seven numeration system?

5. Consider the numeral 426_7. In which base is this numeral written?

What does the '6' represent?

What does the '2' represent?

What do you think the '4' represents?

What place value will there be in the column to the left of the '4'?

6 Here is how our base seven numeration system place value chart will look:

Seven groups of seven sevens $(7\times7\times7)$ 7^3	Seven sevens (7×7) 7^2	Sevens 7 7	Ones 1 7

7 Now we can use the ideas we obtained from a study of base 5 numerals and apply them to a study of base 7 numerals.

Addition

Base Five

$$
\begin{array}{ll}
24_5 = & 2 \text{ fives} + 4 \text{ ones} \\
\underline{13_5} = & \underline{1 \text{ five} + 3 \text{ ones}} \\
42_5 & 3 \text{ fives} + 7 \text{ ones} \\
 & = 3 \text{ fives} + 1 \text{ five} + 2 \text{ ones} \\
 & = 4 \text{ fives} + 2 \text{ ones} \\
 & = 42_5
\end{array}
$$

Base Seven

$$
\begin{array}{ll}
26_7 = & 2 \text{ sevens} + 6 \text{ ones} \\
\underline{35_7} = & \underline{3 \text{ sevens} + 5 \text{ ones}} \\
64_7 & 5 \text{ sevens} + 11 \text{ ones} \\
 & = 5 \text{ sevens} + 1 \text{ seven} + 4 \text{ ones} \\
 & = 6 \text{ sevens} + 4 \text{ ones} \\
 & = 64_7
\end{array}
$$

Subtraction

Base Five

$$
\begin{array}{lll}
30_5 = 3 \text{ fives} + 0 \text{ ones} = 2 \text{ fives} + 1 \text{ five} + 0 \text{ ones} = & 2 \text{ fives} + 5 \text{ ones} & \\
\underline{12_5} = \underline{1 \text{ five} + 2 \text{ ones}} = & \underline{1 \text{ five} + 2 \text{ ones}} & \\
 & 1 \text{ five} + 3 \text{ ones} = 13_5 &
\end{array}
$$

Base Seven

$$
\begin{array}{ll}
43_7 = 4 \text{ sevens} + 3 \text{ ones} = 3 \text{ sevens} + 1 \text{ seven} + 3 \text{ ones} = & 3 \text{ sevens} + 10 \text{ ones} \\
\underline{26_7} = \underline{2 \text{ sevens} + 6 \text{ ones}} = & \underline{2 \text{ sevens} + 6 \text{ ones}} \\
 & 1 \text{ seven} + 4 \text{ ones} \\
 & = 14_7
\end{array}
$$

8. Copy and complete the addition chart for numerals to base 7.

+	0	1	2	3	4	5	6
0							
1							
2				5			
3						11_7	
4							
5			10_7				
6							15_7

9. Add:

a $\begin{array}{r}24_7\\ \underline{13_7}\end{array}$ **b** $\begin{array}{r}46_7\\ \underline{25_7}\end{array}$ **c** $\begin{array}{r}10_7\\ \underline{60_7}\end{array}$ **d** $\begin{array}{r}45_7\\ \underline{25_7}\end{array}$ **e** $\begin{array}{r}14_7\\ \underline{35_7}\end{array}$

f $\begin{array}{r}203_7\\ \underline{106_7}\end{array}$ **g** $\begin{array}{r}115_7\\ \underline{143_7}\end{array}$ **h** $\begin{array}{r}321_7\\ \underline{136_7}\end{array}$ **i** $\begin{array}{r}202_7\\ \underline{146_7}\end{array}$ **j** $\begin{array}{r}520_7\\ \underline{164_7}\end{array}$

k $\begin{array}{r}250_7\\ \underline{340_7}\end{array}$ **l** $\begin{array}{r}132_7\\ \underline{342_7}\end{array}$ **m** $\begin{array}{r}261_7\\ \underline{34_7}\end{array}$ **n** $\begin{array}{r}521_7\\ \underline{641_7}\end{array}$ **o** $\begin{array}{r}406_7\\ \underline{451_7}\end{array}$

p $\begin{array}{r}111_7\\ \underline{603_7}\end{array}$ **q** $\begin{array}{r}214_7\\ \underline{522_7}\end{array}$ **r** $\begin{array}{r}236_7\\ \underline{446_7}\end{array}$ **s** $\begin{array}{r}505_7\\ \underline{416_7}\end{array}$ **t** $\begin{array}{r}244_7\\ \underline{535_7}\end{array}$

u $\begin{array}{r}664_7\\ \underline{556_7}\end{array}$ **v** $\begin{array}{r}1014_7\\ \underline{2045_7}\end{array}$ **w** $\begin{array}{r}3664_7\\ \underline{2065_7}\end{array}$ **x** $\begin{array}{r}4015_7\\ \underline{5625_7}\end{array}$ **y** $\begin{array}{r}6306_7\\ \underline{5464_7}\end{array}$

10. Subtract:

a $\begin{array}{r}33_7\\ \underline{12_7}\end{array}$ **b** $\begin{array}{r}46_7\\ \underline{25_7}\end{array}$ **c** $\begin{array}{r}53_7\\ \underline{2_7}\end{array}$ **d** $\begin{array}{r}65_7\\ \underline{55_7}\end{array}$ **e** $\begin{array}{r}24_7\\ \underline{13_7}\end{array}$

f $\begin{array}{r}52_7\\ \underline{23_7}\end{array}$ **g** $\begin{array}{r}44_7\\ \underline{25_7}\end{array}$ **h** $\begin{array}{r}30_7\\ \underline{16_7}\end{array}$ **i** $\begin{array}{r}61_7\\ \underline{24_7}\end{array}$ **j** $\begin{array}{r}55_7\\ \underline{26_7}\end{array}$

k $\begin{array}{r}543_7\\ \underline{215_7}\end{array}$ **l** $\begin{array}{r}636_7\\ \underline{142_7}\end{array}$ **m** $\begin{array}{r}354_7\\ \underline{235_7}\end{array}$ **n** $\begin{array}{r}554_7\\ \underline{236_7}\end{array}$

o $\begin{array}{r} 402_7 \\ \underline{114_7} \end{array}$	**p** $\begin{array}{r} 501_7 \\ \underline{204_7} \end{array}$	**q** $\begin{array}{r} 203_7 \\ \underline{105_7} \end{array}$	**r** $\begin{array}{r} 402_7 \\ \underline{135_7} \end{array}$
s $\begin{array}{r} 200_7 \\ \underline{113_7} \end{array}$	**t** $\begin{array}{r} 600_7 \\ \underline{264_7} \end{array}$	**u** $\begin{array}{r} 500_7 \\ \underline{111_7} \end{array}$	**v** $\begin{array}{r} 400_7 \\ \underline{204_7} \end{array}$
w $\begin{array}{r} 3462_7 \\ \underline{1063_7} \end{array}$	**x** $\begin{array}{r} 1000_7 \\ \underline{325_7} \end{array}$	**y** $\begin{array}{r} 6214_7 \\ \underline{3556_7} \end{array}$	**z** $\begin{array}{r} 5030_7 \\ \underline{4041_7} \end{array}$

CHANGING TO AND FROM BASE SEVEN NUMERALS

1 Consider the base seven place value chart:

$(7\times7\times7\times7)$'s	$(7\times7\times7)$'s	(7×7)'s	7's	1's
7^4	7^3	7^2	7	7

Now
$$\begin{aligned} 1 &= 1 \\ 7 &= 7 \\ 7^2 &= 7\times7 = 49 \\ 7^3 &= 7\times7\times7 = 343 \\ 7^4 &= 7\times7\times7\times7 = 2401 \end{aligned}$$

Change 3519 to a base seven numeral.

a How many times does 3519 contain 2401?

$$\begin{array}{r} 1 \\ 2401\overline{)3519} \\ \underline{2401} \\ 1118 \end{array}$$

b How many times does 1118 contain 343?

$$\begin{array}{r} 3 \\ 343\overline{)1118} \\ \underline{1029} \\ 89 \end{array}$$

c How many times does 89 contain 49?

$$\begin{array}{r} 1 \\ 49\overline{)89} \\ \underline{49} \\ 40 \end{array}$$

d How many times does 40 contain 7?

$$\begin{array}{r} 5 \\ 7\overline{)40} \\ \underline{35} \\ 5 \end{array}$$

What is the remainder?

e We see that:

$$\begin{aligned} 3519 &= (1\times 2401) + 1118 \\ &= (1\times 2401)+(3\times 343)+89 \\ &= (1\times 2401)+(3\times 343)+(1\times 49)+40 \\ &= (1\times 2401)+(3\times 343)+(1\times 49)+(5\times 7)+5 \\ &= (1\times 7^4)+(3\times 7^3)+(1\times 7^2)+(5\times 7\ )+(5\times 1\ ) \\ &= 13155_7 \end{aligned}$$

2. Change the following decimal numerals to base seven numerals:

a 26	**b** 45	**c** 63	**d** 49	**e** 114
f 343	**g** 344	**h** 737	**i** 1000	**j** 392
k 2401	**l** 4802	**m** 9645	**n** 730	**o** 1648
p 2014	**q** 14516	**r** 13400	**s** 3430	**t** 1709
u 1516	**v** 2400	**w** 10,096	**x** 1029	**y** 11206

3. Express 243_7 as a decimal numeral:

$$\begin{aligned} 243_7 &= (2\times 7^2)+(4\times 7\ )+(3\times\ 1\ ) \\ &= (2\times 49)+(4\times 7)+(3\times 1) \\ &= 98+28+3 \\ &= 129. \end{aligned}$$

4. Express the following base seven numerals as decimal numerals:

a 16_7	**b** 23_7	**c** 45_7	**d** 56_7	**e** 10_7	**f** 60_7
g 100_7	**h** 115_7	**i** 436_7	**j** 206_7	**k** 615_7	**l** 542_7
m 1000_7	**n** 2001_7	**o** 1500_7	**p** 1324_7	**q** 4156_7	

5. Copy and group the number of X's below into groups of (i) ten (ii) seven (iii) five and then write numerals in base 10, base 7 and base 5 to represent this number of X's.

X X X X X X X X X X X X X X X X X X X X X X X X X X X

a (X X X X X X X X X X) (X X X X X X X X X X) X X X X X X X

1 group of ten   1 group of ten   7 ones

Base 10 numeral: 27

b (X X X X X X X) (X X X X X X X) (X X X X X X X) X X X X X X

1 group of seven   1 group of seven   1 group of seven   6 ones

Base 7 numeral: 36_7

c

(X X X X X)(X X X X X)(X X X X X)(X X X X X)(X X X X X)X X

1 group of five   1 group of five   1 group of five   1 group of five   1 group of five   2 ones

1 group of five fives

Base 5 numeral: 102_5

6. Copy the sets of X's below into your workbooks. Put them into groups of (i) ten (ii) seven (iii) five and then write numerals in base 10, base 7, and base 5 to represent the number of X's in each set.

a X X X X X X X X X X X X X X X X

b X X X X X X X X X X X X X X X X X
X X X X X X

c X X X X X X X X X X X X
X X X X X X X X X

d X X X X X X X X X X X X X
X X X X X X X X X X X X X
X X X X X X X X X X X X X
X X X X X X X X X X X X X

e X X X X X X X X X X

f X X X X X X X X X X X X X X X X X X X X
X X X X X X X X X X X X X X X X X X X X
X X X X X X X X X X X X X X X X X X X X
X X X X X X X X X X X X X X X X X X X X

g X X X X X X X X X X X X X X X X X X
X X X X X X X X X X X X X X X X X X
X X X X X X X X X X X X X X X X X X
X X X X X X X X X X X X X X X X X X
X X X X X X X X X X X X X X X X X X

CHAPTER TEST

1. Copy and complete the table below:

Decimal Numeral	Base 5 Numeral	Decimal Numeral	Base 5 Numeral
1	1_5	15	
2		25	
3		50	
4		75	
5	10_5	100	
6		110	
7		125	
8		140	
9		150	
10		625	

2. Add:

a $\begin{array}{r} 3412_5 \\ 2014_5 \\ 333_5 \\ \underline{1301_5} \end{array}$ **b** $\begin{array}{r} 1000_5 \\ 414_5 \\ 321_5 \\ \underline{210_5} \end{array}$ **c** $\begin{array}{r} 3041_5 \\ 2213_5 \\ 4412_5 \\ \underline{44_5} \end{array}$ **d** $\begin{array}{r} 3022_5 \\ 43_5 \\ 134_5 \\ \underline{2012_5} \end{array}$

e $\begin{array}{r} 24_5 \\ 404_5 \\ 13_5 \\ 221_5 \\ \underline{13_5} \end{array}$ **f** $\begin{array}{r} 413_5 \\ 324_5 \\ 12_5 \\ 34_5 \\ \underline{10_5} \end{array}$ **g** $\begin{array}{r} 300_5 \\ 24_5 \\ 43_5 \\ 203_5 \\ \underline{40_5} \end{array}$ **h** $\begin{array}{r} 122_5 \\ 10_5 \\ 102_5 \\ 314_5 \\ \underline{31_5} \end{array}$

Check by changing each base 5 numeral to a decimal numeral and adding.

3. Subtract:

a $\begin{array}{r} 413_5 \\ \underline{302_5} \end{array}$ **b** $\begin{array}{r} 432_5 \\ \underline{422_5} \end{array}$ **c** $\begin{array}{r} 3142_5 \\ \underline{2143_5} \end{array}$ **d** $\begin{array}{r} 1000_5 \\ \underline{412_5} \end{array}$

e $\begin{array}{r} 3122_5 \\ \underline{123_5} \end{array}$ **f** $\begin{array}{r} 2010_5 \\ \underline{1321_5} \end{array}$ **g** $\begin{array}{r} 3142_5 \\ \underline{1133_5} \end{array}$ **h** $\begin{array}{r} 2001_5 \\ \underline{1232_5} \end{array}$

4. Use addition to find the following products: (The first one is done for you.)

a $4_5 \times 3_5$

$$\begin{array}{r} 3_5 \\ 3_5 \\ 3_5 \\ \underline{3_5} \end{array} \qquad \begin{array}{r} 3 \\ 3 \\ 3 \\ \underline{3} \\ 12 \end{array} \text{ in decimal numeration}$$

$$\begin{aligned} 12 &= (2\times 5)+(2\times 1) \\ &= 22_5 \end{aligned}$$

So

$$\begin{array}{r} 3_5 \\ 3_5 \\ 3_5 \\ \underline{3_5} \\ 22_5 \end{array}$$

From this we see that
$4_5 \times 3_5 = 22_5$

b $2_5 \times 3_5$ **c** $3_5 \times 3_5$ **d** $1_5 \times 4_5$

e $4_5 \times 2_5$ **f** $2_5 \times 2_5$ **g** $4_5 \times 4_5$

5. Copy and complete the table of multiplication below. Use addition to find any products of which you are not sure.

$\times$	0_5	1_5	2_5	3_5	4_5
0_5	0_5				
1_5					
2_5				11_5	
3_5					
4_5			13_5		

6. Add:

a $\begin{array}{r} 414_7 \\ \underline{206_7} \end{array}$ **b** $\begin{array}{r} 515_7 \\ \underline{105_7} \end{array}$ **c** $\begin{array}{r} 332_7 \\ \underline{443_7} \end{array}$ **d** $\begin{array}{r} 561_7 \\ \underline{466_7} \end{array}$ **e** $\begin{array}{r} 206_7 \\ \underline{366_7} \end{array}$

f 362_7	**g** 115_7	**h** 232_7	**i** 306_7	**j** 265_7
65_7	423_7	13_7	16_7	626_7
$\underline{104_7}$	$\underline{61_7}$	$\underline{422_7}$	$\underline{535_7}$	$\underline{544_7}$

7. Subtract:

a 462_7	**b** 426_7	**c** 503_7	**d** 620_7	**e** 413_7
$\underline{361_7}$	$\underline{246_7}$	$\underline{112_7}$	$\underline{501_7}$	$\underline{224_7}$
f 1010_7	**g** 3000_7	**h** 2015_7	**i** 3050_7	**j** 6102_7
$\underline{214_7}$	$\underline{160_7}$	$\underline{156_7}$	$\underline{1234_7}$	$\underline{5614_7}$

8. Below are sets of X's. Copy them in your workbook and then group them into (i) tens (ii) sevens (iii) fives. Write numerals to base ten, base seven, base five to represent the number of X's in each set.

a X X X X X X X X X X X X X

b X X X X X X X X X X X X X X
X X X X X X X

c X X X X X X X
X X X X X X X
X X X X X X X
X X X X X X X
X X X X X X X
X X X X X X X

d X X X X X X X X X X X X X X
X X X X X X X X X X X X X X
X X X X X X X X X X X X X X
X X X X X X X X X X X X X X
X X X X X X X X X X X X X X

9. Write the following as expanded numerals using exponential notation to the base indicated:

a 6014_7	**b** 32055_7	**c** 61432_7	**d** 6060_7
e 3124_5	**f** 4041_5	**g** 43214_5	**h** 40040_5
i 6062_7	**j** 34616_7	**k** 60033_7	**l** 213240_5
m 20000_5	**n** 40000_7	**o** 141321_5	**p** 21636_7

10. Make a table of multiplication for base 7 numerals.

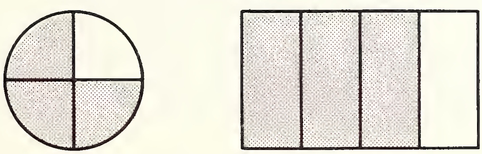
5 Fractions

THE MEANING OF FRACTIONS

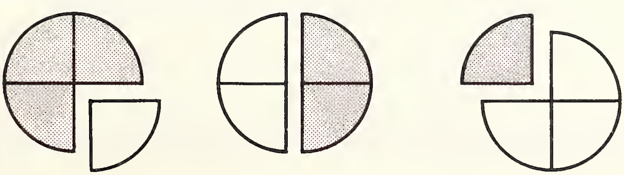
1 When we were talking about our number system in the first chapter, we saw that if we multiplied together two whole numbers our product was a whole number. We said that the system of whole numbers was closed under the operation of multiplication.

Let us have a look at two whole numbers and consider them under the operation of division. Consider the numbers 3 and 4. If we take 3 and divide it by 4, we have $\frac{3}{4}$. If we take the 4 and divide it by 3, we shall have $\frac{4}{3}$. Is either of these a numeral to represent a whole number? We can see just by this one example that the set of whole numbers is not closed under the operation of division, because, if we divide two whole numbers, we do not always get a whole number as our answer. We have special names that we give to numbers that we represent by numerals like $\frac{3}{4}$ and $\frac{4}{3}$. Numbers like these are called **fractional numbers**, and the numerals that we use to represent them are called **fractions** or fraction numerals.

2 **a** Consider the fraction $\frac{3}{4}$. In earlier grades we have seen that this can have several different meanings. One meaning that we have usually taken for it is this: $\frac{3}{4}$ means that we have 3 of the 4 equal parts into which a whole 1 may have been divided.



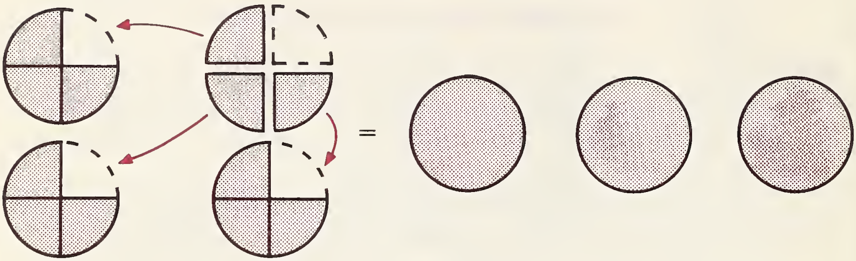
b We have also looked at $\frac{3}{4}$ and taken it to mean 3 whole ones divided by 4. That is, $\frac{3}{4}$ is what we get when we take 3 whole ones and divide them into 4 equal parts.



c Let us think of $\frac{3}{4}$ as being 3 divided by 4.

Then, we can let 3 divided by 4 equal x; i.e., $3 \div 4 = x$, or $\frac{3}{4} = x$.

This means that $3 = 4 \times x$. x must be the number, such that when we multiply it by 4 we have 3 as our answer.



d We think of $\frac{3}{4}$ as being the number which, when multiplied by 4, has 3 as the product.

$\frac{1}{2}$ is the number which, when multiplied by 2, has 1 as the product.

e For any fraction $\frac{a}{b}$, we say $\frac{a}{b}$ is the number which, when multiplied by b, has a as the product.

If $\frac{a}{b}$ is a fraction, then

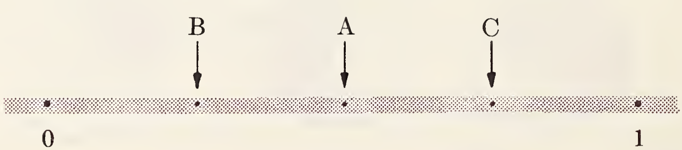
$$b \times \frac{a}{b} = a.$$

a can be any whole number; b can be any natural number. Later, we shall see why b must be a natural number.

f Using the definition above, explain what the following fractions mean: $\frac{2}{3}$, $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{2}$, $\frac{9}{5}$, $\frac{7}{10}$, $\frac{6}{6}$.

FRACTIONS AND NUMBER LINES

1 When we were studying whole numbers, we let points on a line represent the whole numbers. Consider the number line below:



What number is represented by the point "A"? We see that the point lies between the point representing the number 0, and the point representing the number 1. There is no whole number between 0 and 1; therefore point A cannot represent a whole number. Since the point is half way between the point representing 0 and the point representing 1, we agree to let this point represent $\frac{1}{2}$. What numbers do you think the points B and C will represent?

We represent **fractional numbers** with **fractions** like $\frac{1}{2}$, $\frac{3}{4}$, $\frac{0}{2}$, $\frac{5}{6}$, $\frac{15}{3}$, etc. In general we can write a fraction as $\frac{a}{b}$. Now, can a be any whole number? Can a be zero? Can b be any whole number? Can b be zero?

Think of $\frac{0}{2}$ and $\frac{2}{0}$. What does $\frac{0}{2}$ represent? It represents 0. What does $\frac{2}{0}$ represent?

We learned that a number divided by 0 has no meaning; therefore, can $\frac{a}{b}$ represent a fraction if b is 0?

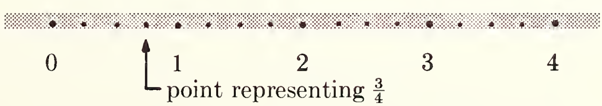
We see that b must be a natural number; it cannot be 0; a can be 0; it can also be a natural number; therefore, a can be any whole number.

$\frac{a}{b}$ represents a fraction when a is any whole number, and b is any natural number; b cannot be 0.

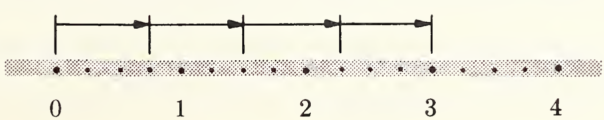
The number represented by a is called the **numerator** of the fraction; the number represented by b is called the **denominator** of the fraction.

2 We have seen that we can think of $\frac{3}{4}$ as being the number which, when multiplied by 4, has 3 as the product.

On the first number line below, the arrow shows the point representing $\frac{3}{4}$. If we take 4 moves equal in length to the distance between the point representing zero and the point representing $\frac{3}{4}$, at what point do we arrive?



4 moves brings us to the point representing the number 3.



3.

a Which point represents the number $\frac{2}{5}$?

b How far is this point from the point representing zero?

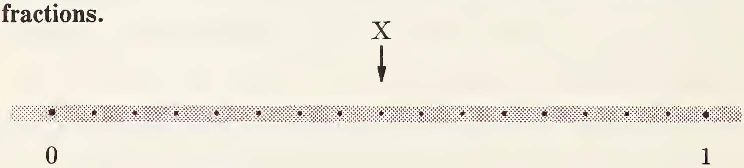
c Take 5 moves on the line equal in distance to this length.

d At what point do you finish?

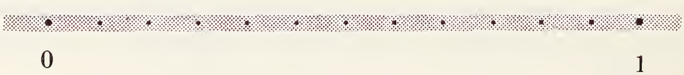
e What number does this point represent?

Now use this idea of the meaning of fractions to show what $\frac{1}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ mean.

4. Below, we have a number line on which we have represented, by points, fractions of different values. If we look at the point "X", we can see that this represents $\frac{1}{2}$. For $\frac{1}{2}$ we can have several names. From the diagram we can see that $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}=\frac{8}{16}$. All these fractional numerals represent the same number. We call fractions like $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ **equivalent fractions.**



5. Make a drawing of the number line below, and complete it by suggesting numerals to name each point on it.



A FIRST LOOK AT MULTIPLICATION OF FRACTIONS

1. In this section we shall discover certain rules about multiplication using fractions. Later in this chapter we shall study multiplication using fractions in more detail.

2 Study the number line below:

0	1	2	3	4	5	6	7	8
$\frac{0}{2}$	$\frac{2}{2}$	$\frac{4}{2}$	$\frac{6}{2}$	$\frac{8}{2}$	$\frac{10}{2}$	$\frac{12}{2}$	$\frac{14}{2}$	$\frac{16}{2}$
$\frac{0}{3}$	$\frac{3}{3}$	$\frac{6}{3}$	$\frac{9}{3}$	$\frac{12}{3}$	$\frac{15}{3}$	$\frac{18}{3}$	$\frac{21}{3}$	$\frac{24}{3}$
$\frac{0}{4}$	$\frac{4}{4}$	$\frac{8}{4}$	$\frac{12}{4}$	$\frac{16}{4}$	$\frac{20}{4}$	$\frac{24}{4}$	$\frac{28}{4}$	$\frac{32}{4}$
$\frac{0}{5}$	$\frac{5}{5}$	$\frac{10}{5}$	$\frac{15}{5}$	$\frac{20}{5}$	$\frac{25}{5}$	$\frac{30}{5}$	$\frac{35}{5}$	$\frac{40}{5}$

Are $\frac{0}{3}$, $\frac{2}{2}$, $\frac{4}{2}$, $\frac{6}{3}$, $\frac{15}{3}$, $\frac{16}{2}$, $\frac{30}{5}$ and so on, fraction numerals? Why? What are the ways in which 1 has been renamed? Are $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$ numerals that we can use to represent 1? Why? What numerals have we used to represent 4? 7? 8? Is $\frac{12}{2}$ another name for 6? Is $\frac{20}{4}$ another name for 5? Think of $\frac{9}{3}$ as $9 \div 3$. Is $9 \div 3$ a numeral for 3? Could we represent 2 by $\frac{24}{12}$? Why? Do you think we could represent the numbers on the number line above by different fraction numerals from those shown? Could we represent all whole numbers by fraction numerals? Is $\frac{32}{2}$ a numeral for 16? Is $\frac{32}{2}$ a fraction numeral? From our examination of the numerals we have used to represent whole numbers on the number line above we are led to believe that:

a all whole numbers can be written as fraction numerals.

b there is no limit to the number of fraction numerals that we can use to represent each of the whole numbers in the set of whole numbers.

3 Let us consider 5×6.

We know that $5 \times 6 = 30$.

Let us now represent 5 by $\frac{15}{3}$ and 6 by $\frac{12}{2}$

Can we write $\frac{15}{3} \times \frac{12}{2}$ for 5×6? Why?

Can we write $\dfrac{15 \times 12}{3 \times 2}$ for $\dfrac{15}{3} \times \dfrac{12}{2}$? Let us see if $\dfrac{15 \times 12}{3 \times 2}$ equals $\dfrac{15}{3} \times \dfrac{12}{2}$.

Now $\dfrac{15 \times 12}{3 \times 2} = \dfrac{180}{6} = 30.$

We know that $5 \times 6 = 30$ and since $\frac{15}{3}$ is a numeral for 5 and $\frac{12}{2}$ is a numeral for 6, then $\frac{15}{3} \times \frac{12}{2}$ must equal 30.

We have just seen that if we compute $\frac{15}{3} \times \frac{12}{2}$ by multiplying the 15 and 12 to give a numerator for a fraction numeral and by multiplying 3 and 2 to give a denominator for a fraction numeral, the fraction numeral we obtain represents the product of $\frac{15}{3}$ and $\frac{12}{2}$.

$$5\times6 = 30$$

$$\frac{15}{3}\times\frac{12}{2}=\frac{15\times12}{3\times2}=\frac{180}{6}=30$$

We have not *proved* a rule for multiplying with fraction numerals, we have shown a method that works when the fraction numerals we use represent whole numbers.

4 Study the examples below:

a $4\times2 = 8$ $\frac{16}{4}=4$ $\frac{10}{5}=2$

$$\frac{16}{4}\times\frac{10}{5}=\frac{16\times10}{4\times5}=\frac{160}{20}=8$$

b $7\times3 = 21$ $\frac{21}{3}=7$ $\frac{12}{4}=3$

$$\frac{21}{3}\times\frac{12}{4}=\frac{21\times12}{3\times4}=\frac{252}{12}=21$$

5. Work the following in the way shown. Check by using the usual decimal numerals for the fraction numerals.

Example: $\frac{21}{7}\times\frac{4}{2}=\frac{21\times4}{7\times2}=\frac{84}{14}=6$

$\frac{21}{7}=3$ $\frac{4}{2}=2$ $3\times2=6$

a $\frac{10}{5}\times\frac{15}{3}$ b $\frac{4}{2}\times\frac{3}{1}$ c $\frac{12}{6}\times\frac{24}{6}$

d $\frac{14}{2}\times\frac{6}{3}$ e $\frac{5}{5}\times\frac{16}{8}$ f $\frac{16}{8}\times\frac{2}{1}$

g $\frac{5}{1}\times\frac{4}{1}$ h $\frac{30}{6}\times\frac{12}{3}$ i $\frac{18}{2}\times\frac{6}{3}$

6. A boy walks $\frac{1}{4}$ mile when he goes to school. How far does he walk when he makes the journey to school and back?

Let the number of miles be x. Then we see that x will be equal to two quarters. For two quarters we can write $2\times\frac{1}{4}$. For two quarters we can write $\frac{2}{4}$.

We know that $2\times\frac{1}{4}=\frac{2}{4}$.

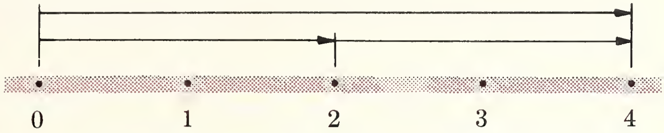
Is $\frac{2}{1}$ another name for 2? For $2\times\frac{1}{4}$ can we write $\frac{2}{1}\times\frac{1}{4}$?

Is $\frac{2\times1}{1\times4}$ another name for $\frac{2}{4}$?

For $2\times\frac{1}{4}$ can we write $2\times\frac{1}{4}=\frac{2}{1}\times\frac{1}{4}=\frac{2\times1}{1\times4}=\frac{2}{4}$?

Does $x=\frac{2}{4}$? Does $x=\frac{2\times1}{1\times4}$?

7 What is $\frac{1}{2}$ of 4? Here, we need to know what we mean mathematically when we use "of". We shall agree to think that "of" and "$\times$" have the same meaning with regard to fractions.



From the diagram we see that $\frac{1}{2}$ of $4=2$.

For $\frac{1}{2}$ of 4 can we write $\frac{1}{2}\times4$?

Is $\frac{1}{2}\times4$ another name for 2? Can we write $\frac{4}{1}$ as another name for 4?

For $\frac{1}{2}$ of 4 can we write $\frac{1}{2}\times\frac{4}{1}$?

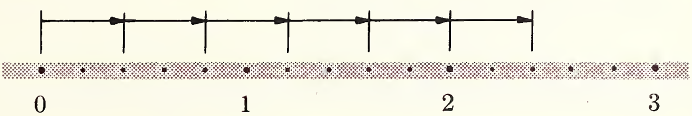
For $\frac{1}{2}\times\frac{4}{1}$ can we write $\dfrac{1\times4}{2\times1}=\dfrac{4}{2}=2$?

8. Write, in as many ways as you can, numerals to represent the following:

a $\frac{1}{4}\times6$ **b** $3\times\frac{1}{3}$ **c** $\frac{1}{8}\times9$ **d** $\frac{1}{3}\times6$ **e** $7\times\frac{1}{2}$

f $8\times\frac{1}{4}$ **g** $\frac{1}{5}\times10$ **h** $\frac{1}{16}\times20$ **i** $\frac{1}{6}\times15$ **j** $12\times\frac{1}{6}$

9 What is $6\times\frac{2}{5}$? Examine the number line below. Can we read off the answer from it?



We can think of $6\times\frac{2}{5}$ as being $\dfrac{6}{1}\times\dfrac{2}{5}=\dfrac{6\times2}{1\times5}=\dfrac{12}{5}=2\frac{2}{5}$.

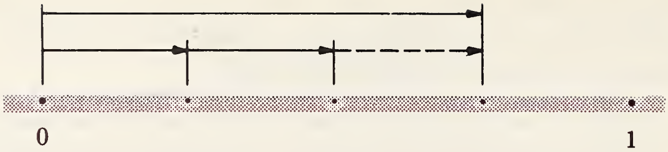
10 Find $\frac{7}{8}$ of 3.

For this we can write $\frac{7}{8}$ of $3=\dfrac{7}{8}\times\dfrac{3}{1}=\dfrac{7\times3}{8\times1}=\dfrac{21}{8}=2\frac{5}{8}$

11 Solve: $\frac{2}{3}\times\frac{3}{4}=x$.

a We can use a number line:

$\frac{2}{3}\times\frac{3}{4}=\frac{2}{3}$ of $\frac{3}{4}$. (We agreed to this previously.)



From the number line we can see that $\frac{1}{3}$ of $\frac{3}{4}$ is $\frac{1}{4}$; so $\frac{2}{3}$ of $\frac{3}{4}$ is $\frac{2}{4}$, or $\frac{1}{2}$. From here we see that $\frac{2}{3}\times\frac{3}{4}=\frac{2}{4}=\frac{1}{2}$.

b $\frac{2}{3}\times\frac{3}{4}=x$. We have seen that $x=\frac{2}{4}=\frac{1}{2}$.

What number is represented by the numeral $\dfrac{2\times3}{3\times4}$?

$\dfrac{2\times3}{3\times4}=\dfrac{6}{12}$ Is $\dfrac{6}{12}$ a numeral for $\dfrac{1}{2}$?

12 Examine the examples below:

a $\dfrac{3}{5}\times\dfrac{2}{7}=\dfrac{3\times2}{5\times7}=\dfrac{6}{35}$

b $\dfrac{3}{10}\times\dfrac{5}{8}=\dfrac{3\times5}{10\times8}=\dfrac{15}{80}=\dfrac{3}{16}$

c $\dfrac{9}{10}\times1\frac{1}{4}=\dfrac{9}{10}\times\dfrac{5}{4}=\dfrac{9\times5}{10\times4}=\dfrac{45}{40}=1\frac{5}{40}=1\frac{1}{8}$

d $2\frac{2}{3}\times1\frac{1}{4}=\dfrac{8}{3}\times\dfrac{5}{4}=\dfrac{8\times5}{3\times4}=\dfrac{40}{12}=3\frac{4}{12}=3\frac{1}{3}$

e $2\frac{5}{12}\times\dfrac{3}{8}=\dfrac{29}{12}\times\dfrac{3}{8}=\dfrac{29\times3}{12\times8}=\dfrac{87}{96}=\dfrac{29}{32}$

13 If $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are any two fractions, then $\dfrac{a}{b}\times\dfrac{c}{d}=\dfrac{a\times c}{b\times d}$, which we can write as $\dfrac{ac}{bd}$.

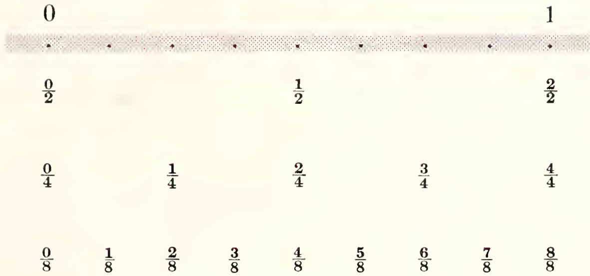
14. Work the following:

a $\frac{4}{3}\times\frac{3}{4}$ **b** $\frac{21}{10}\times15$ **c** $\frac{7}{16}\times6$ **d** $\frac{1}{3}\times7$ **e** $12\times\frac{13}{2}$

f $4\times\frac{5}{16}$ **g** $3\frac{1}{2}\times6$ **h** $4\frac{3}{5}\times10$ **i** $\frac{7}{8}\times1\frac{1}{4}$ **j** $1\frac{4}{5}\times3\frac{1}{3}$

15. Find the cost of $\frac{2}{3}$ pound of meat at 90¢ per pound.

16. A train travelled at an average rate of 55 miles per hour for $\frac{21}{5}$ hours and at an average rate of 58 miles an hour for $\frac{7}{2}$ hours. Find the distance the train travelled.



1 From the diagram we can see that $\frac{1}{2}=\frac{2}{4}=\frac{4}{8}$. These are equivalent fractions.

We call the number represented by the numeral above the line the **numerator** of the fraction, and we call the number represented by the numeral below the line, the **denominator** of the fraction.

2 What names can we give for the number "1"? We see that $1=\frac{1}{1}=\frac{2}{2}=\frac{3}{3}=\ldots$ What is 1 times any number? What is 1×2, 1×4, 1×5?

We remember that when we multiply any number by 1, or 1 by any number, our product is always the number itself. Is this true for fractions?

Consider $\frac{1}{2}$. What is $1\times\frac{1}{2}$? What is $\frac{1}{2}\times1$?

We are led to believe that the product of a fraction and 1 is the fraction itself.

For 1 we can write $\frac{2}{2}$; so $\frac{1}{2}\times1=\frac{1}{2}\times\frac{2}{2}=\frac{1\times2}{2\times2}=\frac{2}{4}$.

Is $\frac{2}{4}$ the same number as $\frac{1}{2}$? We have just seen from our number line that $\frac{1}{2}$ is the same as $\frac{2}{4}$; so we can multiply $\frac{1}{2}$ by 1, and still have $\frac{1}{2}$ for our answer. If we put $\frac{2}{2}$ for 1 when we multiply, we get a different numeral for $\frac{1}{2}$, but the numeral that we get still represents $\frac{1}{2}$.

3 Now consider the following:

$\frac{1}{2}\times1=\frac{1}{2}\times\frac{3}{3}=\frac{1\times3}{2\times3}=\frac{3}{6}$. $\qquad$ $\frac{1}{2}\times1=\frac{1}{2}\times\frac{4}{4}=\frac{1\times4}{2\times4}=\frac{4}{8}$

4 Write equivalent fractions for $\frac{1}{3}$.

$$\frac{1}{3}=\frac{1}{3}\times1=\frac{1}{3}\times\frac{2}{2}=\frac{1\times2}{3\times2}=\frac{2}{6}$$

$$\frac{1}{3}=\frac{1}{3}\times1=\frac{1}{3}\times\frac{3}{3}=\frac{1\times3}{3\times3}=\frac{3}{9}$$

5. Write 6 equivalent fractions for each of the following fractions:

a $\frac{1}{5}$ **b** $\frac{1}{4}$ **c** $\frac{1}{10}$

d $\frac{1}{8}$ **e** $\frac{1}{12}$ **f** $\frac{1}{6}$

6. Write equivalent fractions for $\frac{2}{5}$.

$$\frac{2}{5}=\frac{2}{5}\times1=\frac{2}{5}\times\frac{2}{2}=\frac{2\times2}{5\times2}=\frac{4}{10}$$

Write four other equivalent fractions for $\frac{2}{5}$.

7 If we study what we have done on equivalent fractions, we shall see that we can find a simple rule about making equivalent fractions. If we multiply the numerator by a number, and the denominator of that fraction by the same number, we have a new numeral that represents the same number as the original fraction. In multiplying the numerator and the denominator of the fraction by the same number, we do not alter the size of the fraction. We cannot, however, use zero as the multiplier. Why not?

8. Copy and complete the following:

a $\frac{1}{3}=\frac{?}{12}$ **b** $\frac{?}{6}=\frac{9}{18}$ **c** $\frac{?}{2}=\frac{2}{4}$ **d** $\frac{3}{2}=\frac{6}{?}$

e $\frac{3}{4}=\frac{?}{28}$ **f** $\frac{4}{?}=\frac{24}{30}$ **g** $\frac{7}{?}=\frac{35}{30}$ **h** $\frac{7}{14}=\frac{42}{?}$

i $\frac{8}{?}=\frac{72}{81}$ **j** $\frac{7}{20}=\frac{?}{100}$ **k** $\frac{3}{?}=\frac{15}{40}$ **l** $\frac{6}{7}=\frac{?}{42}$

FRACTIONS IN THEIR LOWEST TERMS

1 We call the numerator and the denominator of a fraction the "terms" of the fraction.

2 Consider the fraction $\frac{9}{12}$.

For 9 we can write 3×3; for 12 we can write 4×3; so

$$\frac{9}{12}=\frac{3\times3}{4\times3}=\frac{3}{4}\times\frac{3}{3}=\frac{3}{4}\times1=\frac{3}{4}$$

We see that $\frac{9}{12}$ and $\frac{3}{4}$ are numerals representing the same numbers, but the terms of the fraction $\frac{3}{4}$ are smaller than the terms of the fraction $\frac{9}{12}$. We say that the fraction $\frac{3}{4}$ is in its lowest terms because (except for the number 1) we cannot find a factor of 3 that is also a factor of 4.

3 Write the following fraction in its lowest terms: $\frac{15}{20}$.

$$\frac{15}{20}=\frac{3\times5}{4\times5}=\frac{3}{4}\times\frac{5}{5}=\frac{3}{4}\times1=\frac{3}{4}$$

Can you think of a way to shorten this working? What operation gives us 3 from 15 at the same time as it gives us 4 from 20?

We see that we can get the same answer if we divide the 15 by 5 and the 20 by 5.

$$\frac{15}{20}=\frac{15\div 5}{20\div 5}=\frac{3}{4}$$

4 $\frac{14}{21}$. $\frac{14}{21}=\frac{14\div 7}{21\div 7}=\frac{2}{3}$

Check: $\frac{14}{21}=\frac{2\times 7}{3\times 7}=\frac{2}{3}\times\frac{7}{7}=\frac{2}{3}\times 1=\frac{2}{3}$

5 $\frac{12}{32}$. $\frac{12}{32}=\frac{12\div 2}{32\div 2}=\frac{6}{16}=\frac{6\div 2}{16\div 2}=\frac{3}{8}$

or $\frac{12}{32}=\frac{12\div 4}{32\div 4}=\frac{3}{8}$

Which of the above is the quicker way of changing $\frac{12}{32}$ to its lowest terms?

6. Write the following fractions in their lowest terms:

a $\frac{22}{44}$	**b** $\frac{24}{72}$	**c** $\frac{18}{27}$	**d** $\frac{12}{30}$
e $\frac{3}{12}$	**f** $\frac{15}{35}$	**g** $\frac{32}{48}$	**h** $\frac{24}{40}$
i $\frac{36}{48}$	**j** $\frac{45}{60}$	**k** $\frac{121}{132}$	**l** $\frac{132}{144}$
m $\frac{39}{65}$	**n** $\frac{6}{8}$	**o** $\frac{72}{108}$	**p** $\frac{81}{153}$

7. Copy and complete the following:

a $\frac{3}{5}=\frac{?}{10}=\frac{?}{15}=\frac{?}{20}$ **b** $\frac{18}{36}=\frac{?}{18}=\frac{?}{6}=\frac{?}{2}$

c $\frac{?}{45}=\frac{3}{?}=\frac{1}{3}$ **d** $\frac{16}{?}=\frac{?}{12}=\frac{2}{3}$

COMPARING FRACTIONS

1 Which is larger, $\frac{1}{4}$ or $\frac{3}{4}$?

We can think of this as "Which is larger, 1 fourth or 3 fourths?" We see that 3 fourths is larger than 1 fourth.

2. Look at the fractional numerals below. Which of them represents the largest fraction?

$$\frac{7}{8},\quad \frac{3}{8},\quad \frac{4}{8},\quad \frac{6}{8},\quad \frac{1}{8}.$$

What do we notice about the denominators of these fractional numerals? How then can we easily choose the largest of this group?

3. Copy the fractional numerals below and arrange them in order of size of the numbers that they represent. Put the numeral representing the largest fraction first.

a $\frac{1}{5}, \frac{4}{5}, \frac{2}{5}, \frac{3}{5}$ b $\frac{3}{10}, \frac{8}{10}, \frac{4}{10}, \frac{1}{10}, \frac{9}{10}$

c $\frac{11}{16}, \frac{9}{16}, \frac{10}{16}, \frac{12}{16}, \frac{8}{16}$ d $\frac{1}{12}, \frac{8}{12}, \frac{10}{12}, \frac{4}{12}, \frac{9}{12}$

e $\frac{17}{20}, \frac{16}{20}, \frac{15}{20}, \frac{18}{20}, \frac{19}{20}$ f $\frac{4}{15}, \frac{9}{15}, \frac{1}{15}, \frac{14}{15}, \frac{15}{15}$

4 Fractions greater than 1.

0 1 2

Look at the number line above. We see that the points that lie on the line between the points representing the numbers 1 and 2 cannot represent whole numbers, for there is no whole number that is both bigger than 1 and less than 2. We agree to let these points represent numbers that we call fractions; but these fractions will be larger than 1. We also notice that as we move to the *right* on the number line, the points represent *larger* numbers.

0 1 2 3

$\frac{0}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{2}$ $\frac{5}{2}$ $\frac{6}{2}$

$\frac{0}{4}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{5}{4}$ $\frac{6}{4}$ $\frac{7}{4}$ $\frac{8}{4}$ $\frac{9}{4}$ $\frac{10}{4}$ $\frac{11}{4}$ $\frac{12}{4}$

$\frac{0}{8}$ $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$ $\frac{9}{8}$ $\frac{10}{8}$ $\frac{11}{8}$ $\frac{12}{8}$ $\frac{13}{8}$ $\frac{14}{8}$ $\frac{15}{8}$ $\frac{16}{8}$ $\frac{17}{8}$ $\frac{18}{8}$ $\frac{19}{8}$ $\frac{20}{8}$ $\frac{21}{8}$ $\frac{22}{8}$ $\frac{23}{8}$ $\frac{24}{8}$

This number line shows the names that we can give to some fractions larger than 1. Use it to find equivalent fractions for: $\frac{14}{8}, \frac{8}{4}, \frac{22}{8}, \frac{6}{4}$.

5. What are some of the fraction numerals that we can give to represent the number 2? We see that $2=\frac{2}{1}=\frac{4}{2}=\frac{6}{3}=\frac{8}{4}=\ldots=\frac{30}{15}=\ldots$
What fraction numerals can you give to represent the numbers 3, 6, 5, 4?

6 Find another name for the number represented by $\frac{17}{4}$.

$$\frac{17}{4}=\frac{16+1}{4}=\frac{16}{4}+\frac{1}{4}=4+\frac{1}{4}=4\tfrac{1}{4}.$$

For $4+\frac{1}{4}$ we often write $4\frac{1}{4}$; but remember, when we write $4\frac{1}{4}$, we really mean $4+\frac{1}{4}$. Sometimes we call numbers like $4\frac{1}{4}$ mixed numbers, because they can be thought of as consisting of a whole number and a fraction.

The name that we give to a fraction that is equal to, or greater than, 1 is "improper fraction".

7. Write the following improper fractions as either mixed numbers or whole numbers: (The first two are done for you.)

a $\frac{12}{3}=4.$

We can think of this as $\dfrac{12}{3}=\dfrac{4\times3}{3}=4\times\dfrac{3}{3}=4\times1=4.$

b $\dfrac{19}{8}=\dfrac{16+3}{8}=\dfrac{16}{8}+\dfrac{3}{8}=2+\dfrac{3}{8}=2\frac{3}{8}$

c $\frac{27}{2}$	**d** $\frac{35}{35}$	**e** $\frac{21}{9}$	**f** $\frac{17}{6}$
g $\frac{12}{4}$	**h** $\frac{15}{5}$	**i** $\frac{29}{9}$	**j** $\frac{38}{14}$
k $\frac{126}{50}$	**l** $\frac{300}{260}$	**m** $\frac{150}{75}$	

8. Look at the fraction numeral $\frac{24}{5}$.

We can think of this as being 24 parts, each of which consists of $\frac{1}{5}$ of a whole 1. How many fifths do we need to make a whole 1?

So every time we can take $\frac{5}{5}$ out of $\frac{24}{5}$ we have a whole 1.

How many times can 5 fifths be taken out of 24 fifths?

We see that we can take 5 fifths four times; so $\frac{24}{5}$ will make 4 whole 1's and 4 fifths will remain. Therefore $\frac{24}{5}=4\frac{4}{5}$.

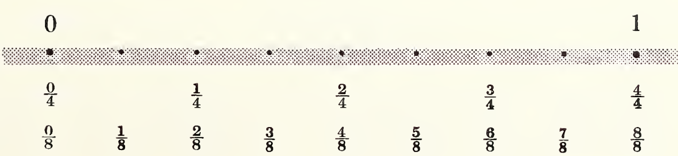
Also, $\dfrac{24}{5}=\dfrac{5+5+5+5+4}{5}=\dfrac{5}{5}+\dfrac{5}{5}+\dfrac{5}{5}+\dfrac{5}{5}+\dfrac{4}{5}=1+1+1+1+\dfrac{4}{5}=4\frac{4}{5}.$

Now use these methods to change the following improper fractions to mixed numbers or whole numbers:

a $\frac{37}{9}$	**b** $\frac{49}{7}$	**c** $\frac{42}{8}$	**d** $\frac{44}{5}$
e $\frac{63}{20}$	**f** $\frac{100}{80}$	**g** $\frac{10000}{1000}$	**h** $\frac{68}{60}$
i $\frac{20}{14}$	**j** $\frac{500}{250}$	**k** $\frac{144}{12}$	**l** $\frac{81}{54}$

9 Which is larger, $\frac{3}{4}$ or $\frac{5}{8}$?

We see that these two fractions have numerals with different denominators. It is therefore difficult to compare them. Look at the number line below:



From this, we can see that $\frac{3}{4}$ is greater than $\frac{5}{8}$; but how could we have discovered this without using the number line? We would have to change the fractions to equivalent fractions, both of them having the same denominator. $\frac{3}{4}=\frac{6}{8}$ and $\frac{5}{8}=\frac{5}{8}$, and obviously $\frac{6}{8}$ is greater than $\frac{5}{8}$.

10 Which is larger, $\frac{5}{6}$ or $\frac{11}{12}$?
$\frac{5}{6}=\frac{10}{12}$; $\frac{11}{12}$ is greater than $\frac{10}{12}$; therefore, $\frac{11}{12}$ is greater than $\frac{5}{6}$.

11. Arrange the fractions below in order of size, putting the largest first:

a $\frac{1}{4}, \frac{3}{8}, \frac{1}{8}$	**b** $\frac{9}{10}, \frac{4}{5}, \frac{7}{10}$	**c** $\frac{11}{12}, \frac{5}{6}, \frac{7}{12}$
d $\frac{3}{4}, \frac{9}{16}, \frac{11}{16}$	**e** $\frac{7}{20}, \frac{2}{5}, \frac{9}{20}$	**f** $\frac{7}{8}, \frac{3}{4}, \frac{5}{6}$

12. Which of the following fractions is the greater: $\frac{2}{3}$ or $\frac{3}{5}$? If we look at the number line below, what do we notice?

How can we find the larger of these two fractions without using the number line? We can change them to equivalent fractions having the same denominator. What common denominator can we have for thirds and fifths?

The common denominator for thirds and fifths is fifteenths; so $\frac{2}{3}=\frac{10}{15}$, and $\frac{3}{5}=\frac{9}{15}$.

Which is greater: $\frac{10}{15}$ or $\frac{9}{15}$? Which is greater: $\frac{2}{3}$ or $\frac{3}{5}$?

13. Arrange the following fractions in order of size, putting the smallest first:

a $\frac{1}{3}, \frac{1}{4}, \frac{2}{3}$	**b** $\frac{3}{5}, \frac{3}{4}, \frac{4}{5}$	**c** $\frac{2}{3}, \frac{1}{2}, \frac{1}{3}$
d $\frac{9}{10}, \frac{2}{3}, \frac{7}{10}$	**e** $\frac{7}{8}, \frac{4}{5}, \frac{5}{8}$	**f** $\frac{9}{10}, \frac{1}{3}, \frac{7}{10}$

COMMON DENOMINATORS

1 What is a common denominator that we can use to compare $\frac{7}{8}$ and $\frac{11}{12}$?

We can easily compare these two fractions if we can change them to equivalent fractions having the same denominator. One number that is obviously a multiple of both 8 and 12 is 8×12, or 96. So 96 can be used as a common denominator for comparing eighths and twelfths.

Why would it be easier to use 48 than 96? We need to find a way of discovering the smallest common denominator.

Let us have a look at the numbers 8 and 12 and express them as products of prime factors. We see then that $8=2\times2\times2$ and $12=2\times2\times3$. The number that will divide exactly by both 8 and 12 must have:

3 factors of 2 in order that 8 will divide it;

2 factors of 2 plus one factor of 3 in order that 12 will divide it.

We have certainly two factors of 2 in 8; so now we must have one factor of 3.

We now have a number that is represented by $2\times2\times2\times3$, or 24. Since $2\times2\times2\times3$ contains three factors of 2 and also two factors of 2 and one factor of 3, it will divide by both 8 and 12. It is therefore the lowest common denominator that we can use for 8ths and 12ths.

2 Find the least common denominator for 10ths, 15ths, and 9ths.

$10=2\times5$ $\quad$ $15=3\times5$ $\quad$ $9=3\times3$

For our common denominator we need the factors 2 and 5 in order that 10 will divide it. Let us begin then with 2×5. For 15 we need the factors 3 and 5. Since 2×5 does not have the factor 3, let us add one factor 3. We now have $2\times5\times3$. In order that 9 should divide the number, we must have two factors of 3, but in $2\times5\times3$ there is only one factor of 3; so now we must add one factor 3. We therefore have $2\times5\times3\times3$, or 90, and this will be the common denominator for 10, 15, and 9.

It will be the lowest common denominator. Why? How is the work we have done like the work we did in finding least common multiples?

3. $\frac{1}{18}$, $\frac{1}{12}$, $\frac{1}{8}$

$18=2\times3\times3$; $12=2\times2\times3$; $8=2\times2\times2$

Lowest common denominator is $2\times3\times3\times2\times2=72$.

Explain what has been done.

4. Arrange the following in order of size, putting the smallest first:

a $\frac{11}{12}$, $\frac{15}{16}$, $\frac{9}{10}$ $\quad$ **b** $\frac{7}{10}$, $\frac{5}{8}$, $\frac{3}{4}$ $\quad$ **c** $\frac{5}{6}$, $\frac{7}{8}$, $\frac{2}{3}$

d $\frac{1}{2}$, $\frac{5}{8}$, $\frac{7}{16}$ $\quad$ **e** $\frac{2}{3}$, $\frac{14}{15}$, $\frac{11}{12}$ $\quad$ **f** $\frac{7}{8}$, $\frac{9}{10}$, $\frac{13}{16}$

5. **a** $\frac{1}{5}\times\frac{1}{2}$ $\quad$ **b** $\frac{8}{9}\times\frac{3}{4}$ $\quad$ **c** $\frac{1}{3}\times\frac{6}{7}$ $\quad$ **d** $\frac{7}{10}\times\frac{5}{8}$

e $\frac{2}{5}\times\frac{1}{4}$ $\quad$ **f** $\frac{3}{8}\times\frac{8}{9}$ $\quad$ **g** $\frac{7}{16}\times\frac{4}{15}$ $\quad$ **h** $\frac{11}{12}\times\frac{4}{5}$

1 Find the value of $\frac{3\times5}{5\times8}$.

We can write: $\frac{3\times5}{5\times8}=\frac{3\times5}{8\times5}$ (why?)

$$=\tfrac{3}{8}\times\tfrac{5}{5}$$
$$=\tfrac{3}{8}\times1$$
$$=\tfrac{3}{8}$$

2. Work as in the example above to find the value of:

a $\frac{2\times9}{9\times7}$ **b** $\frac{4\times5}{3\times5}$ **c** $\frac{6\times10}{10\times11}$ **d** $\frac{7\times4}{4\times9}$

e $\frac{4\times3}{3\times4}$ **f** $\frac{11\times12}{12\times20}$ **g** $\frac{13\times14}{14\times15}$ **h** $\frac{7\times23}{23\times16}$

i $\frac{16\times2}{3\times16}$ **j** $\frac{3\times25}{25\times4}$ **k** $\frac{11\times4}{5\times11}$ **l** $\frac{19\times7}{8\times19}$

3 **a** Consider the example

$2\tfrac{2}{3}\times\tfrac{3}{8}=x.$

$\frac{8}{3}\times\frac{3}{8}=\frac{8\times3}{3\times8}=\frac{24}{24}=1$

If we look at the numeral $\frac{8\times3}{3\times8}$, we can rewrite it as $\frac{8\times3}{8\times3}=\frac{8}{8}\times\frac{3}{3}$ $=1\times1=1.$

We can divide the numerator and the denominator by 8 before we multiply, and the numerator and denominator by 3 before we multiply, and still get the same answer.

For $2\tfrac{2}{3}\times\tfrac{3}{8}$ we can write

$$2\tfrac{2}{3}\times\frac{3}{8}=\frac{8}{3}\times\frac{3}{8}=\frac{\overset{1}{\cancel{8}}\times\overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}}\times\underset{1}{\cancel{8}}}=1.$$

b $1\tfrac{7}{8}\times3\tfrac{1}{3}=x$

$1\tfrac{7}{8}\times3\tfrac{1}{3}=\tfrac{15}{8}\times\tfrac{10}{3}$

(We can divide the 15 and the 3 by 3. We can divide the 10 and the 8 by 2.)

$$=\frac{\cancelto{5}{15}\times\cancelto{5}{10}}{\cancelto{4}{8}\times\cancelto{1}{3}}=\frac{25}{4}=6\tfrac{1}{4}$$

$$x=6\tfrac{1}{4}$$

4. Work the following: (Simplify the work, when you can, by dividing the numerator and the denominator of the fraction numerals by the same number.)

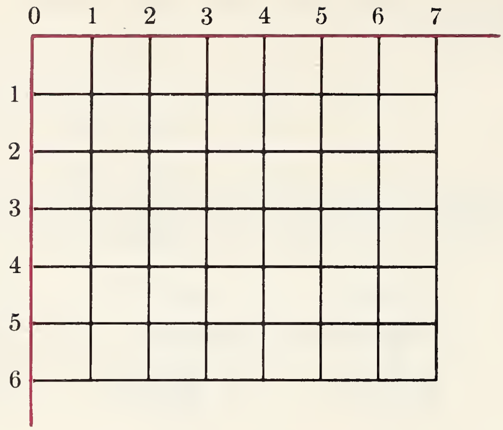
a $\frac{5}{32}\times\frac{8}{15}$	**b** $\frac{3}{16}\times\frac{8}{9}$	**c** $1\frac{1}{3}\times\frac{3}{5}$	**d** $2\frac{2}{5}\times1\frac{5}{6}$
e $\frac{19}{20}\times1\frac{2}{3}$	**f** $1\frac{1}{18}\times7\frac{1}{5}$	**g** $\frac{1}{2}\times4$	**h** $\frac{9}{15}\times10$
i $\frac{7}{8}\times24$	**j** $\frac{8}{9}\times4\frac{4}{5}$	**k** $\frac{5}{6}\times80$	**l** $\frac{7}{9}\times108$
m $\frac{5}{6}\times84$	**n** $\frac{11}{25}\times75$	**o** $\frac{20}{21}\times189$	**p** $1\frac{7}{8}\times\frac{13}{30}$

ANOTHER LOOK AT MULTIPLICATION

1 When we were thinking of the operation of multiplication with whole numbers, we were able to make a picture of the operation by using arrays. For 6×7 we could put

	7 in each row						
	×	×	×	×	×	×	×
6 rows	×	×	×	×	×	×	×
	×	×	×	×	×	×	×
	×	×	×	×	×	×	×
	×	×	×	×	×	×	×
	×	×	×	×	×	×	×

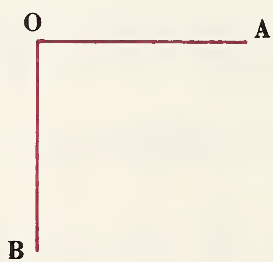
Above is an array to show that $6\times7=42$. We could also picture the operation of multiplication by using a different kind of array. We could make an array of squares.



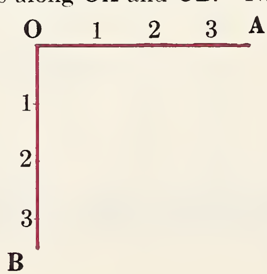
We see that six sets of seven squares give a product of 42 squares, or $6 \times 7 = 42$.

2 Use a square array to find 1×1.

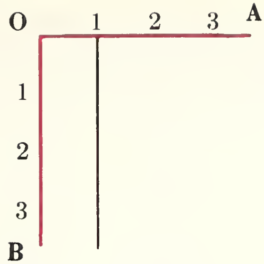
a Draw two lines at right angles to each other.



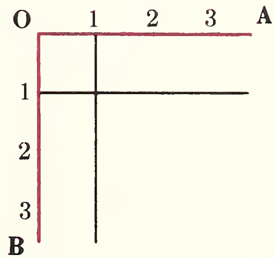
b Label as **O** the point where these two lines meet and put points at equal distances along **OA** and **OB**. Name these points 1, 2, 3, . . .



c Through the point 1 on **OA** draw a line parallel to **OB**.

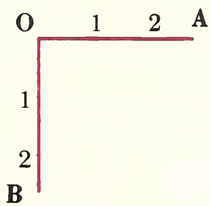


d Through the point 1 on **OB** draw a line parallel to **OA**.



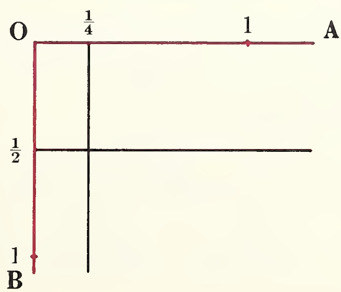
e How many squares have been formed? We see that $1 \times 1 = 1$. We think of the square representing 1×1 as 1 UNIT square.

3 Find the product of $\frac{1}{2}$ and $\frac{1}{4}$, using an array of squares.

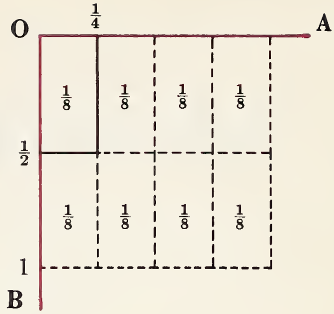


a We need to find $\frac{1}{2} \times \frac{1}{4}$. Put a point on **OB** to represent $\frac{1}{2}$ and a point on **OA** to represent $\frac{1}{4}$.

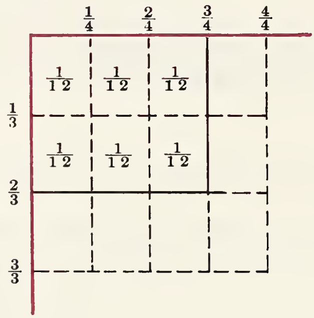
b Through the point on **OB** which represents $\frac{1}{2}$, draw a line parallel to **OA**, and through the point on **OA** which represents $\frac{1}{4}$, draw a line parallel to **OB**.



c How many units are formed by these two lines? How does the diagram below help us to find $\frac{1}{2} \times \frac{1}{4}$?

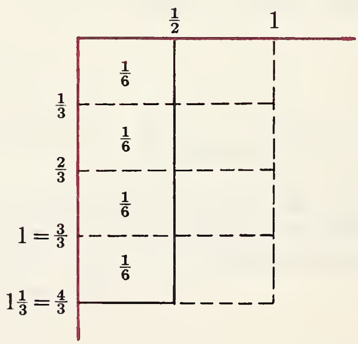


4 Use an array of units to find $\frac{2}{3} \times \frac{3}{4}$:



We see that $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$. Another numeral for $\frac{6}{12}$ is $\frac{1}{2}$.

5 Use an array to find $1\frac{1}{3} \times \frac{1}{2}$:



We see that $1\frac{1}{3} \times \frac{1}{2} = \frac{4}{3} \times \frac{1}{2} = \frac{4}{6}$;
for $\frac{4}{6}$ we can write $\frac{2}{3}$.

6. Use arrays of units to find the following:

a $1\frac{1}{2} \times \frac{2}{3}$	**b** $\frac{3}{4} \times \frac{5}{6}$	**c** $1\frac{1}{5} \times 1\frac{1}{3}$	**d** $2\frac{1}{2} \times \frac{9}{10}$	**e** $1\frac{2}{3} \times \frac{4}{5}$
f $1\frac{1}{8} \times \frac{1}{6}$	**g** $1\frac{1}{2} \times 1\frac{1}{4}$	**h** $2\frac{1}{4} \times 2\frac{1}{3}$	**i** $3 \times 4\frac{1}{4}$	**j** $2\frac{1}{2} \times 2\frac{1}{2}$

1 Study the following:

a $\frac{1}{2}\times\frac{3}{4}$; $\frac{3}{4}\times\frac{1}{2}$

$$=\frac{1\times3}{2\times4} \qquad =\frac{3\times1}{4\times2}$$

$$=\frac{3}{8} \qquad =\frac{3}{8}$$

b $1\frac{1}{3}\times2\frac{1}{4}$; $2\frac{1}{4}\times1\frac{1}{3}$

$$=\frac{4\times9}{3\times4} \qquad =\frac{9\times4}{4\times3}$$

$$=\frac{\overset{1}{\cancel{4}}\times\overset{3}{\cancel{9}}}{\underset{1}{\cancel{3}}\times\underset{1}{\cancel{4}}} \qquad =\frac{\overset{3}{\cancel{9}}\times\overset{1}{\cancel{4}}}{\underset{1}{\cancel{4}}\times\underset{1}{\cancel{3}}}$$

$$=3 \qquad =3$$

c $\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$

$$=\frac{ac}{bd}$$

$\frac{c}{d}\times\frac{a}{b}=\frac{c\times a}{d\times b}$

$$=\frac{a\times c}{b\times d} \quad \text{(Why?)}$$

$$=\frac{ac}{bd}$$

2 If $\frac{a}{b}$ and $\frac{c}{d}$ are any two fractions, then $\frac{a}{b}\times\frac{c}{d}=\frac{c}{d}\times\frac{a}{b}$.

Multiplication of fractions is commutative.

3. Work the following. What do you notice about the answers you get for each example? (The first one is done for you.)

a $(\frac{2}{3}\times1\frac{3}{4})\times\frac{5}{7}$

$$=\left(\frac{2}{3}\times\frac{7}{4}\right)\times\frac{5}{7}$$

$$=\left(\frac{\overset{1}{\cancel{2}}\times7}{3\times\underset{2}{\cancel{4}}}\right)\times\frac{5}{7}$$

$$=\frac{7}{6}\times\frac{5}{7}$$

$$=\frac{\overset{1}{\cancel{7}}\times5}{6\times\underset{1}{\cancel{7}}}$$

$$=\frac{5}{6}$$

$\frac{2}{3}\times(1\frac{3}{4}\times\frac{5}{7})$

$$=\frac{2}{3}\times\left(\frac{7}{4}\times\frac{5}{7}\right)$$

$$=\frac{2}{3}\times\left(\frac{\overset{1}{\cancel{7}}\times5}{4\times\underset{1}{\cancel{7}}}\right)$$

$$=\frac{2}{3}\times\frac{5}{4}$$

$$=\frac{\overset{1}{\cancel{2}}\times5}{3\times\underset{2}{\cancel{4}}}$$

$$=\frac{5}{6}$$

b $(\frac{5}{8}\times1\frac{1}{3})\times2\frac{1}{2};\ \frac{5}{8}\times(1\frac{1}{3}\times2\frac{1}{2})$

c $(1\frac{1}{4}\times1\frac{2}{3})\times1\frac{1}{2};\ 1\frac{1}{4}\times(1\frac{2}{3}\times1\frac{1}{2})$

d $(\frac{7}{9}\times\frac{2}{3})\times\frac{5}{8};\ \frac{7}{9}\times(\frac{2}{3}\times\frac{5}{8})$

e $(3\frac{1}{3}\times\frac{4}{5})\times\frac{7}{8};\ 3\frac{1}{3}\times(\frac{4}{5}\times\frac{7}{8})$

f $(\frac{6}{7}\times1\frac{4}{10})\times3\frac{1}{3};\ \frac{6}{7}\times(1\frac{4}{10}\times3\frac{1}{3})$

g $(6\frac{1}{3}\times\frac{3}{5})\times1\frac{2}{3};\ 6\frac{1}{3}\times(\frac{3}{5}\times1\frac{2}{3})$

h $(8\frac{1}{6}\times2\frac{1}{7})\times1\frac{1}{2};\ 8\frac{1}{6}\times(2\frac{1}{7}\times1\frac{1}{2})$

i $(\frac{2}{3}\times\frac{5}{6})\times\frac{11}{12};\ \frac{2}{3}\times(\frac{5}{6}\times\frac{11}{12})$

4. If $\frac{a}{b}$, $\frac{c}{d}$, $\frac{e}{f}$ are any three fractions, then $\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$.

Multiplication of fractions is associative.

5. a $\left(\frac{3}{4}\times\frac{3}{5}\right)\times\frac{1}{2}=\frac{3\times3}{4\times5}\times\frac{1}{2}=\frac{3\times3\times1}{4\times5\times2}=\frac{9}{40}$

$\frac{3}{4}\times\left(\frac{3}{5}\times\frac{1}{2}\right)=\frac{3}{4}\times\frac{3\times1}{5\times2}=\frac{3\times3\times1}{4\times5\times2}=\frac{9}{40}$

From this we can see that $\frac{3}{4}\times\frac{3}{5}\times\frac{1}{2}=\frac{3\times3\times1}{4\times5\times2}=\frac{9}{40}$.

b $\frac{2}{5}\times\frac{3}{4}\times2\frac{1}{2}=\frac{\overset{1}{2}}{\underset{1}{5}}\times\frac{3}{\underset{2}{4}}\times\frac{\overset{1}{5}}{2}=\frac{3}{4}$

6. $\frac{a}{b}\times\frac{c}{d}\times\frac{e}{f}=\frac{a\times c\times e}{b\times d\times f}$

7. Work the following: (Simplify your computation by dividing the numerator and the denominator by the same number wherever you can.)

a $\frac{3}{4}\times\frac{5}{6}\times\frac{4}{10}$ b $\frac{8}{9}\times\frac{4}{5}\times\frac{15}{16}$ c $\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}$

d $\frac{9}{16}\times\frac{5}{12}\times\frac{8}{15}$ e $\frac{3}{3}\times\frac{4}{4}\times\frac{5}{5}$ f $\frac{19}{20}\times\frac{5}{6}\times\frac{4}{5}$

g $\frac{3}{7}\times\frac{8}{9}\times\frac{21}{24}$ h $\frac{9}{10}\times\frac{5}{8}\times\frac{1}{6}$ i $\frac{14}{25}\times\frac{1}{2}\times\frac{5}{7}$

j $\frac{6}{6}\times\frac{1}{6}\times\frac{2}{2}$ k $\frac{9}{10}\times\frac{5}{18}\times\frac{2}{3}$ l $2\frac{1}{2}\times\frac{4}{5}\times\frac{2}{3}$

m $6\frac{1}{4}\times1\frac{1}{2}\times\frac{3}{5}$ n $\frac{5}{8}\times2\frac{2}{3}\times1\frac{1}{5}$ o $1\frac{3}{10}\times1\frac{1}{8}\times13\frac{1}{3}$

p $\frac{5}{18}\times\frac{11}{12}\times7\frac{1}{5}$ q $20\times1\frac{1}{5}\times\frac{1}{6}$ r $8\times\frac{1}{5}\times15$

s $\frac{8}{21}\times\frac{3}{4}\times14$ t $\frac{11}{15}\times10\times2\frac{1}{4}$ u $350\times1\frac{1}{7}\times1\frac{9}{10}$

v $\frac{14}{50}\times33\frac{1}{3}\times\frac{4}{51}$ w $8\frac{1}{3}\times2\frac{4}{5}\times1\frac{9}{21}$ x $5\times9\frac{1}{15}\times1\frac{7}{8}$

y $2\frac{2}{3}\times1\frac{1}{5}\times\frac{5}{24}$ z $140\times7\frac{1}{7}\times3\frac{4}{14}$

PROBLEMS INVOLVING MULTIPLICATION OF FRACTIONS

1. The exhaust chamber of a rocket contains 25 pieces of metal tubing. If each piece of metal tubing is $5\frac{5}{8}$ feet long, what number of feet of metal tubing would be needed in the construction of 12 similar rockets?

2. A ship had an average speed of $24\frac{2}{5}$ miles an hour. How many miles would it travel in 3 hours 20 minutes at this speed?

3. Find the total cost of the following goods:
$2\frac{3}{4}$ pounds of steak at 96¢ a pound;
$3\frac{1}{2}$ dozen eggs at 58¢ a dozen;
$1\frac{9}{16}$ pounds of cheese at 54¢ a pound;
$2\frac{11}{16}$ pounds of bananas at 16¢ a pound;
$4\frac{5}{8}$ pounds of minced beef at 64¢ a pound.

4. A carpenter made bookshelves that contained three shelves each. Each shelf was $4\frac{2}{3}$ feet long. How many feet of lumber would he need to make the shelves for a score of these bookcases?

5. An airplane flew at a speed of 680 miles an hour with a following wind. When it flew into the wind it flew at $\frac{3}{4}$ of this speed. How many miles would it fly in 4 hours 15 minutes if it flew into the wind?

6. A girl could embroider $1\frac{2}{3}$ yards of material in 1 day. Her mother could work $2\frac{1}{5}$ times as quickly. What number of yards of material could the mother embroider in $1\frac{1}{2}$ days?

7. An engine uses $\frac{3}{16}$ pint of oil in 1 hour. What number of pints of oil will it use in 3 hours 10 minutes at this rate?

8. A man runs a mile in $4\frac{1}{2}$ minutes. It takes him $3\frac{1}{3}$ times as long to walk a mile. How many hours will it take him to walk $5\frac{3}{10}$ miles?

9. Find the product of $5\frac{1}{9}$, $2\frac{2}{13}$, and $6\frac{2}{7}$.

RECIPROCALS

1 What pairs of numbers are there that, when multiplied together, give 1 as the product? One pair is (1, 1). We see immediately that $1\times1=1$. By what must we multiply $\frac{1}{2}$ to get 1 as the product? $2\times\frac{1}{2}=1$. We have seen that another numeral for 2 is $\frac{2}{1}$ and we can see that $\frac{2}{1}\times\frac{1}{2}=1$.

2. By what must we multiply each of the following numbers to get 1 as the product? $\frac{1}{3}$, 4, $\frac{1}{5}$, 5, 3, $\frac{1}{4}$, $\frac{1}{10}$, 8.

3 We see that $3\times\frac{1}{3}=1$ and $\frac{1}{3}\times3=1$,
and that $\frac{3}{1}\times\frac{1}{3}=1$ and $\frac{1}{3}\times\frac{3}{1}=1$,
and that $\frac{4}{1}\times\frac{1}{4}=1$, $\frac{1}{5}\times\frac{5}{1}=1$, . . .

4 We have a name for pairs of numbers that have 1 as their product. We call them **reciprocals.**

5. We say, "3 is the reciprocal of $\frac{1}{3}$"; and also we say, "$\frac{1}{3}$ is the reciprocal of 3." What are the reciprocals of $\frac{1}{9}$? 15? 35? $\frac{1}{12}$? $\frac{1}{16}$? $\frac{1}{10}$? 21?

6 What is the reciprocal of $\frac{3}{5}$? If we let $\frac{a}{b}$ be the reciprocal of $\frac{3}{5}$,

then $\frac{3}{5}\times\frac{a}{b}=1$

or $\frac{3a}{5b}=1$

If $\frac{3a}{5b}=1$, then $3a$ and $5b$ must have the same value.

$3a=5b$. Some solutions to this open number sentence are $\{(a=5,\ b=3),\ (a=10,\ b=6),\ (a=15,\ b=9),\ \ldots\}$

If $a=5$ and $b=3$, then $\frac{a}{b}=\frac{5}{3}$.

If $a=10$ and $b=6$, then $\frac{a}{b}=\frac{10}{6}=\frac{5}{3}$.

If $a=15$ and $b=9$, then $\frac{a}{b}=\frac{15}{9}=\frac{5}{3}$.

Whatever pair of numbers we take for a and b, we see that $\frac{a}{b}=\frac{5}{3}$; so the the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$. Check: $\frac{3}{5}\times\frac{5}{3}=\frac{3\times5}{5\times3}=\frac{15}{15}=1$.

7. Write the reciprocals of the following:
$\frac{3}{4}$, $\frac{2}{9}$, $\frac{4}{5}$, $\frac{11}{10}$, $\frac{5}{8}$, $\frac{9}{10}$, $\frac{7}{16}$, $\frac{21}{5}$, $\frac{7}{2}$, $\frac{14}{15}$.

8 What is the reciprocal of $1\frac{1}{2}$? We can rename $1\frac{1}{2}$ as $\frac{3}{2}$. By what number must we multiply $\frac{3}{2}$ to get 1 as the product? If we work in the same way as we did in No. 6, we see that $\frac{3}{2}\times\frac{2}{3}=1$; so $\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$ or $1\frac{1}{2}$.

9. Write the reciprocals of the following: $1\frac{2}{5}$, $3\frac{1}{3}$, $4\frac{3}{4}$, $2\frac{9}{10}$, $5\frac{4}{9}$.
For $1\frac{2}{5}$ we can write $\frac{7}{5}$.
$\frac{7}{5}\times\frac{5}{7}=\frac{35}{35}=1$. So $\frac{5}{7}$ is the reciprocal of $\frac{7}{5}$ or $1\frac{2}{5}$. Now work the remaining examples.

10 If $\frac{a}{b}$ is a fraction with a and b natural numbers, what is its reciprocal?

Since $\frac{a}{b} \times \frac{b}{a} = \frac{ab}{ba} = \frac{ab}{ab} = 1$, we see that the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

11. Why did we limit a and b to natural numbers? Suppose we take $a=0$ and let $b=2$. Then $\frac{0}{2}$ is a fraction but $\frac{2}{0}$ has no meaning.

DIVISION OF FRACTIONS

1 Compute: $\frac{3}{4} \div \frac{5}{8} = n$

Does $\dfrac{\frac{3}{4}}{\frac{5}{8}} = \frac{3}{4} \div \frac{5}{8}$?

For $\frac{3}{4} \div \frac{5}{8}$ we can write $\dfrac{\frac{3}{4}}{\frac{5}{8}}$.

Consider $\dfrac{\frac{3}{4}}{\frac{5}{8}}$. The reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$. What is the value of $\dfrac{\frac{8}{5}}{\frac{8}{5}}$?

Do we alter the value of $\dfrac{\frac{3}{4}}{\frac{5}{8}}$ if we multiply it by 1?

Does $\dfrac{\frac{3}{4}}{\frac{5}{8}}$ have the same value as $\dfrac{\frac{3}{4}}{\frac{5}{8}} \times \dfrac{\frac{8}{5}}{\frac{8}{5}}$? Why?

For $\dfrac{\frac{3}{4}}{\frac{5}{8}}$ we can write $\dfrac{\frac{3}{4}}{\frac{5}{8}} \times \dfrac{\frac{8}{5}}{\frac{8}{5}} = \dfrac{\frac{3}{4} \times \frac{8}{5}}{\frac{5}{8} \times \frac{8}{5}}$. But the product of $\frac{5}{8}$ and $\frac{8}{5} = 1$; so we can write

$\dfrac{\frac{3}{4}}{\frac{5}{8}} = \dfrac{\frac{3}{4} \times \frac{8}{5}}{\frac{5}{8} \times \frac{8}{5}} = \dfrac{\frac{3}{4} \times \frac{8}{5}}{1} = \frac{3}{4} \times \frac{8}{5}$. $\frac{8}{5}$ is the reciprocal of $\frac{5}{8}$. We see that to divide $\frac{3}{4}$ by $\frac{5}{8}$ we get the answer if we *multiply* together $\frac{3}{4}$ and the *reciprocal* of $\frac{5}{8}$.

$\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \times \frac{8}{5} = \frac{24}{20} = 1\frac{4}{20} = 1\frac{1}{5} = n$;

$$\text{or } \frac{3}{4} \div \frac{5}{8} = \frac{3}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{5} = \frac{6}{5} = 1\frac{1}{5} = n$$

Check: $\frac{3}{4} \div \frac{5}{8} = 1\frac{1}{5}$ means $\frac{3}{4} = 1\frac{1}{5} \times \frac{5}{8}$.

$$1\tfrac{1}{5} \times \tfrac{5}{8} = \frac{\overset{3}{\cancel{6}} \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \times \underset{4}{\cancel{8}}} = \tfrac{3}{4}$$

In this way we prove our answer.

2 $2\frac{2}{3} \div 1\frac{5}{9} = x$

$$2\frac{2}{3} \div 1\frac{5}{9} = \frac{2\frac{2}{3}}{1\frac{5}{9}} = \frac{\frac{8}{3}}{\frac{14}{9}}$$

The reciprocal of $\frac{14}{9}$ is $\frac{9}{14}$. Multiply together $\frac{\frac{8}{3}}{\frac{14}{9}}$ and $\frac{\frac{9}{14}}{\frac{9}{14}}$ (i.e., 1).

$$\frac{2\frac{2}{3}}{1\frac{5}{9}} = \frac{\frac{8}{3}}{\frac{14}{9}} \times \frac{\frac{9}{14}}{\frac{9}{14}} = \frac{\frac{8}{3} \times \frac{9}{14}}{\frac{14}{9} \times \frac{9}{14}} = \frac{\frac{8}{3} \times \frac{9}{14}}{1} = \frac{8}{3} \times \frac{9}{14} = \frac{\overset{4}{\cancel{8}} \times \overset{3}{\cancel{9}}}{\underset{1}{\cancel{3}} \times \underset{7}{\cancel{14}}} = \frac{12}{7} = 1\frac{5}{7}$$

To find $2\frac{2}{3} \div 1\frac{5}{9}$, which is the same as $\frac{8}{3} \div \frac{14}{9}$, we obtain the product of $2\frac{2}{3}$ and the reciprocal of $1\frac{5}{9}$. We can write the operation thus:

$$2\frac{2}{3} \div 1\frac{5}{9} = \frac{8}{3} \div \frac{14}{9} = \frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{7}{\cancel{14}}} = \frac{12}{7} = 1\frac{5}{7}$$

Check: $2\frac{2}{3} \div 1\frac{5}{9} = 1\frac{5}{7}$ means $2\frac{2}{3} = 1\frac{5}{7} \times 1\frac{5}{9}$

$$1\frac{5}{7} \times 1\frac{5}{9} = \frac{12}{7} \times \frac{14}{9} = \frac{\overset{4}{\cancel{12}} \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}} \times \underset{3}{\cancel{9}}} = \frac{8}{3} = 2\frac{2}{3}$$

Our answer is correct.

3 If $\frac{a}{b}$ and $\frac{c}{d}$ are two fractions,

then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$.

4. Show that in No. 3 the fraction $\frac{c}{d}$ cannot equal zero.

Can $\frac{a}{b}$ equal zero? Why?

5 Find x if $x = \frac{3}{4} \div 2$.

$$\frac{3}{4} \div 2 = \frac{\frac{3}{4}}{2} = \frac{\frac{3}{4} \times \frac{1}{2}}{2 \times \frac{1}{2}} = \frac{\frac{3}{4} \times \frac{1}{2}}{1} = \frac{3}{8}; \text{ or}$$

$\frac{3}{4} \div 2 = \frac{3}{4} \times \frac{1}{2}$ (since $\frac{1}{2}$ is the reciprocal of 2)

$= \frac{3}{8}$;

so $x = \frac{3}{8}$

6. Work the following:

a $\frac{5}{8} \div \frac{3}{16}$ b $\frac{1}{3} \div \frac{3}{4}$ c $\frac{7}{8} \div 2$

d	$1\frac{2}{3}\div 5$	**e**	$6\div 4\frac{1}{2}$	**f**	$\frac{3}{4}\div 1\frac{3}{5}$
g	$\frac{5}{6}\div\frac{5}{12}$	**h**	$3\frac{3}{10}\div 2\frac{1}{8}$	**i**	$3\frac{1}{5}\div 1\frac{1}{3}$
j	$6\div\frac{3}{4}$	**k**	$7\div 2\frac{1}{3}$	**l**	$5\div 9$
m	$1\frac{1}{2}\div 2\frac{3}{4}$	**n**	$3\frac{5}{6}\div\frac{11}{12}$	**o**	$5\frac{5}{9}\div 10$
p	$\frac{5}{8}\div\frac{15}{16}$	**q**	$\frac{9}{10}\div 1\frac{4}{15}$	**r**	$\frac{7}{12}\div 5$
s	$8\div\frac{8}{21}$	**t**	$11\div 1\frac{5}{9}$	**u**	$2\div 2\frac{4}{5}$
v	$3\frac{1}{4}\div\frac{5}{6}$	**w**	$6\frac{1}{4}\div 6\frac{1}{4}$	**x**	$18\frac{1}{3}\div 15$
y	$\frac{7}{8}\div\frac{5}{12}$	**z**	$\frac{2}{5}\div 9$	**aa**	$\frac{3}{5}\div 2\frac{2}{5}$

7 We have seen, using natural numbers, that $7\times 4=28$ can be used to show us an example of the operation of division. Division is the inverse operation of multiplication. $7\times 4=28$ tells us that $28\div 4=7$. Now, $\frac{2}{3}\times\frac{4}{5}=\frac{8}{15}$. The inverse operation tells us that $\frac{8}{15}\div\frac{4}{5}=\frac{2}{3}$.

$$\frac{2}{3}=\frac{40}{60}=\frac{8\times 5}{15\times 4}=\frac{8}{15}\times\frac{5}{4}.$$

$\frac{5}{4}$ is the reciprocal of $\frac{4}{5}$.

Therefore, $\frac{8}{15}\div\frac{4}{5}=\frac{8}{15}\times$ reciprocal of $\frac{4}{5}$.

This is another way of demonstrating the rule for the division of fractions.

8 $1\frac{1}{3}\times 1\frac{7}{8}=\frac{4}{3}\times\frac{15}{8}$

$$\frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{8}}}=\frac{5}{2}$$

$\frac{4}{3}\times\frac{15}{8}=\frac{5}{2}$ tells us that

$$\frac{5}{2}\div\frac{15}{8}=\frac{4}{3}=\frac{40}{30}=\frac{5\times 8}{2\times 15}=\frac{5}{2}\times\frac{8}{15}=\frac{5}{2}\times\text{ the reciprocal of }\frac{15}{8}.$$

9. Use the multiplication fact, $\frac{3}{4}\times\frac{3}{8}=\frac{9}{32}$, to show that $\frac{9}{32}\div\frac{3}{8}=\frac{9}{32}\times$ reciprocal of $\frac{3}{8}$.

PROBLEMS INVOLVING DIVISION OF FRACTIONS

1. A strip of carpeting 25 yds. long is to be cut into 18 inch samples for salesmen. What number of salesmen will be able to take a piece each?
2. About $2\frac{1}{2}$ lb. of strawberries will fill one basket. What number of baskets can be filled from 3 cwt. of strawberries?
3. A man takes $3\frac{1}{2}$ hours to build $\frac{7}{8}$ of a fence. What fraction of the fence would be done if he had worked for only one hour?
4. A time sheet shows that a total of $45\frac{3}{4}$ hours was worked by a gang of men. If each man worked for 45 minutes, give the number of men involved.

5. $\frac{7}{8}$ of a concrete driveway contains 18 tons of mix. What fraction of the driveway would 1 ton of mix make?

6. $27\frac{3}{4}$ cran of herring are caught by a fishing boat with a crew of 3. If each man takes an equal share of the catch, what amount of herring belongs to each man?

7. How many $1\frac{1}{2}$ lb. bags of sugar can be filled from 5 sacks, each containing 1 cwt?

8. $\frac{2}{3}$ of an inch on a map represents 1 mile. How long a line segment would be needed to represent 32 miles?

9. A man leaves $\frac{2}{3}$ of his fortune to be shared equally among his wife, son and daughter. What fraction of the fortune should each receive?

10. $39\frac{7}{8}$ miles is the distance between Farm A and Farm B. There is a boundary fence $\frac{2}{3}$ of the way from A to B. What is the distance from Farm B to the fence?

ADDITION OF FRACTIONS

1. If you add $\frac{1}{4}$ and $\frac{2}{4}$, how many 4ths do you have?
2. Add $\frac{3}{10}$ and $\frac{1}{10}$. How many 10ths do you have?
3. What is $\frac{2}{9}$ and $\frac{5}{9}$?
4. Add:

a $\frac{3}{8}+\frac{2}{8}$	**b** $\frac{3}{8}+\frac{1}{8}$	**c** $\frac{9}{10}+\frac{7}{10}$
d $\frac{7}{8}+\frac{9}{8}$	**e** $\frac{11}{12}+\frac{11}{12}$	**f** $\frac{15}{18}+\frac{21}{18}$
g $\frac{9}{15}+\frac{7}{15}+\frac{8}{15}$	**h** $\frac{7}{3}+\frac{2}{3}+\frac{8}{3}$	**i** $\frac{5}{4}+\frac{7}{4}+\frac{1}{4}$
j $\frac{7}{10}+\frac{12}{10}+\frac{1}{10}$	**k** $\frac{5}{12}+\frac{6}{12}+\frac{3}{12}$	**l** $\frac{4}{9}+\frac{33}{9}+\frac{5}{9}$

Do you remember how we add fractions when the denominators are alike? We merely add the numerators using the same denominator.

5 If a and c represent any whole numbers, and b represents any natural number, $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$.

6 Add $\frac{2}{3}+\frac{3}{5}$.

$\frac{2}{3}=\frac{2}{3}\times\frac{2}{2}=\frac{2}{3}\times\frac{3}{3}=\frac{2}{3}\times\frac{4}{4}=\frac{2}{3}\times\frac{5}{5}=\ \dots$

$\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\frac{8}{12}=\frac{10}{15}=\ \dots$

$\frac{3}{5}=\frac{3}{5}\times\frac{2}{2}=\frac{3}{5}\times\frac{3}{3}=\frac{3}{5}\times\frac{4}{4}=\frac{3}{5}\times\frac{5}{5}=\ \dots$

$\frac{3}{5}=\frac{6}{10}=\frac{9}{15}=\frac{12}{20}=\frac{15}{25}=\ \dots$

What common denominator can we use to add $\frac{2}{3}$ and $\frac{3}{5}$?

$\frac{2}{3}=\frac{10}{15}$; $\frac{3}{5}=\frac{9}{15}$; so $\frac{2}{3}+\frac{3}{5}=\frac{10}{15}+\frac{9}{15}=\frac{19}{15}$.

We can rename $\frac{19}{15}$: $\frac{19}{15}=\frac{15+4}{15}=\frac{15}{15}+\frac{4}{15}=1+\frac{4}{15}=1\frac{4}{15}$

To obtain the common denominator, we find that we can multiply 3 and 5 together. Can we always find the common denominator by multiplying the denominators together? Let us consider $\frac{1}{8}+\frac{1}{4}$. Multiply the two denominators and we have 8×4. 4 will divide 8×4, and 8 will divide 8×4; so 8×4 is a common denominator for eighths and fourths.

$\frac{5}{6}$ and $\frac{3}{10}$. Multiply the denominators and we have $6\times10=60$. Will both 6 and 10 divide 6×10? Is 6×10 a common denominator for sixths and tenths? Now let us examine the examples below:

$\frac{1}{3}+\frac{3}{4}=(\frac{1}{3}\times\frac{4}{4})+(\frac{3}{4}\times\frac{3}{3})=\frac{4}{12}+\frac{9}{12}=\frac{13}{12}$

$\frac{3}{8}+\frac{2}{5}=(\frac{3}{8}\times\frac{5}{5})+(\frac{2}{5}\times\frac{8}{8})=\frac{15}{40}+\frac{16}{40}=\frac{31}{40}$

7 $$\frac{a}{b}+\frac{c}{d}=\frac{a}{b}\times\frac{d}{d}+\frac{c}{d}\times\frac{b}{b}=\frac{a\times d}{b\times d}+\frac{c\times b}{d\times b}=\frac{a\times d}{b\times d}+\frac{c\times b}{b\times d}$$
$$=\frac{(a\times d)+(c\times b)}{b\times d}=\frac{ad+cb}{bd}$$

(Remember, for $a\times d$ we can write ad, and for $c\times b$ we can write cb, and for $b\times d$ we can write bd.)

8. Add:

a $\frac{7}{9}+\frac{2}{5}$	**b** $\frac{11}{7}+\frac{3}{4}$	**c** $\frac{5}{3}+\frac{7}{8}$	**d** $\frac{18}{11}+\frac{9}{7}$	**e** $\frac{7}{8}+\frac{4}{5}$
f $\frac{7}{8}+\frac{9}{5}$	**g** $\frac{2}{3}+\frac{5}{12}$	**h** $\frac{8}{11}+\frac{3}{2}$	**i** $\frac{7}{4}+\frac{11}{9}$	**j** $\frac{3}{4}+\frac{3}{10}$
k $\frac{10}{21}+\frac{19}{28}$	**l** $\frac{6}{3}+\frac{5}{4}$	**m** $\frac{3}{10}+\frac{7}{15}$	**n** $\frac{2}{5}+\frac{5}{4}$	**o** $\frac{2}{6}+\frac{7}{6}$

9. Find $\frac{3}{4}+\frac{1}{2}$; $\frac{1}{2}+\frac{3}{4}$;

$\frac{3}{4}+\frac{1}{2}=\frac{3}{4}+\frac{2}{4}=\frac{5}{4}$ $\qquad$ $\frac{1}{2}+\frac{3}{4}=\frac{2}{4}+\frac{3}{4}=\frac{5}{4}$

What do you notice about the sum in each case?

10 Find the following sums:

a $\frac{2}{3}+\frac{1}{2}$; $\frac{1}{2}+\frac{2}{3}$	**b** $\frac{9}{10}+\frac{3}{5}$; $\frac{3}{5}+\frac{9}{10}$	**c** $\frac{7}{8}+\frac{9}{2}$; $\frac{9}{2}+\frac{7}{8}$
d $\frac{6}{5}+\frac{5}{11}$; $\frac{5}{11}+\frac{6}{5}$	**e** $\frac{9}{10}+\frac{8}{10}$; $\frac{8}{10}+\frac{9}{10}$	**f** $\frac{15}{16}+\frac{21}{16}$; $\frac{21}{16}+\frac{15}{16}$
g $\frac{1}{2}+\frac{3}{8}$; $\frac{3}{8}+\frac{1}{2}$	**h** $\frac{17}{20}+\frac{5}{6}$; $\frac{5}{6}+\frac{17}{20}$	**i** $\frac{4}{2}+\frac{12}{4}$; $\frac{12}{4}+\frac{4}{2}$

What do you notice about the sum in each case? We see that for all fractions we can make a general rule:

If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions, then $\frac{a}{b}+\frac{c}{d}=\frac{c}{d}+\frac{a}{b}$.

Addition with fractions is commutative.

11. Find the following:

a $(\frac{1}{4}+\frac{3}{4})+\frac{3}{4}$; $\frac{1}{4}+(\frac{3}{4}+\frac{3}{4})$

$\frac{4}{4}+\frac{3}{4}$ $\qquad$ $\frac{1}{4}+\frac{6}{4}$

$=\frac{7}{4}$ $\qquad$ $=\frac{7}{4}$

b $\frac{2}{3}+(\frac{1}{2}+\frac{1}{3})$; $\qquad$ $(\frac{2}{3}+\frac{1}{2})+\frac{1}{3}$

$\frac{4}{6}+(\frac{3}{6}+\frac{2}{6})$ $\qquad$ $(\frac{4}{6}+\frac{3}{6})+\frac{2}{6}$

$\frac{4}{6}+\frac{5}{6}=\frac{9}{6}=1\frac{1}{2}$ $\qquad$ $\frac{7}{6}+\frac{2}{6}=\frac{9}{6}=1\frac{1}{2}$

What do we notice about the two answers?

12. Work the following:

a $\frac{7}{2}+(\frac{3}{4}+\frac{1}{4})$; $(\frac{7}{2}+\frac{3}{4})+\frac{1}{4}$

b $\frac{2}{5}+(\frac{9}{10}+\frac{3}{5})$; $(\frac{2}{5}+\frac{9}{10})+\frac{3}{5}$

c $\frac{11}{12}+(\frac{7}{8}+\frac{5}{6})$; $(\frac{11}{12}+\frac{7}{8})+\frac{5}{6}$

d $(\frac{4}{5}+\frac{3}{5})+\frac{7}{8}$; $\frac{4}{5}+(\frac{3}{5}+\frac{7}{8})$

e $\frac{9}{10}+(\frac{4}{9}+\frac{1}{8})$; $(\frac{9}{10}+\frac{4}{9})+\frac{1}{8}$

f $(\frac{1}{3}+\frac{4}{9})+\frac{3}{4}$; $\frac{1}{3}+(\frac{4}{9}+\frac{3}{4})$

What do we notice about the sums in each case?
What are we led to believe about the addition of fractions?

13 If $\frac{a}{b}$ and $\frac{c}{d}$ and $\frac{e}{f}$ are three fractions, then

$$\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right).$$

Addition of fractions is associative.

14 Find the sum of $3\frac{1}{4}$ and $1\frac{3}{4}$:

$3\frac{1}{4}+1\frac{3}{4}=3+\frac{1}{4}+1+\frac{3}{4}=3+1+\frac{1}{4}+\frac{3}{4}=4+\frac{4}{4}=4+1=5$

When we add mixed numbers, we see that we can add the whole numbers and then the fractions and that the two sums together give us our final answer.

15. Work the following:

a $\frac{3}{4}+\frac{4}{4}+\frac{2}{3}$

b $1\frac{1}{24}+3\frac{7}{36}+5\frac{7}{12}$

c $\frac{13}{15}+1\frac{9}{10}+2\frac{3}{5}$

d $\frac{5}{6}+1\frac{1}{2}+2\frac{2}{3}$

e $2\frac{3}{8}+4\frac{5}{12}+\frac{8}{5}$

f $5\frac{1}{2}+1\frac{7}{8}+3\frac{1}{4}$

g $3\frac{6}{7}+4\frac{3}{10}+5\frac{3}{8}$

h $2\frac{5}{9}+1\frac{11}{12}+3\frac{3}{4}$

16 Add $1\frac{3}{4}+2\frac{5}{12}+\frac{7}{2}$:

$$\begin{aligned}1\tfrac{3}{4}+2\tfrac{5}{12}+\tfrac{7}{2}&=1\tfrac{3}{4}+2\tfrac{5}{12}+3\tfrac{1}{2}\\&=1\tfrac{9}{12}+2\tfrac{5}{12}+3\tfrac{6}{12}\\&=6\tfrac{9+5+6}{12}\end{aligned}$$

$=6\frac{20}{12}$
$=6\frac{12+8}{12}$
$=6+\frac{12}{12}+\frac{8}{12}$
$=6+1+\frac{8}{12}$
$=7+\frac{2}{3}$
$=7\frac{2}{3}$

The answer, $7\frac{2}{3}$, is in its simplest form; it is written as a mixed number with the fractional part in its lowest terms. It is often best for us to put our answers in their simplest forms.

17. Work the following, putting the answers in their simplest forms:

a $3\frac{9}{10}+2\frac{3}{8}+1\frac{3}{4}$ **b** $5\frac{1}{10}+1\frac{2}{5}+3\frac{1}{2}$

c $1\frac{3}{8}+2\frac{5}{6}+5\frac{2}{8}$ **d** $4\frac{2}{3}+3\frac{1}{5}+5\frac{7}{12}$

e $4\frac{1}{3}+2\frac{3}{10}+2\frac{1}{2}$ **f** $3\frac{1}{4}+3\frac{3}{8}+1\frac{9}{16}$

g $2\frac{3}{8}+1\frac{1}{4}+1\frac{7}{12}$ **h** $9\frac{1}{5}+\frac{1}{3}+8$

i $\frac{3}{2}+\frac{9}{4}+\frac{11}{6}$ **j** $3\frac{4}{8}+9\frac{7}{8}+1\frac{1}{9}$

PROBLEMS INVOLVING ADDITION OF FRACTIONS

1. A tent maker requires $23\frac{1}{2}$ yds. of canvas for the walls and $14\frac{2}{5}$ yds. for the roof of a tent he is making. How many yards of canvas does he require to make the complete tent?

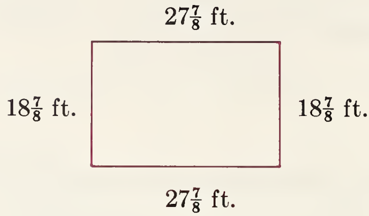
2. A group of scouts joined short lengths of rope together in order to descend a steep bank. The lengths measured $3\frac{1}{2}$ ft., $2\frac{3}{4}$ ft., 18 inches, and $4\frac{3}{8}$ ft. How many feet long was the joined rope? (Allow 1 foot for the joinings).

3. What is the distance in feet around a triangular flower-bed whose sides measure $9\frac{2}{5}$ ft., $8\frac{3}{4}$ ft., and $7\frac{1}{3}$ ft.?

4. The ages of three children are $12\frac{1}{4}$ years, $11\frac{2}{3}$ years, and $9\frac{3}{4}$ years. What is the sum of their ages?

5. A dress requires $56\frac{3}{8}$ inches of binding for the hem and $8\frac{5}{16}$ inches for each sleeve. Altogether how many inches of binding are needed?

6. Find the cost, at 12¢ a foot, of wooden moulding to go all around a room which is $18\frac{7}{8}$ ft. wide and $27\frac{7}{8}$ ft. long.



7. A passenger is travelling from Toronto, Canada to Belfast, in Northern Ireland. The times taken for each stage of the journey are shown below:

(*Car*)	Royal York Hotel to Toronto International Airport	$\frac{3}{4}$ hr.
(*Plane*)	Toronto to Shannon	$12\frac{1}{3}$ hrs.
(*Car*)	Shannon to Belfast	$7\frac{3}{8}$ hrs.

How many hours did the whole journey take?

8. Five partners in a company ask for the following shares of the profit: $\frac{1}{4}$, $\frac{1}{6}$, $\frac{2}{5}$, $\frac{1}{3}$, $\frac{1}{6}$. Is this reasonable? Explain your answer.

9. A plumber needs four lengths of copper piping to install a hot water tank. The lengths required are as follows:

   $3\frac{7}{8}$ ft., $2\frac{1}{4}$ ft., 18 inches and $2\frac{3}{4}$ ft.

   How many feet of pipe does he need in all?

10. A boat builder bores a $\frac{5}{16}$ inch hole through the gunwale of a boat. He wishes to fasten a metal plate $\frac{3}{16}$ inches thick to the gunwale, using a $\frac{1}{8}$ inch washer and a $\frac{3}{8}$ inch nut. Find the length in inches of the bolt he will require.

MULTIPLICATION AND ADDITION OF FRACTIONS

1 Find $6\frac{1}{2}\times 4$.

a We can think of this as a multiplication example, using fractions as the factors: $6\frac{1}{2}=\frac{13}{2}$, $4=\frac{4}{1}$

$$\text{So } 6\tfrac{1}{2}\times 4=\frac{13}{2}\times\frac{4}{1}=\frac{13\times 4}{2\times 1}=\frac{52}{2}=26$$

b Does the distributive property apply to fractions? We think of $6\frac{1}{2}$ as being $6+\frac{1}{2}$. Then $6\frac{1}{2}\times 4=(6+\frac{1}{2})\times 4=(6\times 4)+(\frac{1}{2}\times 4)$;
$6\times 4=24$; $\frac{1}{2}\times 4=2$
So $(6\times 4)+(\frac{1}{2}\times 4)=24+2=26$
By using the distributive property, we get the same answer as we did when we used the rule we had learned for multiplying fractions.

2 Study the examples below:

a

$$3\tfrac{1}{2}\times\frac{5}{8}=\frac{7}{2}\times\frac{5}{8}=\frac{7\times 5}{2\times 8}=\frac{35}{16}=2\tfrac{3}{16}$$

$$3\tfrac{1}{2}\times\frac{5}{8}=\left(3+\frac{1}{2}\right)\times\frac{5}{8}=\left(\frac{3}{1}\times\frac{5}{8}\right)+\left(\frac{1}{2}\times\frac{5}{8}\right)=\frac{15}{8}+\frac{5}{16}=\frac{30}{16}+\frac{5}{16}=\frac{35}{16}=2\tfrac{3}{16}$$

b

$$\left(\frac{3}{5}\times\frac{1}{2}\right)+\left(\frac{2}{5}\times\frac{1}{2}\right)=\frac{3\times 1}{5\times 2}+\frac{2\times 1}{5\times 2}=\frac{3}{10}+\frac{2}{10}=\frac{5}{10}=\frac{1}{2}$$

$$\left(\frac{3}{5}\times\frac{1}{2}\right)+\left(\frac{2}{5}\times\frac{1}{2}\right)=\left(\frac{3}{5}+\frac{2}{5}\right)\times\frac{1}{2}=\frac{5}{5}\times\frac{1}{2}=1\times\frac{1}{2}=\frac{1}{2}$$

c

$$\left(5\tfrac{1}{4}\times\frac{2}{3}\right)+\left(\frac{3}{4}\times\frac{2}{3}\right)=\left(\frac{21}{4}\times\frac{2}{3}\right)+\left(\frac{3}{4}\times\frac{2}{3}\right)=\frac{42}{12}+\frac{6}{12}=\frac{48}{12}=4$$

$$\left(5\tfrac{1}{4}\times\frac{2}{3}\right)+\left(\frac{3}{4}\times\frac{2}{3}\right)=\left(5\tfrac{1}{4}+\frac{3}{4}\right)\times\frac{2}{3}=6\times\frac{2}{3}=\frac{12}{3}=4$$

How do the pairs of examples above illustrate the distributive property with respect to fractions?

The distributive property holds with respect to fractions.

3. Work the following: (Use the distributive property where you can in order to find the simplest computation. The first is done for you.)

a $(1\frac{3}{4}\times 2)+(\frac{1}{4}\times 2)=(1\frac{3}{4}+\frac{1}{4})\times 2=2\times 2=4$

b $(5\times\frac{2}{3})+(\frac{1}{4}\times\frac{1}{3})$ **c** $(\frac{1}{4}\times\frac{5}{6})+(\frac{3}{4}\times\frac{5}{6})$

d $9\frac{1}{5}\times 125$ **e** $64\times 7\frac{1}{8}$

f $106\times 2\frac{1}{2}$ **g** $8\frac{3}{4}\times 24$

h $(1\frac{4}{9}\times 3\frac{1}{3})+(2\frac{5}{9}\times 3\frac{1}{3})$ **i** $(\frac{5}{6}\times\frac{2}{3})+(1\frac{1}{2}\times\frac{1}{3})$

SUBTRACTION OF FRACTIONS

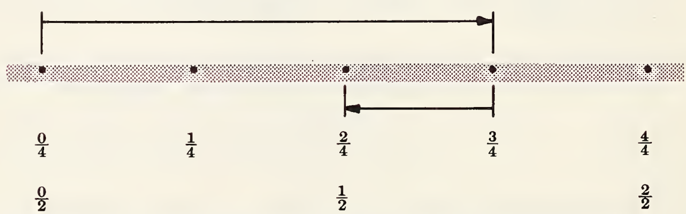
1 **a** Find $\frac{3}{4}-\frac{1}{4}$.

Let the answer to this question be x. Then $\frac{3}{4}-\frac{1}{4}=x$. When we were talking about whole numbers, we learned that subtraction was the inverse operation to addition. We learned that sentences $7-5=x$, mean the same as $7=x+5$.

Accordingly, we can think of $\frac{3}{4}-\frac{1}{4}=x$ as meaning $\frac{3}{4}=x+\frac{1}{4}$. Now the question is, "What number must we add to $\frac{1}{4}$ to make $\frac{3}{4}$?" We see that the number is $\frac{2}{4}$.

So $x=\frac{2}{4}=\frac{1}{2}$, and $\frac{3}{4}-\frac{1}{4}=\frac{1}{2}$.

b



From the number line above we can see that $\frac{3}{4}-\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$.

2. Work the following:

a $\frac{9}{4}-\frac{5}{4}$ **b** $\frac{7}{3}-\frac{4}{3}$ **c** $\frac{5}{8}-\frac{1}{8}$ **d** $\frac{19}{16}-\frac{3}{16}$ **e** $\frac{7}{10}-\frac{4}{10}$

f $\frac{11}{12}-\frac{11}{12}$ **g** $\frac{4}{2}-\frac{1}{2}$ **h** $\frac{19}{20}-\frac{15}{20}$ **i** $\frac{8}{9}-\frac{5}{9}$ **j** $\frac{15}{8}-\frac{8}{8}$

3 $\frac{4}{5}-\frac{1}{2}=x$. This means $\frac{4}{5}=x+\frac{1}{2}$

What must we add to $\frac{1}{2}$ to make $\frac{4}{5}$? Why is it difficult to find the answer by inspection? What common denominator can we use for fifths and halves? We see that we can use tenths.

$\frac{4}{5}=\frac{8}{10}$, $\frac{1}{2}=\frac{5}{10}$; so for $\frac{4}{5}-\frac{1}{2}=x$, we can have $\frac{8}{10}-\frac{5}{10}=x$, or $\frac{8}{10}=x+\frac{5}{10}$

What must we add to $\frac{5}{10}$ to make $\frac{8}{10}$? $x=\frac{3}{10}$.

We see that $\frac{4}{5}-\frac{1}{2}=\frac{3}{10}$.

4 $3\frac{1}{4} - 1\frac{2}{3} = x.$

This means that $3\frac{1}{4} = x + 1\frac{2}{3}$

or $\frac{13}{4} = x + \frac{5}{3}$

or $\frac{39}{12} = x + \frac{20}{12}$;

so $x = \frac{19}{12} = 1\frac{7}{12}$

5 $4\frac{3}{8} - 1\frac{5}{6} = x$ means $4\frac{3}{8} = x + 1\frac{5}{6}$

or $\frac{35}{8} = x + \frac{11}{6}$

or $\frac{105}{24} = x + \frac{44}{24}$;

so $x = \frac{61}{24} = 2\frac{13}{24}$

6. Work the following:

a $1\frac{2}{3} + x = 2\frac{5}{6}$

b $2\frac{5}{6} = x + 1\frac{7}{12}$

c $1\frac{4}{5} + x = 3\frac{9}{10}$

d $3\frac{7}{10} - 2\frac{3}{4} = x$

e $\frac{25}{6} - 1\frac{7}{8} = x$

f $4\frac{13}{20} - 1\frac{7}{8} = x$

g $6\frac{11}{12} - 3\frac{13}{16} = x$

h $7\frac{2}{3} - 6\frac{4}{5} = x$

i $x + 1\frac{5}{9} = 3\frac{5}{6}$

j $2\frac{11}{24} + x = 4\frac{9}{20}$

7 **a** We can also do our computation in a different way:

$$\frac{5}{3} - \frac{5}{6} = x$$

For this we can write $\frac{10}{6} - \frac{5}{6} = x$

or $\frac{10-5}{6} = x$

or $\frac{5}{6} = x$

b $3\frac{2}{3} - 1\frac{3}{10} = x$

or $\frac{11}{3} - \frac{13}{10} = x$

or $\frac{110}{30} - \frac{39}{30} = x$

or $\frac{110-39}{30} = x$

or $\frac{71}{30} = x$; so $x = 2\frac{11}{30}$

8 We can prove each of the answers above by doing addition, which is the inverse operation to subtraction.

We can prove that $\frac{5}{3}-\frac{5}{6}=\frac{5}{6}$ because $\frac{5}{3}-\frac{5}{6}=\frac{5}{6}$ means $\frac{5}{3}=\frac{5}{6}+\frac{5}{6}$; and $\frac{5}{6}+\frac{5}{6}=\frac{10}{6}$; $\frac{10}{6}=\frac{5}{3}$ so our subtraction is right.

9. Use the operation of addition to prove that the answer to example 7(**b**) is correct.

10. Solve the following:

a $\frac{4}{9}=x+\frac{3}{9}$	**b** $\frac{7}{8}+x=\frac{9}{8}$
c $\frac{5}{2}+x=\frac{13}{4}$	**d** $\frac{2}{2}=\frac{4}{4}-x$
e $x+\frac{1}{6}=\frac{4}{9}$	**f** $1\frac{7}{18}-x=\frac{11}{12}$
g $4\frac{17}{30}-x=2\frac{11}{24}$	**h** $1\frac{7}{8}-x=\frac{8}{9}$
i $\frac{4}{15}-x=\frac{1}{12}$	**j** $\frac{7}{8}-x=\frac{1}{5}$

11 If we let $\frac{a}{b}$ and $\frac{c}{d}$ be any two fractions,

then $\frac{a}{b}-\frac{c}{d}=\frac{ad}{bd}-\frac{bc}{bd}=\frac{ad-bc}{bd}$.

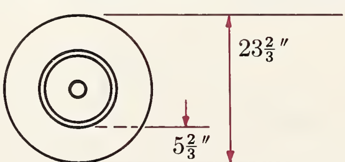
PROBLEMS INVOLVING SUBTRACTION OF FRACTIONS

1. From a theatre ticket $2\frac{3}{4}$ inches long, a doorman tears $1\frac{1}{8}$ inches. How many inches are left?

2. A man fitting a plastic top to a table discovers that it is $\frac{5}{16}$ of an inch too long. The sheet of plastic is 4 ft. $8\frac{1}{4}$ in. long. What is the length of the table?

3. A plane removes $\frac{1}{32}$ inch thickness of wood with every stroke. A carpenter takes three shavings from a piece of wood 4 inches thick. What thickness of wood remains?

4. The diagram below shows the measurements of a wheel and tire. Find the size in inches across the wheel without the tire.



5. The water in a rain barrel is $27\frac{7}{10}$ inches deep. The farmer's daughter takes a bowlful to wash her hair and leaves a depth of $24\frac{3}{5}$ inches. By how many inches has the level fallen?

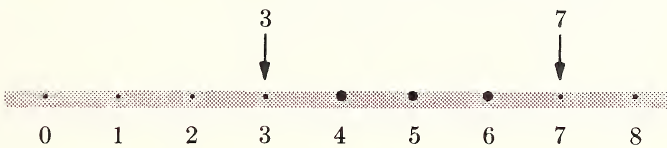
6. A bulrush measures $28\frac{7}{12}$ inches in length. The brown part at the top measures $5\frac{5}{6}$ inches. What is the length in inches, of the rest of the stem?

7. A man is in his office for $8\frac{2}{3}$ hours one day. Allowing $1\frac{5}{6}$ hours for lunch and coffee breaks how many hours does he actually work?

8. A cooler holds $4\frac{1}{3}$ gallons of water when full. By noon it has $3\frac{2}{5}$ gallons in it. How many gallons have been used?

FRACTIONS BETWEEN FRACTIONS

1 **a** What whole numbers are there between 3 and 7? We can draw a number line and graph the whole numbers that lie between 3 and 7.



We see that the numbers make the set $\{4, 5, 6\}$

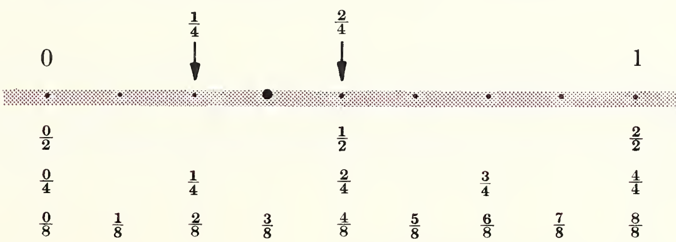
b What whole numbers are there between 2 and 3?

Study the number line. Are there any whole numbers between 2 and 3? We see that there are no whole numbers between 2 and 3. The solution is the empty set, $\{\ \}$.

c We see that it is not always possible to find a whole number between two other whole numbers.

2 What fractions are there between $\frac{1}{4}$ and $\frac{2}{4}$?

a Study the number line below:



We see one fraction that lies between. It is the fraction $\frac{3}{8}$. $\frac{3}{8}$ is midway between $\frac{1}{4}$ and $\frac{2}{4}$. Are there any other fractions? Can we find a fraction midway between $\frac{3}{8}$ and $\frac{2}{4}$? Would such a fraction be between $\frac{1}{4}$ and $\frac{2}{4}$?

b We have learned in earlier grades how to find the average of two numbers. The average of 4 and 6 is $\frac{4+6}{2}=\frac{10}{2}=5$. Then 5 is between 4 and 6. It is midway between 4 and 6. Let us use this method of finding averages to find the average of $\frac{1}{4}$ and $\frac{2}{4}$.

$\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$. The average of $\frac{1}{4}$ and $\frac{2}{4}$ is $\dfrac{\frac{3}{4}}{2}$.

$$\frac{\frac{3}{4}}{2}=\frac{\frac{3}{4}}{2}\times\frac{\frac{1}{2}}{\frac{1}{2}}=\frac{\frac{3}{4}\times\frac{1}{2}}{2\times\frac{1}{2}}=\frac{\frac{3}{4}\times\frac{1}{2}}{1}=\frac{\frac{3}{8}}{1}=\frac{3}{8}$$

This gives us the number $\frac{3}{8}$. From our number line we had already seen that $\frac{3}{8}$ was midway between $\frac{1}{4}$ and $\frac{2}{4}$.

c Now let us find the average of $\frac{3}{8}$ and $\frac{2}{4}$. The average will give us a number which is midway between $\frac{3}{8}$ and $\frac{2}{4}$. This number will certainly be between $\frac{1}{4}$ and $\frac{2}{4}$. Why?

$$\frac{\frac{3}{8}+\frac{2}{4}}{2}=\frac{\frac{3}{8}+\frac{4}{8}}{2}=\frac{\frac{7}{8}}{2}=\frac{\frac{7}{8}}{2}\times\frac{\frac{1}{2}}{\frac{1}{2}}=\frac{\frac{7}{8}\times\frac{1}{2}}{1}=\frac{\frac{7}{16}}{1}=\frac{7}{16}$$

We see that $\frac{7}{16}$ lies between $\frac{3}{8}$ and $\frac{2}{4}$, and it *must* lie between $\frac{1}{4}$ and $\frac{2}{4}$.

d Now find the number midway between $\frac{7}{16}$ and $\frac{2}{4}$.

The number is $\dfrac{\frac{7}{16}+\frac{2}{4}}{2}$. Why?

$$\frac{\frac{7}{16}+\frac{2}{4}}{2}=\frac{\frac{7}{16}+\frac{8}{16}}{2}=\frac{\frac{15}{16}}{2}=\frac{\frac{15}{16}}{2}\times\frac{\frac{1}{2}}{\frac{1}{2}}=\frac{\frac{15}{16}\times\frac{1}{2}}{2\times\frac{1}{2}}=\frac{\frac{15}{32}}{1}=\frac{15}{32}$$

e Can we find a number midway between $\frac{15}{32}$ and $\frac{3}{4}$? How?

f Can we go on finding numbers that lie between $\frac{1}{4}$ and $\frac{2}{4}$?

g How long can we continue doing this?

h Can we *always* find a fraction between two other fractions?

3 We have seen that we can always find a fraction between two other fractions by finding the average of these two fractions and hence finding the fraction midway between them. Because we can go on for ever finding the fraction which lies midway between two fractions we see that there is an unlimited number of fractions between two given fractions.

4 Let $\frac{a}{b}$ and $\frac{c}{d}$ be any two fractions.

a Between $\frac{a}{b}$ and $\frac{c}{d}$ there is an unlimited number of fractions.

b The number midway between $\frac{a}{b}$ and $\frac{c}{d}$ is given by $\dfrac{\frac{a}{b}+\frac{c}{d}}{2}$.

5 Here is another way we might use to find fractions between two given fractions. Name three fractions between $\frac{1}{5}$ and $\frac{1}{3}$.

a Change the two fractions to two equivalent fractions having the same denominator:

$$\frac{1}{5}=\frac{1}{5}\times\frac{3}{3}=\frac{1\times 3}{5\times 3}=\frac{3}{15} \qquad \frac{1}{3}=\frac{1}{3}\times\frac{5}{5}=\frac{1\times 5}{3\times 5}=\frac{5}{15}$$

What fraction lies between $\frac{3}{15}$ and $\frac{5}{15}$?

b Now let us change $\frac{1}{5}$ and $\frac{1}{3}$ to two equivalent fractions with a denominator that is a multiple of 15. Let us take 60 $(=4\times 15)$.

$$\frac{1}{5}=\frac{3}{15}=\frac{3}{15}\times\frac{4}{4}=\frac{3\times 4}{15\times 4}=\frac{12}{60} \qquad \frac{1}{3}=\frac{5}{15}=\frac{5}{15}\times\frac{4}{4}=\frac{5\times 4}{15\times 4}=\frac{20}{60}$$

What fractions lie between $\frac{12}{60}$ and $\frac{20}{60}$?

Do these fractions lie between $\frac{1}{5}$ and $\frac{1}{3}$?

Between $\frac{12}{60}$ and $\frac{20}{60}$ we can choose $\frac{13}{60}$, $\frac{14}{60}$, $\frac{15}{60}$.

These are three fractions that lie between $\frac{12}{60}$ and $\frac{20}{60}$. They lie between $\frac{1}{5}$ and $\frac{1}{3}$. Why?

6 Find a fraction between $\frac{1}{5}$ and $\frac{2}{5}$.

$$\frac{1}{5}=\frac{1}{5}\times\frac{2}{2}=\frac{1\times 2}{5\times 2}=\frac{2}{10} \qquad \frac{2}{5}=\frac{2}{5}\times\frac{2}{2}=\frac{2\times 2}{5\times 2}=\frac{4}{10}$$

One fraction between $\frac{2}{10}$ and $\frac{4}{10}$ is $\frac{3}{10}$. If we write the fractions in order we have $\frac{1}{5}$, $\frac{3}{10}$, $\frac{2}{5}$.

We can show the order by writing: $\frac{1}{5}<\frac{3}{10}<\frac{2}{5}$.

7. Find the fractions which lie midway between the pairs of fractions below: The first one is done for you.

a $\frac{3}{4}$ and $\frac{8}{9}$.

The average is $\dfrac{\frac{3}{4}+\frac{8}{9}}{2}$.

$$\frac{\frac{3}{4}+\frac{8}{9}}{2}=\frac{\frac{27}{36}+\frac{32}{36}}{2}=\frac{\frac{59}{36}}{2}=\frac{\frac{59}{36}}{2}\times\frac{\frac{1}{2}}{\frac{1}{2}}=\frac{\frac{59}{36}\times\frac{1}{2}}{1}=\frac{59}{72}$$

We can write $\frac{3}{4}<\frac{59}{72}<\frac{8}{9}$.

b $\frac{1}{4}$, $\frac{1}{3}$ **c** $\frac{1}{2}$, $\frac{7}{8}$ **d** $\frac{3}{2}$, $\frac{5}{2}$ **e** $\frac{9}{10}$, $\frac{15}{16}$

f $\frac{8}{3}$, $\frac{14}{3}$ **g** $\frac{5}{2}$, $\frac{11}{1}$ **h** $\frac{15}{7}$, $\frac{22}{5}$ **i** $\frac{6}{1}$, $\frac{7}{1}$

j $\frac{9}{8}$, $\frac{11}{8}$ **k** $\frac{2}{5}$, $\frac{7}{8}$ **l** $\frac{7}{12}$, $\frac{11}{16}$ **m** $\frac{11}{15}$, $\frac{17}{15}$

n $\frac{23}{6}$, $\frac{27}{3}$ **o** $\frac{19}{2}$, $\frac{10}{1}$ **p** $\frac{5}{1}$, $\frac{15}{2}$ **q** $\frac{4}{7}$, $\frac{8}{9}$

8. Find three fractions between each of the pairs of fractions below using the denominator indicated. Write the set of fractions you now have in order. The first is done for you.

a $\frac{3}{5}, \frac{4}{5}$ (20)

$\frac{3}{5}=\frac{3}{5}\times\frac{4}{4}=\frac{12}{20}$

$\frac{4}{5}=\frac{4}{5}\times\frac{4}{4}=\frac{16}{20}$

Three of the fractions between $\frac{12}{20}$ and $\frac{16}{20}$ are $\frac{13}{20}$, $\frac{14}{20}$ and $\frac{15}{20}$.

We can write $\frac{3}{5}<\frac{13}{20}<\frac{14}{20}<\frac{15}{20}<\frac{4}{5}$

If we write equivalent fractions in their lowest terms for $\frac{14}{20}$ and $\frac{15}{20}$ we have $\frac{3}{5}<\frac{13}{20}<\frac{7}{10}<\frac{3}{4}<\frac{4}{5}$

b $\frac{1}{2}, \frac{2}{2}$ (8) **c** $\frac{3}{5}, \frac{7}{5}$ (5) **d** $\frac{1}{4}, \frac{3}{8}$ (32) **e** $\frac{2}{7}, \frac{9}{14}$ (14)

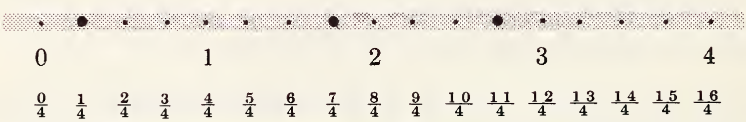
f $\frac{5}{8}, \frac{9}{10}$ (40) **g** $\frac{2}{3}, \frac{5}{6}$ (24) **h** $\frac{7}{2}, \frac{8}{2}$ (8) **i** $\frac{4}{3}, \frac{7}{5}$ (60)

j $\frac{11}{2}, \frac{27}{4}$ (4) **k** $\frac{7}{15}, \frac{7}{9}$ (45) **l** $\frac{1}{1}, \frac{2}{1}$ (8) **m** $\frac{3}{1}, \frac{7}{2}$ (16)

GRAPHING FRACTIONS ON A NUMBER LINE

1 We have seen how we can graph sets of whole numbers on a number line. We can also graph sets of fractions on a number line.

Example: Graph the set $\{\frac{1}{4}, \frac{7}{4}, \frac{11}{4}\}$.



We draw a number line with points representing the set of fractions: $\{\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \ldots\}$ then we draw big dots on the points that represent $\frac{1}{4}$, $\frac{7}{4}$ and $\frac{11}{4}$.

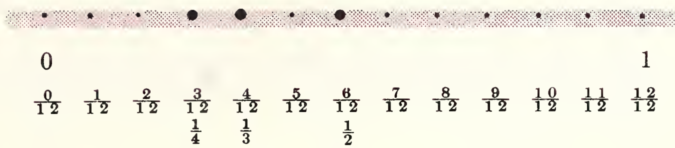
Study the graph; which of the numbers $\frac{1}{4}$, $\frac{7}{4}$ and $\frac{11}{4}$ is the largest? How can you tell? What is the order of these three fractions? We can write $\frac{1}{4}<\frac{7}{4}<\frac{11}{4}$.

2 Graph the set $\{\frac{1}{4}, \frac{1}{3}, \frac{1}{2}\}$.

We notice that the fractions have different denominators. If we write the fractions with the same denominator what common denominator can we use?

For $\{\frac{1}{4}, \frac{1}{3}, \frac{1}{2}\}$ we can write $\{\frac{3}{12}, \frac{4}{12}, \frac{6}{12}\}$

Now draw a number line with points representing the set $\{\frac{0}{12}, \frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \dots\}$



On this we put dots on the points representing $\frac{3}{12}$, $\frac{4}{12}$ and $\frac{6}{12}$. These dots will show the coordinates of $\frac{3}{12}$, $\frac{4}{12}$ and $\frac{6}{12}$ and hence of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$. (Why?)

What is the order of these three fractions?

We can write $\frac{1}{4} < \frac{1}{3} < \frac{1}{2}$.

3. Graph the following sets and then write the order of the fractions in each set.

a $\{\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}\}$

b $\{\frac{1}{8}, \frac{7}{8}, \frac{3}{8}, \frac{5}{8}\}$

c $\{\frac{3}{5}, \frac{1}{5}, \frac{4}{5}, \frac{2}{5}\}$

d $\{\frac{4}{6}, \frac{2}{6}, \frac{1}{6}, \frac{7}{6}\}$

e $\{\frac{3}{8}, \frac{10}{8}, \frac{12}{8}, \frac{11}{8}\}$

f $\{\frac{4}{5}, \frac{11}{5}, \frac{20}{5}, \frac{14}{5}\}$

g $\{\frac{1}{2}, \frac{7}{2}, \frac{13}{2}, \frac{5}{3}\}$

h $\{\frac{14}{10}, \frac{11}{10}, \frac{20}{10}, \frac{7}{10}\}$

i $\{\frac{13}{3}, \frac{4}{3}, \frac{9}{3}, \frac{3}{3}\}$

j $\{\frac{7}{4}, \frac{13}{4}, \frac{5}{4}, \frac{8}{4}\}$

k $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\}$

l $\{\frac{2}{3}, \frac{5}{6}, \frac{1}{2}\}$

m $\{\frac{4}{5}, \frac{1}{10}, \frac{7}{10}\}$

n $\{\frac{5}{6}, \frac{11}{12}, \frac{3}{4}\}$

o $\{\frac{1}{5}, \frac{14}{10}, \frac{3}{2}\}$

p $\{\frac{5}{8}, \frac{11}{16}, \frac{9}{4}\}$

q $\{\frac{2}{3}, \frac{5}{2}, \frac{7}{4}\}$

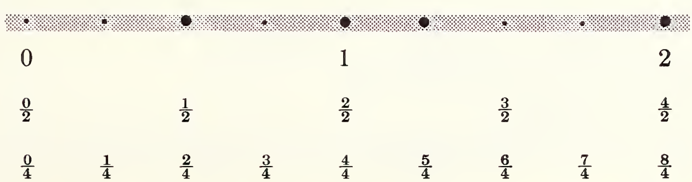
r $\{\frac{4}{9}, \frac{11}{3}, \frac{8}{9}\}$

s $\{\frac{5}{4}, \frac{5}{8}, \frac{5}{2}\}$

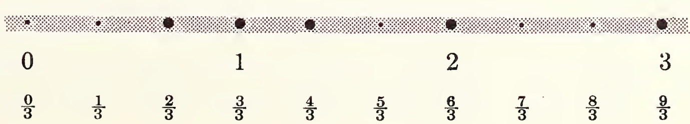
t $\{\frac{4}{3}, \frac{6}{5}\}$

4. Write the set of fractions illustrated by each graph below:

a



b



c

0 1 2 3 4

$\frac{0}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{2}$ $\frac{5}{2}$ $\frac{6}{2}$ $\frac{7}{2}$ $\frac{8}{2}$

$\frac{0}{4}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{5}{4}$ $\frac{6}{4}$ $\frac{7}{4}$ $\frac{8}{4}$ $\frac{9}{4}$ $\frac{10}{4}$ $\frac{11}{4}$ $\frac{12}{4}$ $\frac{13}{4}$ $\frac{14}{4}$ $\frac{15}{4}$ $\frac{16}{4}$

d

0 1 2

$\frac{0}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{2}$

$\frac{0}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{5}{3}$ $\frac{6}{3}$

$\frac{0}{6}$ $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{6}{6}$ $\frac{7}{6}$ $\frac{8}{6}$ $\frac{9}{6}$ $\frac{10}{6}$ $\frac{11}{6}$ $\frac{12}{6}$

e

0 1

$\frac{0}{2}$ $\frac{1}{2}$ $\frac{2}{2}$

$\frac{0}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$

$\frac{0}{4}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$

$\frac{0}{6}$ $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{6}{6}$

$\frac{0}{12}$ $\frac{1}{12}$ $\frac{2}{12}$ $\frac{3}{12}$ $\frac{4}{12}$ $\frac{5}{12}$ $\frac{6}{12}$ $\frac{7}{12}$ $\frac{8}{12}$ $\frac{9}{12}$ $\frac{10}{12}$ $\frac{11}{12}$ $\frac{12}{12}$

5. Graph the solution set for $x < \frac{3}{4}$. *Replacement set: the set of fractions*

First we must find what fractions are less than $\frac{3}{4}$.

It is true that all fractions which lie between 0 and $\frac{3}{4}$ are less than $\frac{3}{4}$. How many fractions are there between 0 and $\frac{3}{4}$? Can we count them? Is $\frac{0}{2}$ a fraction? are $\frac{0}{3}$, $\frac{0}{4}$, $\frac{0}{5}$, . . . fractions?

What number do they all equal?

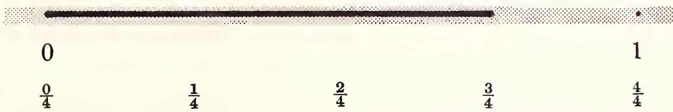
Is 0 less than $\frac{3}{4}$?

Here is how we might attempt to draw a graph of all the fractions less than $\frac{3}{4}$.

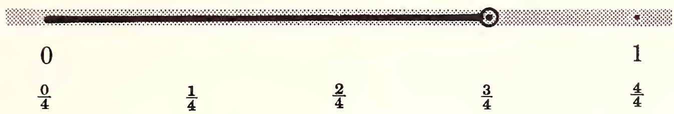
0 1

$\frac{0}{4}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$

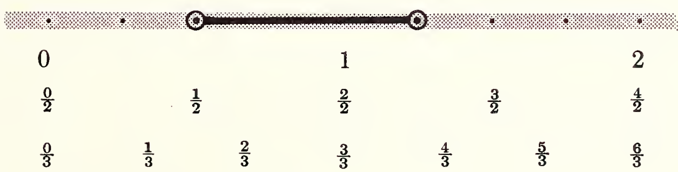
Would these dots represent *all* the fractions that are less than $\frac{3}{4}$? Could we continue to draw dots for ever and ever? To show that there are innumerable dots that we could draw, we can thicken the number line so:



Is $\frac{3}{4}$ less than $\frac{3}{4}$? To show that the point representing $\frac{3}{4}$ is not in the solution set for $x < \frac{3}{4}$ we can draw a circle around the point representing $\frac{3}{4}$.



6 Draw a graph of all the fractions which are greater than $\frac{1}{2}$ but less than $\frac{4}{3}$.



How many fractions are there between $\frac{1}{2}$ and $\frac{4}{3}$? Is $\frac{1}{2}$ in this set? Is $\frac{4}{3}$ in this set? Why have circles been drawn around the points representing $\frac{1}{2}$ and $\frac{4}{3}$?

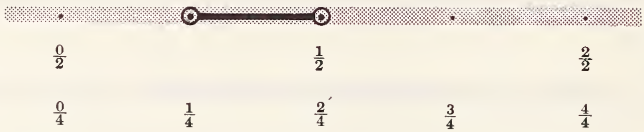
7. Graph solution sets to the following: *Replacement set: the set of fractions*

a $x < \frac{2}{3}$	**b** $y < \frac{5}{8}$	**c** $m < \frac{5}{6}$	**d** $n < \frac{4}{2}$
e $x < \frac{5}{4}$	**f** $z < \frac{7}{5}$	**g** $y > \frac{1}{2}$	**h** $m > \frac{3}{4}$
i $p > \frac{5}{6}$	**j** $x > \frac{7}{3}$	**k** $z > \frac{9}{5}$	**l** $n > \frac{4}{2}$

8. Graph the following: *Replacement set: the set of fractions*

a All fractions greater than $\frac{1}{4}$ but less than $\frac{3}{4}$
b All fractions greater than $\frac{2}{5}$ but less than $\frac{4}{5}$
c All fractions greater than $\frac{1}{3}$ but less than $\frac{5}{3}$
d All fractions greater than $\frac{5}{8}$ but less than $\frac{5}{4}$
e All fractions greater than $\frac{2}{3}$ but less than $\frac{3}{2}$

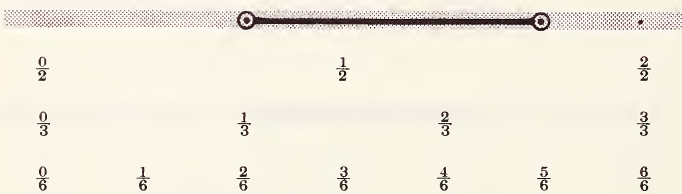
9. Graph the solution set for $x < \frac{1}{2}$ but $x > \frac{1}{4}$.



Is $\frac{1}{4}$ in the solution set? How is this shown on the graph? Is $\frac{1}{2}$ in the solution set? How is this shown on the graph? Why is the line segment between $\frac{1}{4}$ and $\frac{1}{2}$ made thicker than the rest of the line?

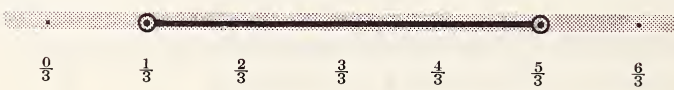
10. Describe the sets that are graphed on the number lines below:

a

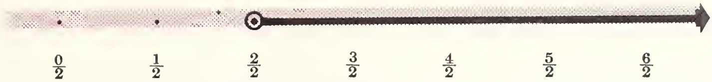


b

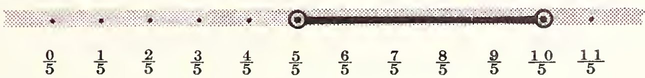
c



d



e



11. Graph solution sets for the following:

a $x < \frac{2}{3}$ **b** $x < 1\frac{1}{8}$ **c** $x > \frac{1}{2}$ **d** $x > 1\frac{2}{3}$

e $x < 1$ but $x > \frac{1}{2}$ **f** $x < 1\frac{1}{5}$ but $x > \frac{1}{3}$

g $x < 2\frac{1}{2}$ but $x > 1\frac{3}{4}$ **h** $x < 3\frac{1}{5}$ but $x > \frac{3}{5}$

PROBLEMS USING FRACTIONS

1. If 23 out of every 30 persons at a theatre first night paid for their tickets, what fraction of the audience came free of charge or were paid to attend?

2. A total of 6 bowls of varying sizes were filled with potato chips before a party. The bowls held $\frac{1}{3}$ of a box, $\frac{1}{4}$ of a box, $\frac{5}{6}$ of a box, $\frac{1}{2}$ of a box, $\frac{2}{3}$ of a box and $\frac{3}{4}$ of a box respectively. How many boxes of chips had to be opened for the occasion?

3. $\frac{1}{10}$ of the marbles in a bag are made of steel. Of the remainder, $\frac{2}{3}$ are made of glass and the rest are made of clay. What fraction of the whole do the clay marbles constitute?

4. One press of the *Vanilla* button on a soda fountain delivers $\frac{1}{24}$ of a pint. The vanilla container holds $3\frac{1}{4}$ pints of syrup. How many vanilla drinks, each taking one press of the button, can be served from the full container?

5. An espresso coffee machine holds $4\frac{3}{4}$ gallons of hot water. 68 cups of coffee are served and the tank is now empty. If there are 160 oz. in 1 gallon, how many ounces does one cup hold?

6. A whole candle will burn for 10 hrs. Some cave explorers have 3 pieces of candle, $\frac{5}{12}$, $\frac{2}{3}$, and $\frac{3}{4}$ of a whole candle in length. For how many hours will the pieces burn if they are lit one after the other?

7. A bar of chocolate weighs $7\frac{1}{2}$ oz. A brother and two sisters decide to have $\frac{1}{3}$ of the bar each. How many ounces will each get?

8. How many patio stones each $\frac{5}{6}$ yd. long will be required to make a path $54\frac{1}{6}$ yds. long?

9. How many tablets, each $\frac{3}{8}$ inch across, will fit into a box $3\frac{3}{8}$ inches long?

10. From Sleepyville post office to the firehall is $\frac{2}{7}$ mi. From the firehall to the hospital is $1\frac{3}{14}$ mi. and from there to the railroad station is $\frac{1}{2}$ mile. What is the distance in miles from the post office to the railroad station?

11. How much more money is $\frac{1}{2}$ share worth than a $\frac{1}{3}$ share of a $35,000.00 business?

12. A splice uses up 8 ft. of a ship's mooring rope. What fraction is this of a 120 fathom rope?

13. $\frac{5}{8}$ of a congregation is female. $\frac{2}{5}$ of these are under 16 years old. What fraction of the whole congregation are girls under 16?

14. A man spends $\frac{1}{3}$ of a certain day working, $\frac{1}{12}$ eating and $\frac{5}{24}$ in amusements. What number of hours does he spend sleeping?

15. A hand saw removes a $1\frac{13}{16}$ inch strip from a piece of lumber $6\frac{7}{8}$ inches wide. What is the width of the remaining piece of lumber?

16. A hay baler uses $22\frac{5}{8}$ ft. of string for each bale of hay. What number of yards will it use for 160 bales?

17. A man spends $\frac{1}{20}$ of his salary on food, $\frac{1}{12}$ on rent and $\frac{1}{120}$ on clothes. What fraction of his salary do these three items take?

18. If a spring measures $2\frac{5}{12}$ feet when stretched and $\frac{18}{29}$ of this when at rest, what is its length in feet before it is stretched?

19. A large rock stands in a garden pond. The water in the pond is $15\frac{7}{16}''$ deep. When the rock is taken out the water level falls $4\frac{5}{8}$ inches. Find the new depth of the pond, in inches.

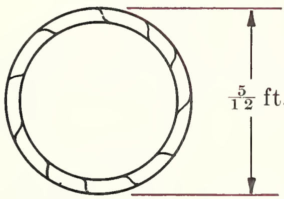
20. The new rope on the flag pole at summer camp was $46\frac{2}{3}$ feet long when new. After a rain storm it measured $44\frac{3}{8}$ feet. By what number of feet had it shrunk?

21. In mid-winter the expansion gap in a bridge measures $2\frac{1}{7}$ inches. In mid-summer the gap measures $\frac{25}{28}$ inches. By what number of inches has the bridge expanded?

22. A newly cut out dress length measures $38\frac{1}{2}$ inches from shoulder to bottom edge. What will its length be in inches if a $\frac{3}{8}$ inch seam allowance is made at the shoulder seam and a $2\frac{1}{4}$ inch hem at the bottom edge?

23. 4 books are lying in a pile on a library table. The bottom one in the pile is $1\frac{7}{16}$ inches thick and each succeeding one is $\frac{1}{8}$ inch thicker than the one below it. Find the thickness of the pile of books, in inches.

24. To make a deck quoit requires one strand of rope $3\frac{1}{2}$ times the distance across the quoit in length. How many quoits, each $\frac{5}{12}$ foot across can be made from a three-stranded rope $17\frac{1}{2}$ feet long?



25. How many complete wooden dowel pins $\frac{7}{16}$ inch long can be cut from a rod 36 inches long?

26. From the edge of a board $6\frac{3}{4}$ in. wide, a carpenter removes $1\frac{7}{8}$ in. by hand saw, $\frac{3}{16}$ in. by plane and $\frac{4}{128}$ in. by sander. Find the width of the remaining board in inches.

27. A pencil sharpener removes $\frac{3}{16}$ inch from an $8\frac{1}{2}$ inch pencil with each sharpening. How many inches of pencil will be left after 4 sharpenings?

28. The top of a coffee table is $\frac{5}{6}$ inch thick. The vertical legs are $10\frac{3}{4}$ inches long. Find the distance in inches from the floor to the surface of the table.

29. The span of the fingers of my left hand is $8\frac{7}{16}$ inches. That of my right hand is $8\frac{1}{3}$ inches. Find the distance in inches from my left little finger to my right when my thumbs are just touching.

30. The radio antenna on a car measures $18\frac{2}{5}$ inches when closed and $37\frac{1}{3}$ inches when extended. Find the number of inches in the extension.

CHAPTER TEST

1. Write the next three terms in the sets of equivalent fractions shown below:

a $\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \ldots\}$ **b** $\{\frac{1}{12}, \frac{2}{24}, \frac{3}{36}, \ldots\}$ **c** $\{\frac{1}{20}, \frac{2}{40}, \frac{3}{60}, \ldots\}$

d $\{\frac{3}{8}, \frac{6}{16}, \frac{9}{24}, \ldots\}$ **e** $\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \ldots\}$ **f** $\{\frac{7}{16}, \frac{14}{32}, \frac{21}{48}, \ldots\}$

g $\{\frac{4}{15}, \frac{8}{30}, \frac{12}{45}, \ldots\}$ **h** $\{\frac{9}{10}, \frac{18}{20}, \frac{27}{30}, \ldots\}$ **i** $\{\frac{2}{7}, \frac{4}{14}, \frac{6}{21}, \ldots\}$

2. Copy and complete the following:

a $\frac{2}{5}=\frac{12}{?}$ b $\frac{?}{8}=\frac{9}{24}$ c $\frac{6}{10}=\frac{?}{5}$ d $\frac{48}{60}=\frac{?}{15}$

e $\frac{27}{?}=\frac{3}{4}$ f $\frac{?}{10}=\frac{81}{90}$ g $\frac{72}{?}=\frac{1}{2}$ h $\frac{?}{280}=\frac{1}{14}$

3. Write the following fractions in their lowest terms:

a $\frac{36}{144}$ b $\frac{21}{49}$ c $\frac{39}{65}$ d $\frac{76}{95}$ e $\frac{121}{143}$ f $\frac{261}{372}$

4. Draw number lines and graph the following sets:

a $\{\frac{2}{9}, \frac{3}{9}, \frac{1}{9}, \frac{5}{9}, \frac{8}{9}\}$ b $\{\frac{13}{23}, \frac{15}{23}, \frac{11}{23}, \frac{20}{23}, \frac{16}{23}\}$

c $\{\frac{32}{16}, \frac{2}{16}, \frac{48}{16}, \frac{54}{16}, \frac{17}{16}\}$ d $\{\frac{9}{15}, \frac{23}{15}, \frac{37}{15}, \frac{11}{15}, \frac{8}{15}\}$

5. Write the following as whole numbers or as mixed numbers:

a $\frac{36}{4}$ b $\frac{48}{14}$ c $\frac{27}{13}$ d $\frac{88}{22}$ e $\frac{100}{15}$ f $\frac{96}{18}$

6. Arrange the following in order of size putting the smallest first:

a $\frac{1}{5}, \frac{7}{10}, \frac{4}{5}$ b $\frac{2}{3}, \frac{9}{24}, \frac{5}{6}$ c $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}$

d $\frac{5}{9}, \frac{11}{12}, \frac{3}{4}$ e $\frac{3}{5}, \frac{3}{10}, \frac{3}{4}$ f $\frac{3}{10}, \frac{4}{15}, \frac{5}{20}$

7. Add:

a $\frac{5}{6}+\frac{2}{3}+\frac{11}{12}$ b $\frac{9}{10}+\frac{3}{5}+\frac{11}{30}$ c $\frac{9}{10}+\frac{3}{4}+\frac{2}{5}$

d $\frac{6}{25}+\frac{3}{10}+\frac{4}{5}$ e $\frac{3}{8}+\frac{4}{9}+\frac{11}{16}$ f $\frac{9}{10}+\frac{11}{12}+\frac{3}{4}$

g $\frac{5}{8}+\frac{11}{16}+\frac{2}{3}$ h $\frac{4}{9}+\frac{11}{18}+\frac{1}{2}$ i $\frac{7}{15}+\frac{3}{4}+\frac{3}{10}$

j $\frac{3}{4}+\frac{11}{15}+\frac{7}{12}$ k $1\frac{2}{3}+1\frac{3}{4}+2\frac{5}{12}$ l $9\frac{2}{3}+3\frac{4}{5}+5\frac{3}{4}$

m $6\frac{1}{2}+4\frac{9}{10}+2\frac{4}{15}$ n $3\frac{11}{12}+4\frac{1}{10}+2\frac{3}{8}$ o $5\frac{5}{6}+3\frac{9}{10}+2\frac{3}{8}$

p $2\frac{3}{4}+4\frac{11}{16}+3\frac{1}{9}$ q $2\frac{5}{9}+1\frac{1}{15}+3\frac{7}{10}$ r $4\frac{5}{8}+2\frac{2}{3}+4\frac{3}{5}$

8. Find n:

a $\frac{2}{3}-\frac{1}{6}=n$ b $1\frac{3}{8}-1\frac{1}{4}=n$ c $6\frac{2}{5}-5\frac{4}{9}=n$

d $15-11\frac{5}{29}=n$ e $6\frac{2}{3}-1\frac{7}{8}=n$ f $9\frac{1}{5}-4\frac{1}{10}=n$

g $2\frac{14}{15}-1\frac{5}{6}=n$ h $11\frac{19}{30}-4\frac{3}{4}=n$ i $2\frac{5}{8}-1\frac{11}{12}=n$

j $6\frac{4}{9}-2\frac{7}{8}=n$ k $1\frac{2}{13}-\frac{5}{7}=n$ l $4\frac{11}{16}-2\frac{9}{10}=n$

9. Find x:

a $\frac{2}{5}\times1\frac{1}{2}\times2\frac{2}{3}=x$ b $(1\frac{1}{4}\times\frac{3}{5})+(2\frac{3}{4}\times\frac{3}{5})=x$

c $1\frac{5}{8}\times\frac{8}{9}\times1\frac{1}{26}=x$ d $4\frac{1}{2}\times3\frac{1}{7}\times\frac{8}{9}=x$

e $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=x$ f $(3\frac{2}{3}\times\frac{1}{10})+(3\frac{2}{3}\times\frac{9}{10})=x$

g $\frac{4}{15}\times22\frac{1}{2}\times2\frac{2}{3}=x$ h $\frac{11}{13}\times\frac{2}{5}\times\frac{13}{22}=x$

i $1\frac{2}{9}\times\frac{3}{4}\times3\frac{2}{3}=x$ j $\frac{14}{19}\times3\frac{1}{7}\times2\frac{3}{8}=x$

10. Write reciprocals for the following:

a $\frac{2}{3}$ b $\frac{3}{8}$ c $\frac{1}{5}$ d 5 e $1\frac{1}{2}$ f $\frac{3}{9}$ g 12 h $2\frac{2}{3}$

11. Find n:

a $3 \div \frac{1}{2} = n$　　b $1\frac{1}{2} \div \frac{1}{4} = n$　　c $n \div 3 = 1\frac{1}{2}$

d $\frac{3}{4} \div n = \frac{1}{4}$　　e $1\frac{5}{6} \div \frac{2}{3} = n$　　f $1\frac{7}{8} \div \frac{5}{6} = n$

g $n \div 1\frac{2}{5} = 1\frac{1}{2}$　　h $4\frac{1}{3} \div n = 2\frac{1}{5}$　　i $16 \div \frac{2}{3} = n$

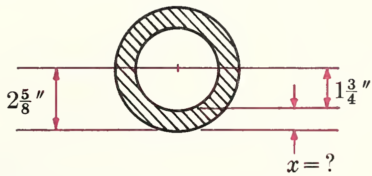
j $2\frac{3}{16} \div 3\frac{5}{9} = n$　　k $4\frac{1}{2} \div 18 = n$　　l $n \div 1\frac{15}{20} = 2$

m $\frac{1}{2} \div 1\frac{1}{3} = n$　　n $5\frac{2}{15} \div 3\frac{1}{10} = n$　　o $n \div \frac{11}{20} = 2\frac{1}{2}$

p $5\frac{3}{8} \div 12 = n$　　q $21 \div 49 = n$　　r $15 \div n = \frac{1}{3}$

12. A rubber ball is allowed to fall from a height of $12\frac{1}{2}$ ft. On hitting the ground, it bounces to a height that is $\frac{2}{5}$ of the distance it was allowed to fall. To what height did the ball bounce?

13. Here is a drawing of a cross-section of a metal pipe. Find the missing measurement.



14. By how much is 10 greater than the sum of $2\frac{5}{9}$ and $3\frac{7}{8}$?

15. An airplane travelled 280 miles in 35 minutes. How far would it travel in 1 hour at this speed? (Think: What fraction of 1 hour is 35 minutes?)

16. A knot is a rate of speed. It is a rate of 1 nautical mile per hour. If a ship travels at $22\frac{1}{2}$ knots, how many nautical miles will it travel in $21\frac{3}{5}$ hr.?

17. The scale on a map is 1″ representing 10 miles. What distance is represented by a distance of $\frac{3}{4}''$ on the map?

18. An airplane pilot reckons on completing a trip in $5\frac{1}{2}$ hours. The trip consists of 3 legs. If the first leg takes $\frac{3}{4}$ hr. and the second leg takes $1\frac{5}{12}$ hr., in how many hours must the airplane fly the third leg to complete the trip on time?

19. A carpenter used pieces of wood $3\frac{5}{8}$ feet, $2\frac{1}{2}$ feet, and $1\frac{7}{12}$ feet long. These pieces were cut from a plank that was 10 feet long. If $\frac{1}{8}$ inch of wood was lost on each cut, what length of the plank remained?

20. One part of a rocket was made from three pieces of metal tubing. If the lengths of the three pieces were $1\frac{5}{8}''$, $2\frac{3}{4}''$, and $1\frac{1}{16}''$, what total length of metal tubing was used in making the rocket part?

6 Decimal Fractions

DECIMAL NUMERATION AND FRACTIONS

1 We have learned that the numeration system we use is a decimal system and that the size of the number a digit represents depends on its position in a decimal numeral.

4 4 4 4

4 ones	4
4 tens	40
4 hundreds	400
4 thousands	4000

2. 132 2× ?

129 2× ?

208 2× ?

When we move a digit from one column to the next column on its left, by what number is the number it represents multiplied?

3. 329 3× ?

436 3× ?

703 3× ?

When we move a digit from one column to the next column on its right, by what number is the number it represents divided?

4 Let us look at a numeral with the column for each digit named, as below:

Hundreds ($\frac{1}{10}$'s of 1 thousand)	**Tens** ($\frac{1}{10}$'s of 1 hundred)	**Ones** ($\frac{1}{10}$'s of 1 ten)
4	5	6

What might we expect a column to the right of the ones' column to be named?

What is one tenth of 1?

Write 456. Now place a '9' to the *right* of the '6'.

What numeral do we have now?

What can we do to make sure that we read this numeral as "four hundred fifty-six and nine tenths"?

We use a decimal point, and we write 456.9.

5. Copy the chart below and name the columns that are headed by a question mark.

10's ($\frac{1}{10}$ of 1 hundred)	**1's** ($\frac{1}{10}$ of 1 ten)	**Point** .	($\frac{1}{10}$ of 1 one)	?	?	?
	1	.	0	4	6	5
2	9	.	1	3	0	8
	3	.	2	6	8	

6 Write 3.268 as an expanded numeral.

We have $3.268 = 3 + \frac{2}{10} + \frac{6}{100} + \frac{8}{1000}$.

By using equivalent fractions we can write this as a mixed number.

$$3 + \frac{2}{10} + \frac{6}{100} + \frac{8}{1000} = 3 + \frac{200}{1000} + \frac{60}{1000} + \frac{8}{1000}$$
$$= 3\tfrac{268}{1000}$$

One way in which we can read 3.268 is "Three *and* two hundred sixty-eight thousandths". Other ways are "Three *point* two, six, eight" and "Three *decimal* two, six, eight." Whichever method we use, we are able to see where the decimal point belongs.

7 We have seen that for 100 we can write $10\times 10=10^2$; for 1000 we can write $10\times 10\times 10=10^3$, for 10,000 we can write $10\times 10\times 10\times 10=10^4$. Using exponential notation we can write decimal-fraction place values as follows:

	10	1	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$
	10	1		$\frac{1}{10}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$	$\frac{1}{10^4}$
a	4	2	.	3	4	6	9
b	3	5	.	0	5	7	2
c		9	.	6	0	0	5
d		0	.	3	2	8	7

Written as an expanded numeral, we have for **a** $42.3469=(4\times 10\ )+(2\times 1)+\left(3\times\frac{1}{10}\right)+\left(4\times\frac{1}{10^2}\right)+\left(6\times\frac{1}{10^3}\right)+\left(9\times\frac{1}{10^4}\right)$

Now write expanded numerals for the decimal numerals **b, c,** and **d,** above.

8. Write expanded numerals for the following. The first is done for you.

a $3.016=3+\frac{0}{10}+\frac{1}{100}+\frac{6}{1000}$

$=(3\times 1)+\frac{0}{10}+\frac{1}{10^2}+\frac{6}{10^3}$

b 4.367	**c** 12.98	**d** 5.4071	**e** 27.27
f 0.1650	**g** 12.3041	**h** .0001	**i** .105
j 0.0105	**k** 1.05	**l** 2.675	**m** 41.983
n 620.1	**o** 5.3477	**p** 5.0941	**q** 6.0000
r 7.8818	**s** 8.0808	**t** 19.684	**u** 23.6083

9. Write the following as expanded numerals; then write them as mixed numbers. Read them in two ways.

a 6.94	**b** 17.8	**c** 1.04	**d** 5.008
e 7.018	**f** 6.204	**g** 8.109	**h** 19.6147
i 5.4014	**j** 1.3306	**k** 2.0104	**l** 5.0006

10 We have seen that we can rewrite decimal numerals as fractions or mixed numbers. Do you think that all decimal numerals can be written as fractions? Let us consider the numeral 35.064.

$$35.064=(3\times10)+(5\times1)+\left(0\times\frac{1}{10}\right)+\left(6\times\frac{1}{10^2}\right)+\left(4\times\frac{1}{10^3}\right)$$

$$=35+\frac{0}{10}+\frac{6}{100}+\frac{4}{1000}$$

$$=\frac{35000}{1000}+\frac{0}{1000}+\frac{60}{1000}+\frac{4}{1000}$$

$$=\frac{35064}{1000}$$

11. Write the following in the form $\frac{a}{b}$, the form for fractions:

a 7.85 $\quad 7.85=7+\frac{8}{10}+\frac{5}{100}=\frac{700}{100}+\frac{80}{100}+\frac{5}{100}=\frac{785}{100}$

b 8.46 **c** 23.048 **d** 9.649

e 306.8 **f** 1,000.6042 **g** 7.0415

h 0.00014 **i** 96.405 **j** .0105

12 We see that decimal fractions can be written as fraction numerals. We can say, **"Decimal-fraction numerals are another way of expressing fractions."**

13. Write decimal numerals for each of these fraction numerals:

a $\frac{415}{100}$ **b** $\frac{65}{10}$ **c** $\frac{14}{1000}$ **d** $\frac{3685}{1000}$

e $\frac{5}{10000}$ **f** $\frac{4659}{10}$ **g** $\frac{73149}{1000}$ **h** $\frac{694506}{100000}$

14. Write decimal numerals for each of the following:

a $\frac{4}{10}+\frac{7}{100}$ **b** $(4\times10)+(3\times1)+\frac{9}{10}$

c $\frac{8}{100}+\frac{4}{1000}$ **d** $(7\times100)+(6\times1)+\frac{5}{10}+\frac{3}{100}$

e $(3\times1{,}000)+(9\times1)$ **f** $(4\times100)+(3\times1)+\frac{8}{100}+\frac{2}{1000}$

g $(5\times100)+\frac{5}{1000}$ **h** $(3\times10)+(4\times1)+\frac{7}{100}+\frac{1}{10000}$

15. Write .387 as an expanded numeral.

$.387=\frac{3}{10}+\frac{8}{100}+\frac{7}{1000}$

But $100=10\times10$, and $1000=10\times10\times10$

so $.387=\dfrac{3}{10}+\dfrac{8}{(10\times10)}+\dfrac{7}{(10\times10\times10)}$

$$=\frac{3\times10\times10}{10\times10\times10}+\frac{8\times10}{10\times10\times10}+\frac{7}{10\times10\times10}=\frac{300}{1000}+\frac{80}{1000}+\frac{7}{1000}=\frac{387}{1000}$$

Why is it easy to change any decimal-fraction numeral to a fraction numeral?

16. Change $\frac{7}{8}$ to a decimal-fraction numeral. Here the denominator is 8. What denominator would make it easy for us to change $\frac{7}{8}$ to a decimal-fraction numeral? Our denominator must be 10 or 10 multiplied by itself any number of times.

$\frac{7}{8}=\frac{7}{2\times2\times2}$ Now multiply together this fraction and $\frac{5\times5\times5}{5\times5\times5}$.

We have $\frac{7}{8}=\frac{7\times5\times5\times5}{2\times2\times2\times5\times5\times5}$. Does $\frac{5\times5\times5}{5\times5\times5}=1$?

Does $\frac{7}{8}=\frac{7\times5\times5\times5}{2\times2\times2\times5\times5\times5}$? Why?

$$\frac{7}{8}=\frac{7\times5\times5\times5}{2\times2\times2\times5\times5\times5}=\frac{7\times5\times5\times5}{(2\times5)\times(2\times5)\times(2\times5)}=\frac{7\times5\times5\times5}{10\times10\times10}=$$

$$\frac{875}{10\times10\times10}=\frac{875}{1000}$$

What is $\frac{875}{1000}$ as a decimal-fraction numeral?

Does $\frac{875}{1000}=.875$?

17. Change the following to decimal-fraction numerals: (The first is done for you.)

a $\frac{7}{20}=\frac{7}{2\times2\times5}=\frac{7}{(2\times5)\times2}=\frac{7}{10\times2}$

$=\frac{7}{10\times2}\times\frac{5}{5}=\frac{7\times5}{10\times2\times5}$

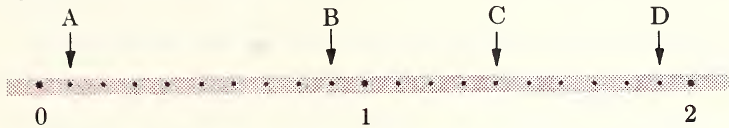
$=\frac{7\times5}{10\times10}=\frac{35}{100}=.35$

b $\frac{5}{40}$	**c** $\frac{11}{20}$	**d** $\frac{3}{8}$	**e** $\frac{13}{40}$	**f** $\frac{7}{25}$
g $\frac{9}{16}$	**h** $\frac{19}{50}$	**i** $\frac{27}{80}$	**j** $\frac{15}{16}$	**k** $\frac{31}{50}$
l $\frac{11}{16}$	**m** $\frac{3}{20}$	**n** $\frac{5}{8}$	**o** $\frac{4}{25}$	**p** $\frac{5}{32}$

18. We have just seen that if we can change the denominator of a fraction numeral to a denominator of 10, 10×10, $10\times10\times10$, or some similar number, we can easily change the fraction numeral to a decimal-fraction numeral. Later we shall learn about another method of changing fraction numerals to decimal numerals.

DECIMAL FRACTIONS AND NUMBER LINES

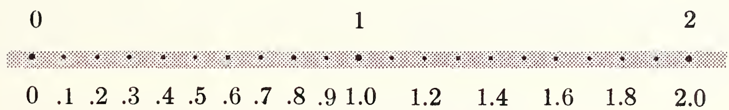
1. We have seen that we can use points on a number line to represent whole numbers and also to represent fractions. Consider the number line below:



Into how many equal lengths has the segment between 0 and 1 been divided? What number could we let the point labelled **A** represent?

A names the point which is $\frac{1}{10}$ of the distance from 0 to 1. We could label it $\frac{1}{10}$ or .1.

What numbers are represented by the points labelled **B**? **C**? **D**? Let us name the points on the number line using decimal fraction numerals. We have:



2. From the number line we can see that as we move to the right of zero, the points marked represent larger and larger numbers. We see that the point representing 1.4 is to the *right* of the point representing .9. We can write $1.4 > .9$ Similarly, since the point representing .9 is to the *left* of the point representing 1.4, we can write $.9 < 1.4$

3. Study the number line above and arrange the following sets of numbers in order. The first one is done for you:

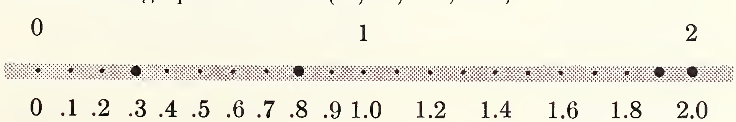
a $\{1.5, .6, 1.2, .9\}$
$.6 < .9 < 1.2 < 1.5$

b $\{.7, .5, .1\}$ **c** $\{.0, 1.0, .9, .8\}$

d $\{.1, 1.0, .2, 2.0\}$ **e** $\{.8, .7, .9, .6\}$

f $\{.6, .7, .1, 1.02\}$ **g** $\{1.9, 1.4, .9, .4\}$

4. We can also use decimal-fraction number lines to graph sets of numbers. Below is the graph of the set $\{.3, .8, 1.9, 2.0\}$



5. Draw graphs of the sets below:

a {.3, .4, .5} **b** {.5, 1.0, 1.5, 2.0}
c {.0, 1.0, 2.0} **d** {.4, .9, 1.6, 1.7}
e {.1, .2, .4, .8} **f** {.2, .5, .8, 1.1}

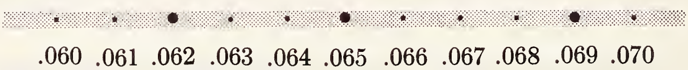
6. Below is the section of the number line from .1 to .2 enlarged: The segment has been divided into 10 equal parts.

Make a copy of this number line and graph the following sets:

a {.11, .14, .19} **b** {.10, .15, .20}
c {.12, .16, .17} **d** {.13, .16, .19, .20}

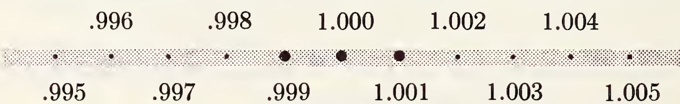
7. We could enlarge any section of the number line and divide the segment into smaller segments in order to illustrate other decimal fractions. Study the number lines below. What set is graphed on each of the number lines?

a

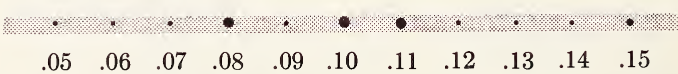


b

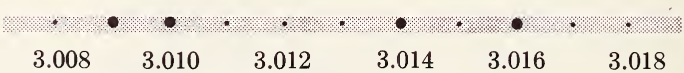
c



d



e



8. Draw number lines and graph the following sets:

a {.04, .08, .10} **b** {.098, .099, .101}
c {2.3, 2.5, 2.7} **d** {1.002, 1.010, 1.012}
e {5.0, 5.5, 6.0} **f** {.888, .890, .895}
g {.05, .07, .11} **h** {.775, .777, .779}
i {3.1, 3.5, 3.9} **j** {.43, .45, .48}

ADDITION AND SUBTRACTION WITH DECIMAL NUMERALS

1. Write the following as expanded numerals. Add. Rewrite the answer as a decimal numeral. (The first is done for you).

a $3.5+2.72$

$3.5=3+\frac{5}{10}=3+\frac{50}{100}$

$2.72=2+\frac{7}{10}+\frac{2}{100}$

$3.5+2.72=3+\frac{50}{100}+2+\frac{70}{100}+\frac{2}{100}$

$=3+2+\frac{50}{100}+\frac{70}{100}+\frac{2}{100}$

$=5+\frac{122}{100}$

$=5+\frac{100}{100}+\frac{22}{100}$

$=6\frac{22}{100}$

$=6.22$

b $6.9+3.5$ **c** $8.41+3.68$

d $7.04+3.09$ **e** $27.8+8.08$

f $1.92+.005$ **g** $1.375+1.869$

h $3.04+.095$ **i** $9.09+9.9$

2. Now add, using the decimal numerals directly:

a $\begin{array}{r} .4 \\ .9 \\ \hline \end{array}$ **b** $\begin{array}{r} 2.56 \\ 1.9 \\ \hline \end{array}$ **c** $\begin{array}{r} 7.15 \\ 6.752 \\ \hline \end{array}$

d $\begin{array}{r} 1.09 \\ .984 \\ \hline \end{array}$ **e** $\begin{array}{r} 5.649 \\ .99 \\ \hline \end{array}$ **f** $\begin{array}{r} 8.015 \\ 7.339 \\ \hline \end{array}$

g $\begin{array}{r} 6.8 \\ 7.245 \\ \hline \end{array}$ **h** $\begin{array}{r} 3.013 \\ 9.38 \\ \hline \end{array}$ **i** $\begin{array}{r} 15.614 \\ 93.2869 \\ \hline \end{array}$

3. How is addition with decimal fractions much like addition with whole numbers?

In computation, it helps if we keep the ones' digits underneath each other.

4. Compare the following subtractions:

a $5.6=5+\frac{6}{10}=4+\frac{10}{10}+\frac{6}{10}=4\frac{16}{10}$

$\underline{-3.9}=3+\frac{9}{10}=3\frac{9}{10}$ $\qquad =\frac{3\frac{9}{10}}{1\frac{7}{10}}$

b $\begin{array}{r} {}^{4\ 1}\cancel{5}.6 \\ -3.9 \\ \hline 1.7 \end{array}$

How is the subtraction in example **b** above like the subtraction of whole numbers?

5 Find x if $4.09-2.495=x$.

a We have:

$$\begin{array}{rll} 4.09 = 4+\frac{0}{10}+\frac{9}{100} = 4+\frac{90}{1000} = 3+1+\frac{90}{1000} & & = 3\frac{1090}{1000} \\ \underline{2.495} = 2+\frac{4}{10}+\frac{9}{100}+\frac{5}{1000} = 2+\frac{495}{1000} = 2\frac{495}{1000} & & = \underline{2\frac{495}{1000}} \\ & & \ 1\frac{595}{1000} \end{array}$$

$1\frac{595}{1000}=1.595$; so $x=1.595$.

b 4.09 has no digit in the thousandths' place. If we put a zero in the thousandths' place, will it alter the size of the number?

Explain each step in the computation below:

$$\begin{array}{r} 4.09 \\ -2.495 \\ \hline \end{array} \qquad \begin{array}{r} 4.090 \\ -2.495 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 8\ 1} \\ 4.0\cancel{9}\cancel{0} \\ -2.495 \\ \hline 5 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ {\scriptstyle 3\ 9\ 8\ 1} \\ \cancel{4}.\cancel{0}\cancel{9}\cancel{0} \\ -2.495 \\ \hline 95 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ {\scriptstyle 3\ 9\ 8\ 1} \\ \cancel{4}.\cancel{0}\cancel{9}\cancel{0} \\ -2.495 \\ \hline 1.595 \end{array}$$

How is this similar to the subtraction of whole numbers? Does it help if we keep the ones' digits underneath each other?

6. Work the following:

a $\begin{array}{r} 7.92 \\ -4.68 \\ \hline \end{array}$ **b** $\begin{array}{r} 6.09 \\ -2.495 \\ \hline \end{array}$ **c** $\begin{array}{r} 3.498 \\ -1.78 \\ \hline \end{array}$

d $\begin{array}{r} 18.6 \\ -\ 7.704 \\ \hline \end{array}$ **e** $\begin{array}{r} 3.207 \\ -3.109 \\ \hline \end{array}$ **f** $\begin{array}{r} 5.64 \\ -3.959 \\ \hline \end{array}$

g $\begin{array}{r} 7.21 \\ -5.045 \\ \hline \end{array}$ **h** $\begin{array}{r} 6 \\ -2.14 \\ \hline \end{array}$ **i** $\begin{array}{r} 3.4 \\ -2.951 \\ \hline \end{array}$

j $\begin{array}{r} 24.0164 \\ -\ 7.9285 \\ \hline \end{array}$ **k** $\begin{array}{r} 32.9815 \\ -16.9826 \\ \hline \end{array}$ **l** $\begin{array}{r} 14.2023 \\ -\ 5.4537 \\ \hline \end{array}$

m $\begin{array}{r} 4.6 \\ -2.9815 \\ \hline \end{array}$ **n** $\begin{array}{r} 9 \\ -4.0724 \\ \hline \end{array}$ **o** $\begin{array}{r} 8.19 \\ -5.2963 \\ \hline \end{array}$

PROBLEMS USING ADDITION AND SUBTRACTION OF FRACTIONS

1. A truck is loaded with 4 crates weighing as follows: 246.9 lb., 615.85 lb., 493.69 lb., and 306.7 lb. What is the total weight of the load in pounds?

2. A boy planed .15 in. from a board that was .7 in. thick. How many inches thick was the board that remained?

3. A rod that is 2.614″ long is to be cut to a length of 2.495″. What number of inches is to be cut off?

4. One year in a Canadian city 32.58 inches of rain fell. The amounts of rain falling in June, July, August and September were as follows: 2.93″, 1.86″, 1.77″, 3.05″. What number of inches of rain fell in the other eight months of the year?

5. A boy and his father were driving from their home to a city 239.6 miles away. When they left home, the odometer on the car read 16,847.3. When they stopped for lunch, the odometer read 17,023.1. What number of miles did they still have to travel?

6. A boy has a steel rod that is .4″ thick and 2 feet long. He needs to cut three pieces of the following lengths: 2.2″, 5.9″ and 4.7″. Each saw cut is .2″ wide. What length of rod will remain when these three pieces of rod have been cut off?

7. The expenses on a motor trip were as follows: gasoline, \$21.95; oil, \$0.79; motel, \$42.00; meals, \$63.86; amusements, \$33.19; other expenses, \$17.48. What was the total cost of the trip?

8. A field is 240 yards long. Its width is 74.5 yards less than its length. What number of yards is it around the whole field?

9. A storekeeper bought a radio for \$19.98 and sold it for \$23.95. What profit did he make on the deal?

10. A boy's allowance is \$2.50 a week. Last week he spent \$1.25 for a hockey stick, 55¢ for movies and 12¢ for a pencil. He saved the rest. What amount of money did he save?

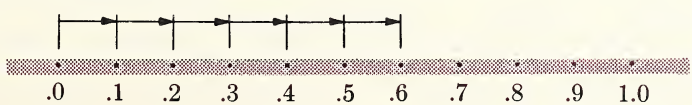
MULTIPLICATION USING DECIMAL FRACTIONS

1 Find n if $6 \times .1 = n$.

a $.1 = \frac{1}{10}$ so $6 \times .1 = 6 \times \frac{1}{10} = \frac{6 \times 1}{10} = \frac{6}{10} = .6 \quad n = .6$

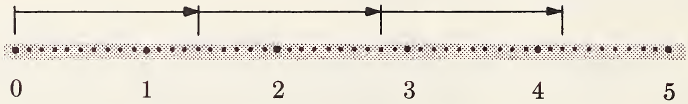
b We can think of $6 \times .1$ as $.1 + .1 + .1 + .1 + .1 + .1$ We see that the sum of six .1's is .6; so $6 \times .1 = .6$ and $n = .6$.

c We can use a number line:



We see that $n = .6$

2 Use a number line to find x if $3 \times 1.4 = x$.



From the number line we see that $3 \times 1.4 = 4.2$; so $x = 4.2$.
We can check by finding $1.4 + 1.4 + 1.4$.

3 Find the product of 1.2 and 3.6.

$1.2 = 1 + \frac{2}{10} = \frac{12}{10} \qquad 3.6 = 3 + \frac{6}{10} = \frac{36}{10}$

$$\begin{aligned} 1.2 \times 3.6 &= \tfrac{12}{10} \times \tfrac{36}{10} \\ &= \tfrac{432}{100} \\ &= \tfrac{400}{100} + \tfrac{32}{100} \\ &= 4 + \tfrac{32}{100} \\ &= 4.32 \end{aligned} \qquad \begin{array}{r} 36 \\ \underline{12} \\ 72 \\ \underline{360} \\ 432 \end{array}$$

Notice that when we multiply tenths by tenths, we get hundredths in the product.

Also: $1.2 \quad \times \quad 3.6 \quad = \quad 4.32$

1.2	3.6	4.32
1 place of decimal	1 place of decimal	2 places of decimal

4 Find the product of 4.3 and 2.56.

$4.3 = 4 + \frac{3}{10} = \frac{40}{10} + \frac{3}{10} = \frac{43}{10}$

$2.56 = 2 + \frac{5}{10} + \frac{6}{100} = \frac{200}{100} + \frac{50}{100} + \frac{6}{100} = \frac{256}{100}$

$$\begin{aligned} 4.3 \times 2.56 &= \tfrac{43}{10} \times \tfrac{256}{100} \\ &= \tfrac{11008}{1000} \\ &= 11\tfrac{8}{1000} \\ &= 11.008 \end{aligned} \qquad \begin{array}{r} 256 \\ \underline{43} \\ 768 \\ \underline{10240} \\ 11008 \end{array}$$

4.3: 1 place of decimal
2.56: 2 places of decimal
11.008: 3 places of decimal

Notice that when we multiply tenths and hundredths, we have thousandths in the product.

5. Find the following products by:

(i) changing the decimal-fraction numerals to common fractions

(ii) multiplying the common fractions

(iii) changing the product to a decimal fraction

a 2.9×1.4 **b** 5.6×2.7 **c** 8.4×9 **d** 17.04×1.1

e 12.41×8 **f** 1.64×1.3 **g** $2.09 \times .26$ **h** $4.16 \times .25$

i 22×3.8 **j** 14.1×9.7 **k** 3.115×7 **l** 1.04×1.6

6. What is $.01 \times .06$?

What is $\frac{1}{100} \times \frac{6}{100}$?

What is $\frac{6}{10000}$ written as a decimal-fraction numeral?

How many decimal places were there in each of the factors?

How many decimal places are there in the product?

Now find the following products by the method used above:

a $.04 \times .15$ **b** $.03 \times .1$ **c** $.06 \times .07$

d $.92 \times .06$ **e** $.07 \times .12$ **f** $.08 \times .08$ **g** $.004 \times .3$

h $.112 \times .09$ **i** $.206 \times .3$ **j** $.001 \times .01$ **k** $.104 \times .03$

l $.56 \times .14$ **m** $.175 \times .15$ **n** $1.36 \times .9$ **o** $.09 \times .12$

p $.54 \times .07$ **q** $.111 \times .71$ **r** $.185 \times .16$ **s** $.99 \times .68$

t $.139 \times .59$ **u** $.187 \times .37$ **v** $.309 \times .7$ **w** $.533 \times .79$

Can you suggest a rule for placing the decimal point when you multiply decimal fractions?

7 **When we multiply two numbers written as decimal numerals, there are as many decimal places in the product as there are in the sum of the decimal places in the factors.**

8. Illustrate the above rule by finding the following products: (Write each factor as a fraction numeral before multiplying. The first is done for you.)

a 3.2×1.049

$3.2 = 3 + \frac{2}{10} = \frac{30}{10} + \frac{2}{10} = \frac{32}{10}$

$1.049 = 1 + \frac{0}{10} + \frac{4}{100} + \frac{9}{1000} = \frac{1000}{1000} + \frac{0}{1000} + \frac{40}{1000} + \frac{9}{1000} = \frac{1049}{1000}$

$$\begin{aligned} 3.2 \times 1.049 &= \frac{32}{10} \times \frac{1049}{1000} \\ &= \frac{33568}{10000} \\ &= 3\tfrac{3568}{10000} \\ &= 3.3568 \end{aligned} \qquad \begin{array}{r} 1049 \\ \underline{32} \\ 2098 \\ \underline{31470} \\ 33568 \end{array}$$

1 place of decimal — 3 places of decimal — 4 places of decimal

b 6.8×2.4

c 2.05×3.4

d 15.1×8

e 7×37.5

f 9.06×1.45

g $.0145 \times .6$

h 12.09×6.182

i $17.014 \times .8$

9. In example 8**a**, we found that $3.2 \times 1.049 = 3.3568$.

a Multiply 3.2 and 1.049 ignoring the decimal points.

$$\begin{array}{r} 1049 \\ \underline{32} \\ 2098 \\ \underline{31470} \\ 33568 \end{array}$$

3.2 is about 3.

1.049 is about 1.

So 3.2×1.049 is about $3 \times 1 = 3$.

Where must we place the decimal point in the numeral 33568 to make it about equal to 3?

10. Work the following multiplication examples, ignoring the decimal points: (Estimate each product and then place decimal points to make your answers correct. Check each answer by using the rule in Number 7 above.)

a $15.8 \times .9$

b 7.01×3.76

c $.04 \times 53.1$

d $.06 \times .9$

e $32.5 \times .205$

f 6.37×12.695

g 14.23×11.8

h 9.9×9.9

11 What is $10 \times .15$?

a $.15 = \frac{15}{100}$; so $10 \times .15 = \frac{\overset{1}{\cancel{10}}}{1} \times \frac{15}{\underset{10}{\cancel{100}}} = \frac{15}{10} = 1 + \frac{5}{10} = 1.5$

b
$$\begin{array}{r} .15 \\ \times\ 10 \\ \hline 1.50 \end{array}$$
$1.50 = 1.5$

c $10 \times .15$. We know that $10 \times 15 = 150$.
.15 is more than one tenth.
$10 \times$ 'more than $\frac{1}{10}$' is more than 1.
So in '150' we put the decimal point after the '1'.
$10 \times .15 = 1.50 = 1.5$

We see that $10 \times .15 = 1.5$. What do you notice about the digits in the products?

12. Work the following:

a $100 \times .04$ b 10×3.1 c 9.43×100
d $1{,}000 \times 5.06$ e 17.4×10 f $14.001 \times 1{,}000$
g $.42 \times 1{,}000$ h $.6 \times 100$ i 100×3.5
j $.05 \times 10$ k $100 \times .01$ l $1{,}000 \times 1.46$

Can you make a rule for multiplying when one of the factors is 10, 100, 1,000, etc., and the other factor is represented by a decimal numeral?

13. What is 200×1.056?

$200 \times 1.056 = (2 \times 100) \times 1.056 = 2 \times (100 \times 1.056)$
$100 \times 1.056 = 105.6$, so $200 \times 1.056 = 2 \times 105.6 = 211.2$

Find: a 300×4.1 b $60 \times .54$ c 9.46×200
d 4.08×30 e 19.01×70 f $800 \times .09$
g $5.6 \times 2{,}000$ h $.008 \times 8{,}000$ i $3{,}000 \times .03$
j 7.014×80 k 19.1×70 l $9{,}000 \times 1.06$

Can you make a rule for multiplying when one of the factors is a number of 10's, 100's, 1,000's, etc.?

14. By exploring, we have found two useful things about multiplying decimal-fraction numerals. Work the following and see whether you can make another useful discovery:

a $6 \times .1$ b $.1 \times 17$ c $26 \times .1$
d $.1 \times 8.4$ e $.1 \times 27.3$ f $15.95 \times .1$
g $6.842 \times .1$ h $.1 \times .6$ i $.1 \times .06$
j $.1 \times .1$ k $.1 \times .009$ l $2.015 \times .1$

What do you notice about the digits in the product in each example?

What do you notice about the position of the decimal point in each product?

Can you make a rule for multiplying when one of the factors is .1?

15. Now that you have discovered a rule, use it to find $.4 \times 3.5$. (Hint: think of .4 as $4 \times .1$.)

Now find the following products:

a $9 \times .8$	**b** $.8 \times 3.14$	**c** $.7 \times 8.961$
d $15.04 \times .5$	**e** $2.39 \times .6$	**f** $.2 \times .143$

16. Work the following:

a $7 \times .01$	**b** $16 \times .01$	**c** $.01 \times 10$
d $12 \times .001$	**e** $4 \times .001$	**f** $.001 \times 8$
g $2.4 \times .01$	**h** $9.14 \times .01$	**i** $1.004 \times .01$
j $1.5 \times .001$	**k** $2.4 \times .001$	**l** $.6 \times .001$
m $1.04 \times .01$	**n** $436.8 \times .01$	**o** $.01 \times .468$
p $.001 \times 3.08$	**q** $17.32 \times .001$	**r** $1{,}000 \times .001$
s $.001 \times 6.958$	**t** $493 \times .01$	**u** $.01 \times .065$

Can you find a rule for multiplying when one of the factors is .01 or .001?

17. Use the rule you have discovered to find the following products: (The first one is done for you.)

a $.006 \times 498.4$

$.006 = 6 \times .001$;

so $.006 \times 498.4 = (6 \times .001) \times 498.4 = 6 \times (.001 \times 498.4) = 6 \times .4984 = 2.9904$

b $.04 \times 73$	**c** $.009 \times 1{,}416.8$	**d** $.08 \times 37.3$
e $644 \times .002$	**f** $.05 \times 480$	**g** $71.42 \times .003$

DIVISION WITH DECIMAL FRACTIONS

1 Find the quotient: $\frac{65}{2}$

2)65	30
60	
5	2
4	
1	32

We see that $\frac{65}{2} = 32\frac{1}{2}$

For $\frac{1}{2}$ we can write:

$\frac{1}{2} = \frac{1}{2} \times \frac{5}{5} = \frac{5}{10} = .5$

So $\frac{65}{2} = 32\frac{1}{2} = 32.5$.

b What is $51 \div 4$?

For $51 \div 4$ we can write $\frac{51}{4}$.

$$\begin{array}{r|r} 4\overline{)51} & 10 \\ \underline{40} & \\ 11 & 2 \\ \underline{8} & \underline{} \\ 3 & 12 \end{array}$$

$\frac{51}{4} = 12\frac{3}{4}$

For $\frac{3}{4}$ we can write

$$\frac{3}{4} = \frac{3}{2 \times 2} = \frac{3}{2 \times 2} \times \frac{5 \times 5}{5 \times 5} = \frac{3 \times 5 \times 5}{(2 \times 5) \times (2 \times 5)}$$

$$= \frac{75}{10 \times 10} = \frac{75}{100} = .75$$

So $51 \div 4 = 12\frac{3}{4} = 12.75$.

2. Study the examples below:

$\frac{65}{2} = \frac{65.00}{2}$

$$\begin{array}{r|r} 2\overline{)65.00} & 30 \\ \underline{60} & \\ 5.00 & 2 \\ \underline{4} & \\ 1.00 & .5 \\ \underline{1.0} & \underline{} \\ & 32.5 \end{array}$$

$\frac{51}{4} = \frac{51.00}{4}$

$$\begin{array}{r|r} 4\overline{)51.00} & 10 \\ \underline{40} & \\ 11.00 & 2 \\ \underline{8} & \\ 3.00 & .7 \\ \underline{2.8} & \\ .20 & .05 \\ \underline{.20} & \underline{} \\ & 12.75 \end{array}$$

How are these two examples similar to other division examples that we have done?

We see that we can obtain decimal fractions in our quotients by putting a decimal point and a series of zeros after the dividend.

3. Work the following: (Give each answer in decimal numeration.)

a $\frac{536}{16}$ **b** $\frac{1795}{40}$ **c** $\frac{579}{25}$ **d** $\frac{631}{20}$

e $\frac{404}{32}$ **f** $\frac{925}{125}$ **g** $\frac{3044}{80}$ **h** $\frac{9148}{64}$

4. Divide: $3.304 \div 1.4$

$$\begin{array}{r|l} 1.4\overline{)3.304} & 2 \\ \underline{2.8} & \\ .504 & .3 \leftarrow \text{How did we obtain this?} \\ \underline{.420} & \text{What is } .3 \times 1.4\text{?} \\ .084 & .06 \leftarrow \text{How did we obtain this?} \\ \underline{.084} & \text{What is } .06 \times 1.4\text{?} \\ & \overline{2.36} \end{array}$$

Did you find this difficult? If you did, can you think of a reason for the difficulty? State the reason.

5 Consider again $3.304 \div 1.4$.

Write the example as a fraction numeral.

We have $3.304 \div 1.4 = \frac{3.304}{1.4}$.

Now multiply this fraction numeral by $\frac{1000}{1000}$.

$$\frac{1000}{1000} \times \frac{3.304}{1.4} = \frac{1000 \times 3.304}{1000 \times 1.4} = \frac{3304}{1400}$$

What does $\frac{1000}{1000}$ equal? Do we alter the value of a fraction if we multiply it by 1?

Does $\frac{3.304}{1.4} = \frac{3304}{1400}$?

Compute $\frac{3304}{1400}$.

$$\begin{array}{r|l}
1400\overline{)3304.00}\ \ldots & 2 \\
\underline{2800} & \\
504.00\ \ldots & .3 \\
\underline{420.00} & \\
84.00 & .06 \\
\underline{84.00} & \underline{} \\
 & 2.36
\end{array}$$

6. Study the examples below. Copy them without the working, and compute. Check your answers with the answers here.

a $\frac{1.25}{2.5} = \frac{100}{100} \times \frac{1.25}{2.5}$

$= \frac{125}{250}$

$$\begin{array}{r|l}
250\overline{)125.00}\ \ldots & .5 \\
\underline{125.00} &
\end{array}$$

$\therefore\ \frac{1.25}{2.5} = .5$

b $2.4 \div .006 = \frac{2.4}{.006}$

$\frac{2.4}{.006} = \frac{1000}{1000} \times \frac{2.4}{.006}$

$= \frac{2400}{6}$

$$\begin{array}{r|l}
6\overline{)2400} & 400 \\
\underline{2400} &
\end{array}$$

$\therefore\ 2.4 \div .006 = 400$

c $4.2 \div 21 = \frac{4.2}{21}$

$\frac{4.2}{21} = \frac{10}{10} \times \frac{4.2}{21}$

$\frac{42}{210}$

$$\begin{array}{r|l}
210\overline{)42.00} & .2 \\
\underline{42.0} & \\
.00 &
\end{array}$$

$\therefore\ 4.2 \div 21 = .2$

d $\frac{.08}{.0008} = \frac{10000}{10000} \times \frac{.08}{.0008}$

$= \frac{800}{8}$

$= 100$

$\therefore\ \frac{.08}{.0008} = 100$

7. Work the following:

a $.015 \div 5$	**b** $3.6\overline{)72}$	**c** $4.8\overline{)9.6}$	**d** $\frac{.57}{.19}$
e $2.5\overline{)325}$	**f** $.07\overline{).049}$	**g** $.025\overline{)6.25}$	**h** $\frac{4.098}{.02}$
i $.3\overline{).003}$	**j** $24\overline{)7.2}$	**k** $14.4 \div 3.6$	**l** $9.6 \div .012$

8 We learned earlier that we could write fraction numerals as decimal-fraction numerals:

$$\begin{aligned}
\frac{7}{8} = \frac{7}{2\times2\times2} &= \frac{7}{2\times2\times2}\times\frac{5\times5\times5}{5\times5\times5} \\
&= \frac{7\times5\times5\times5}{2\times2\times2\times5\times5\times5} \\
&= \frac{7\times5\times5\times5}{(2\times5)\times(2\times5)\times(2\times5)} \\
&= \frac{875}{1000} \\
&= .875
\end{aligned}$$

We can think of $\frac{7}{8}$ as meaning $7 \div 8$.

Now let us find a decimal-fraction numeral for $\frac{7}{8}$ by dividing 7 by 8:

$$\begin{array}{r|l}
8\overline{)7.000}\ \ldots & .8 \\
\underline{6.4} & \\
.600\ \ldots & .07 \\
\underline{.560} & \\
.040 & .005 \\
\underline{.040} & \underline{} \\
 & .875
\end{array}
\qquad\qquad
\begin{array}{l}
.8\times8 = 6.4 \\
.07\times8 = .56 \\
\underline{.005\times8 = .04} \\
.875\times8 = 7.00
\end{array}$$

Similarly, $\frac{3}{20} = 3 \div 20$

$$\begin{array}{r|l}
20\overline{)3.000}\ \ldots & .1 \\
\underline{2.0} & \\
1.000\ \ldots & .05 \\
\underline{1.000} & \underline{} \\
.000\ \ldots & .15
\end{array}
\qquad\qquad
\begin{array}{l}
.1\times20 = 2.0 \\
\underline{.05\times20 = 1.00} \\
.15\times20 = 3.00
\end{array}$$

9 We can also do the computation by using the short division method we have learned.

Express $\frac{1}{8}$ as a decimal-fraction numeral.

$\frac{1}{8} = 1 \div 8.\quad 1 = 1.000\ \ldots$

$$\begin{array}{r}
8\underline{)1.0^{1}0^{2}0^{4}}\ \ldots \\
0.125
\end{array}
\qquad \text{or} \qquad
\begin{array}{r}
8\underline{)1.000}\ \ldots \\
.125
\end{array}$$

Explain each step that has been taken.

10. Write the following as decimal-fraction numerals:

a $\frac{4}{5}$	**b** $\frac{3}{8}$	**c** $\frac{3}{4}$	**d** $\frac{9}{20}$
e $\frac{5}{8}$	**f** $\frac{13}{40}$	**g** $\frac{11}{16}$	**h** $\frac{7}{16}$
i $\frac{3}{80}$	**j** $\frac{17}{80}$	**k** $\frac{1}{40}$	**l** $\frac{5}{16}$

DIVISION OF DECIMALS — SHORTENED METHOD

1. When we were working with division of whole numbers, we saw a shortened way of setting down the computation. This way of computing division we called the 'shortened method'. Study the examples below and see how we can use this shortened method of division with decimal-fraction numerals.

a $3.924 \div 3.6 = n.$

$$n = \frac{3.924}{3.6} = \frac{3.924 \times 10}{3.6 \times 10} = \frac{39.24}{36}$$

(i)
$$
\begin{array}{r|l}
36\overline{)39.24} & 1 \\
\underline{36.00} & \\
3.24 & .09 \\
\underline{3.24} & \underline{} \\
 & 1.09
\end{array}
$$

(ii)
$$
\begin{array}{r}
\left.\begin{array}{r}.09\\1.00\end{array}\right\}1.09 \\
36\overline{)39.24} \\
\underline{36} \\
3.24 \\
\underline{3.24}
\end{array}
$$

(iii)
$$
\begin{array}{r}
1.0 \\
36\overline{)39.24} \\
\underline{36} \\
3.2
\end{array}
\qquad
\begin{array}{r}
1.09 \\
36\overline{)39.24} \\
\underline{36} \\
3.24 \\
\underline{3.24}
\end{array}
$$

b $.1363 \div .47 = x$

$$x = \frac{.1363}{.47} = \frac{.1363 \times 100}{.47 \times 100} = \frac{13.63}{47}$$

(i)
$$
\begin{array}{r|l}
47\overline{)13.63} & .20 \\
\underline{9.40} & \\
4.23 & .09 \\
\underline{4.23} & \underline{} \\
 & .29
\end{array}
$$

(ii)
$$
\begin{array}{r}
\left.\begin{array}{r}.09\\.2\end{array}\right\}.29 \\
47\overline{)13.63} \\
\underline{9.4} \\
4.23 \\
\underline{4.23}
\end{array}
$$

(iii)
$$
\begin{array}{r}
.2 \\
47\overline{)13.63} \\
\underline{9.4} \\
4.2
\end{array}
\qquad
\begin{array}{r}
.29 \\
47\overline{)13.63} \\
\underline{9.4} \\
4.23 \\
\underline{4.23}
\end{array}
$$

Explain each step in the examples above.

2. Study the following: (Copy, without the computation, and do the problems. Check your answers with the answers here.)

a $.763 \div 21.8 = x$

$$\frac{.763}{21.8} = \frac{.763 \times 10}{21.8 \times 10} = \frac{7.63}{218}$$

$$\begin{array}{r} 0.035 \\ 218\overline{)7.630\ldots} \\ \underline{6.54} \\ 1.090 \\ \underline{1.090} \end{array}$$

$x = 0.035$

b $89.873 \div 2.59 = n$

$$\frac{89.873}{2.59} = \frac{89.873 \times 100}{2.59 \times 100} = \frac{8987.3}{259}$$

$$\begin{array}{r} 34.7 \\ 259\overline{)8987.3} \\ \underline{777} \\ 1217 \\ \underline{1036} \\ 181.3 \\ \underline{181.3} \end{array}$$

$n = 34.7$

3. Work the following by:
   (i) the subtractive method
   (ii) the shortened method

a $102.896 \div 47.2$	**b** $.83435 \div .407$	**c** $1,006.647 \div 9.67$
d $.047152 \div 5.6$	**e** $10.3731 \div .487$	**f** $3261.36 \div .0856$

PROBLEMS USING DECIMALS

1. A walking cane is 35.89 inches long. A brass ferrule on the end extends a further .25 inches. What is the length in inches of the complete cane?

2. A calendar is .15 inches thick when new. Each month is printed on a sheet .0125 inches thick. What thickness remains when the calendar shows the month of April?

3. 4 stamps are fixed on an envelope. Each stamp measures .465 inches across. Find the distance in inches from the left border of the first stamp to the right border of the fourth stamp.

4. Each complete turn of a screw takes it .0625 inches into a board. How many turns are needed to drive a 1.5 inch screw completely into a board?

5. A machine cuts slices .15 inches thick from a Polish sausage. How many slices can be taken from a sausage 15.6 inches long? (Ignore the ends.)

6. A motorist travelled 530.4 miles. If his car averaged 33.15 miles to the gallon, what number of gallons would he have used on the trip?

7. A telescope measures 10.75 inches when closed. It has two extensions, the first measuring 5.46 inches and the second measuring 4.8 inches. Find the length of the telescope in inches with:

   a the first section extended

   b both sections extended

   c the second section extended

8. A totem pole consists of six figures of the following descriptions and lengths starting at the top in this order: A raven 10.5 ft., a man 9.25 ft., a grizzly bear 11.75 ft., a woman 8.125 ft., a killer whale 15 ft. and a beaver 4.875 ft. The total length of the cedar pole is 68.5 ft.

   a What number of feet is below ground when the pole is erected?

   b What is the number of feet from the ground level (i) to the top of the grizzly bear, (ii) to the top of the woman, (iii) to the top of the killer whale?

9. How many wooden floor blocks 8.75 inches in length will fit end-to-end on a floor 17.5 ft. long?

10. Toys for midway sideshows cost 1.5 cents each. How much will the showman have to pay for 4.5 gross of these toys?

11. The price of chickens increased by $0.06 per lb. Find the new cost of a $2.75 chicken that weighs 5.5 lb.

12. A steel plate is .56 inches thick. A bracket to be bolted to the plate is .24 inches thick. How much of a 1.50 inch bolt will project if two washers each .125″ thick and a nut .25″ thick are used?

13. The gold contained in a British sovereign is worth $4.78. What is the value of 245 of these coins?

14. Share $1,256.48 equally among 8 people. How much will each receive?

15. How many bars of chocolate, each weighing 2.5 oz., can be made from 1.5 cwt. of liquid chocolate?

16. What length of steel rod will be used to make 5,256 bicycle spokes, each 12.75 inches long?

17. A telephone wire contracts in the winter by 1.325 inches in every 100 yd. of its length. Find the amount of contraction in 1 mile of the wire.

18. A field had an area of 5.5 acres. This field was divided into building lots of .375 acres each. How many building lots were obtained from this field?

19. A car travels 15.5 miles on a gallon of gasoline. (a) What number of miles will it travel on 18 gallons of gasoline? (b) How many gallons of gasoline will the car use in travelling 112.375 miles?

20. A girl bought 3.75 yards of material at \$2.95 a yard. How much change should she receive from a \$20 bill?

21. A machinist cuts 15 pieces each 3.95 inches long from a steel rod that is 10 feet long. The width of each saw cut is 0.12 inches. What length of the steel rod remains?

22. An airplane has a ground speed of 565 knots. What is its speed in miles an hour if 1 knot is a speed of 1.15 miles an hour?

23. A ship travels for 14 hours at a speed of 19.75 knots. What number of miles does it travel in this time?

24. A refrigerator sells for \$395 cash or \$50 down and 12 payments of \$33.29 each. How much money is saved by paying cash?

25. A piece of meat cost \$9.36. The price of the meat was \$0.96 a pound. What number of pounds did the piece of meat weigh?

26. Six students were practising using a micrometer. A micrometer is a very accurate measuring instrument. Each student measured the thickness of a block of metal. The readings obtained were as follows: 0.9984 inches; 1.0006 inches; 0.9998 inches; 0.9995 inches; 1.0002 inches and 1.0002 inches. What was the average of the readings?

27. A mountaineer noted that the temperature decreased by 3.4^0 Fahrenheit for each 1,000 feet that he climbed. The temperature at the foot of the mountain was 65^0F. The temperature at the top of the mountain was 48^0F. How many feet high was the mountain?

28. Potatoes sell for \$0.49 a bag. A bag contains 10 pounds of potatoes. How many pounds of potatoes can be bought for \$3.43?

29. If 3.09 is added to a number, the sum is 5.26×2.07. What is the number?

30. When a number is subtracted from 5.83, the difference is $1.44 \div 1.2$. What is the number?

REPEATING DECIMALS

1 Write $\frac{1}{3}$ as a decimal-fraction numeral.

$\frac{1}{3} = 1 \div 3$

$3\overline{)1.000\ \ldots}$	.3
.9	
.10	.03
.09	
.010	.003
.009	
.0010	.0003
.0009	
.0001	.3333 . . .

We see that the decimal numeral for $\frac{1}{3}$ has no end. Each time we divide by 3 we have 1 as the remainder. The '3' keeps repeating or recurring in the quotient. We say that the decimal numeral for $\frac{1}{3}$ is a **repeating decimal-numeral** or a **recurring decimal numeral.**

For convenience we shall call such decimal numerals, **repeating** decimal numerals.

We saw that the decimal numeral for $\frac{7}{8}$ was .875. Because the numeral came to an end or **terminated**, we call decimal numerals such as .875, **terminating** decimal.

2 We have just seen that the decimal-fraction for $\frac{1}{3}$ is .3333 . . . The three dots (. . .) shows that this numeral can go on for ever. Another way to show that the '3' repeats is to write the decimal-fraction numeral as $.\dot{3}$. The dot over the '3' shows that the '3' repeats.

3 Write $\frac{5}{6}$ as a decimal-fraction numeral.

$\frac{5}{6} = 5 \div 6$

$6\overline{)5.000\ \ldots}$	.8
4.8	
.200	.03
.18	
.020	.003
.018	
.002	.833

$\frac{5}{6} = 5 \div 6 \quad 5 = 5.000\ \ldots$

$6\underline{)5.000.}\ \ldots$

0.833 . . .

Each time the remainder will be '2'.

So '3' will repeat. Why?

$\frac{5}{6} = .8333\ \ldots = .8\dot{3}$

The dot over the '3' shows that it repeats.

4 Write $\frac{1}{7}$ as a decimal-fraction numeral.

$\frac{1}{7} = 1 \div 7$

$7\overline{)1.000000\ldots}$	.1
.7	
.30	.04
.28	
.020	.002
.014	
.0060	.0008
.0056	
.00040	.00005
..00035	
.000050	.000007
.000049	
.0000010	.0000001
.0000007	
3	.1428571

a Note how we shorten the amount of writing we do by using each time only as many zeros as we need.

b How do we know that this numeral will repeat?

c What remainder will come next? How do you know?

$\frac{1}{7} = .1428571\ldots$

If we use the short division method, we have:

$\frac{1}{7} = 1 \div 7. \quad 1 = 1.000\ldots$

$$\begin{array}{r} \scriptstyle 3\,2\,6\,4\,5\,1\,3\,2 \\ 7\overline{)1.000000000}\ldots \\ 0.142857142\ldots \end{array}$$

We may keep the remainders in mind and write $\begin{array}{r} 7\overline{)1.000000000}\ldots \\ .142857142\ldots \end{array}$

5 The figures that repeat are 142857. To show that a group of digits repeats, we put a dot over the first and the last digit of that group of digits.

$$\begin{aligned} .14285714\ldots &= .\dot{1}4285\dot{7} \\ 3.25252\ldots &= 3.\dot{2}\dot{5} \\ 16.159159\ldots &= 16.\dot{1}5\dot{9} \\ 8.7324324\ldots &= 8.7\dot{3}2\dot{4} \end{aligned}$$

6. Write the following as decimal-fraction numerals:

a $\frac{1}{9}$	**b** $\frac{1}{6}$	**c** $\frac{5}{18}$	**d** $\frac{2}{3}$	**e** $\frac{1}{12}$
f $\frac{7}{12}$	**g** $\frac{4}{9}$	**h** $\frac{5}{12}$	**i** $\frac{11}{18}$	**j** $\frac{5}{9}$

ANOTHER LOOK AT MULTIPLICATION AND DIVISION USING MULTIPLES OF TEN

1. Find the following products:

a $10\times.6$	**b** $10\times.4$	**c** 10×2.3	**d** 10×1.9
e $10\times.99$	**f** $10\times.16$	**g** 10×3.98	**h** 10×4.05
i 10×1.416	**j** $10\times.897$	**k** $10\times.055$	**l** $10\times.002$

Can you make a rule that applies when you multiply a decimal fraction by 10? Study the examples below:

(i) $1\times.84=.84$ $10\times.84=8.4$	(ii) $1\times.105=.105$ $10\times.105=1.05$	(iii) $1\times.042=.042$ $10\times.042=.42$
(iv) $1\times.7=.7$ $10\times.7=7$	(v) $1\times1.3=1.3$ $10\times1.3=13$	(vi) $1\times21.06=21.06$ $10\times21.06=210.6$

When we multiply a decimal fraction or a mixed decimal fraction by 10, we can compute the product by moving each figure in the numeral of the number multiplied one place to the left.

We can compute the answer by thinking of moving the decimal point one place to the right.

2. Find $.86\times10$.

$.86\times10=10\times.86$ (Why?)

$10\times.86=8.6$; so $.86\times10=8.6$.

Now compute the following:

a $.7\times10$	**b** $.9\times10$	**c** $.4\times10$	**d** 1.8×10
e $.32\times10$	**f** $.08\times10$	**g** 1.24×10	**h** 3.05×10
i $.529\times10$	**j** 1.063×10	**k** 3.567×10	**l** $.001\times10$

Now make a rule for multiplying 10 by a decimal fraction or a mixed decimal fraction.

3. **When we find the product of two factors one of which is 10 and the other is a decimal fraction, we can compute the answer by moving the decimal point in the decimal fraction one place to the right.**

4. Find the following products:

a $100\times.84$	**b** $100\times.9$	**c** 100×3.016	**d** $100\times.908$
e $.76\times100$	**f** 3.167×100	**g** $.009\times100$	**h** 14.3×100
i 5.8×100	**j** $.032\times100$	**k** 100×15.1	**l** $100\times.65$
m 19.7×100	**n** 100×14.621	**o** $100\times.004$	**p** $100\times.42$

5. Study the following:

(i) $1 \times 3.65 = 3.65$
$100 \times 3.65 = 365$

(ii) $1 \times 14.03 = 14.03$
$100 \times 14.03 = 1403$

(iii) $1 \times .6 = .6$
$100 \times .6 = 60$

(iv) $1 \times .1 = .1$
$100 \times .1 = 10$

(v) $1 \times 1.9 = 1.9$
$100 \times 1.9 = 190$

(vi) $1 \times 3.5 = 3.5$
$100 \times 3.5 = 350$

Now make a rule for finding the product of two factors when one of the factors is a decimal fraction and the other is 100.

6. Now that you have found rules that help you to find 10 or 100 times a decimal fraction, what rule do you think will help you to find quickly 1,000 times a decimal fraction? State your rule and test it. Multiply each of the following by **a** 10, **b** 100, **c** 1,000:

1.5; 2.06; .095; 2.3288; .00008; 2.6; 7.02; 91.3; .00196

7. Compute $\frac{3.6}{10}$.

(i) $\frac{3.6}{10} = 3.6 \div 10$

$$\begin{array}{r} .36 \\ 10\overline{)3.60} \\ \underline{3.0} \\ .60 \\ \underline{.60} \end{array}$$

(ii) $\frac{3.6}{10} = \frac{3\frac{6}{10}}{10} = \frac{\frac{36}{10}}{10} \times \frac{\frac{1}{10}}{\frac{1}{10}} = \frac{\frac{36}{10} \times \frac{1}{10}}{1}$

$\frac{36}{10} \times \frac{1}{10} = \frac{36}{100} = .36$

$\frac{3.6}{10} = .36$

$\frac{3.6}{10} = .36$

Now compute the following:

a $\frac{2.7}{10}$ **b** $\frac{15.9}{10}$ **c** $\frac{3.8}{10}$ **d** $\frac{4.5}{10}$

e $1.35 \div 10$ **f** $4.26 \div 10$ **g** $.57 \div 10$ **h** $.88 \div 10$

i $\frac{12.046}{10}$ **j** $\frac{.964}{10}$ **k** $\frac{.018}{10}$ **l** $\frac{.009}{10}$

8. Study the following:

(i) $1.9 \div 10 = .19$

(ii) $.8 \div 10 = .08$

(iii) $.104 \div 10 = .0104$

(iv) $.315 \div 10 = .0315$

What rule can you make to help you compute quickly the quotient when you divide a decimal fraction by 10?

9 **When we divide a decimal fraction by 10, we move the digits in the dividend one place to the right. If we find that there is no digit in the tenths' place, we put a zero there. To compute the quotient when we divide a decimal fraction by 10, we think of the decimal point being moved one place to the left.**

10. Now that we know how to divide a decimal fraction by 10, we can try to find rules for dividing decimal fractions by 100, 1,000 and so on. Think of rules and then test them by actually computing to see if they work.

11. Work the following:

a $\frac{13.7}{10}$	**b** $\frac{13.7}{100}$	**c** $\frac{13.7}{1000}$
d $6.9 \div 10$	**e** $6.9 \div 100$	**f** $6.9 \div 1{,}000$
g $.7 \div 10$	**h** $.7 \div 100$	**i** $.7 \div 1{,}000$
j $.04 \div 10$	**k** $.04 \div 100$	**l** $.04 \div 1{,}000$
m $1.205 \div 10$	**n** $1.205 \div 100$	**o** $1.205 \div 1{,}000$
p $.003 \div 10$	**g** $.003 \div 100$	**r** $.003 \div 1{,}000$

SCIENTIFIC NOTATION

1 The distance of the earth from the sun is approximately 93,000,000 miles.

Consider the number 93,000,000.

Does $93{,}000{,}000 = 93{,}000{,}000 \times 1$?

Does $93{,}000{,}000 = 93 \times 1{,}000{,}000$?

Does $93{,}000{,}000 = 9.3 \times 10{,}000{,}000$?

Write 10,000,000 in exponential notation with 10 as the base. We have 10^7.

So $93{,}000{,}000 = 9.3 \times 10{,}000{,}000$
$\qquad\qquad\qquad = 9.3 \times 10^7$.

When we write 93,000,000 as 9.3×10^7, we are writing 93,000,000 in *Scientific Notation.* Scientific notation is often used by scientists in writing numerals for large numbers.

Notice $\quad 9.3 \quad \times \quad 10^7$

numeral with *ONE* figure before the decimal point

numeral containing 10 as a base and using an exponent

When we write a number as the product of two factors, one of which is a numeral with one digit before the decimal point and the other is a numeral expressed in exponential notation to base 10, we are expressing the number in *Scientific Notation.*

2 Let us see how we could express 93,000,000 in scientific notation by using our knowledge of multiplying and dividing by 10's, 100's, 1000's, etc. By what number must we divide 93,000,000 in order to have a numeral with the decimal point after the 9?

93,000,000. We must divide by 10,000,000 for the decimal point to "move" 7 places to the left.

$$\text{So } 93{,}000{,}000 = 93{,}000{,}000 \times \frac{10{,}000{,}000}{10{,}000{,}000} \quad \text{(Why?)}$$

$$= \frac{93{,}000{,}000}{10{,}000{,}000} \times 10{,}000{,}000$$

$$= 9.3 \times 10{,}000{,}000$$

$$= 9.3 \times 10^7.$$

3 Express 24,400 in scientific notation. How many places to the left must we "move" the decimal point to have a numeral with one figure before the decimal point?

24,400. By what number must we divide 24,400 in order to have the decimal point between the '2' and the '4'?

$$24{,}400 = 24{,}400 \times \frac{10{,}000}{10{,}000}$$

$$\frac{24{,}400}{10{,}000} \times 10{,}000$$

$$= 2.44 \times 10{,}000$$

$$= 2.44 \times 10^4$$

4 Express 7.5×10^5 in the usual decimal notation.

$7.5 \times 10^5 = 7.5 \times 100{,}000$ ("move" the decimal point 5 places to the right)

$= 750000.$ (notice that we put zeros in as place holders)

$= 750{,}000$

5. Express, in scientific notation, the numerals in the table overleaf:

Facts About the Planets

Planet	Approximate diameter in miles	Approximate distance from the sun in miles
Mercury	3,100	36,000,000
Venus	7,850	67,000,000
Earth	7,930	93,000,000
Mars	4,270	142,000,000
Jupiter	89,300	483,000,000
Saturn	75,000	886,000,000
Uranus	33,200	1,780,000,000
Neptune	30,900	2,795,000,000
Pluto	3,600	3,700,000,000

6. Express the following in the usual decimal notation:

a 6.5×10^4 **b** 7.09×10^5 **c** 5.24×10^3 **d** 6.65×10^2

e 8.04×10^6 **f** 1.11×10^4 **g** 3.32×10[illegible] **h** 2.95×10^7

7. Light travels at a speed of 186,000 miles a second. Express the speed of light, in miles an *hour*, using scientific notation.

8. The earth travels about 584,000,000 miles around the sun in one year. How many miles per day is this? How many miles per hour? Express each answer in scientific notation.

ROUNDING OFF DECIMALS

1 A Grade 7 boy was asked to find n if $37.24 \times 59.13 = n$. His answer was 2,202.0012. How can we tell fairly quickly if this is a reasonable answer? We can round off the numbers and multiply. This will give us an *estimate* of the product.

$37.24 \simeq 37 \simeq 40$ ($\simeq$ means "is approximately equal to")

$59.13 \simeq 59 \simeq 60$

$40 \times 60 = 2{,}400$. The boy's answer was 2,202.012.

$2{,}202.012 \simeq 2{,}202 \simeq 2{,}200$.

We are led to believe that his answer is correct. We are sure that if it is wrong, it is not wrong by a great amount. Rounding off numbers and estimating is a useful way of making sure that we have not made serious errors in our computation.

2 To help us round off we have fairly simple rules to follow. Let us see if we can discover the rules by studying the examples below:

a Round off 6915 to the nearest hundred.

$6{,}915 \simeq 6{,}900$

hundreds' digit

b Round off .016202 to the nearest thousandth

$0.016202 \simeq 0.016000 = 0.016$

thousandths' digit $\quad 0.016202 \simeq 0.016$

c Round off 1.73124 to the nearest tenth

$1.73124 \simeq 1.70000 = 1.7$

tenths' digit $\quad 1.73124 \simeq 1.7$

d Round off 3,596 to the nearest hundred

3,596 $\quad$ Is 96 closer to 100 than to zero?

hundreds' digit $\quad 3{,}596 \simeq 3{,}600$

e Round off .681483 to the nearest ten-thousandth or round off .681483 to four places of decimal.

$.681483 \simeq .681500 = .6815$

fourth place of decimal

f Round off 795 to the nearest ten.

795 $\quad$ Is 5 nearer to 10 or to 0?

It is halfway between 0 and 10

tens' digit $\quad 795 \simeq 800$

g Round off 0.0185 to the nearest thousandth or round off 0.0185 to three places of decimal.

0.0185 $\quad$ Is 0.0005 nearer to 0.001 or 0.000?

It is halfway between.

third place of decimal $\quad 0.0185 \simeq 0.0190 = 0.019$

3 Compare the rules you have discovered from the examples above with these rules:

a When rounding off a numeral to a given place value, rewrite the numeral using all the digits up to and including the digit for the required place value. Discard all the digits to the right of the required place value and replace them with zeros.

b If the first digit to be discarded after the digit in the required place value is 5 or more, increase the digit in the required place value by one.

Examples:

(i) Round off 0.914 to one decimal place

$0.914 \qquad \simeq 0.900$ (this equals .9)

first place of decimal — 1 is less than 5

(ii) Round off 0.01653 to three places of decimal

$0.01653 \qquad \simeq 0.01700$ (this equals .017)

third place of decimal — 5 or more?

4 Round off $0.\dot{1}6\dot{5}$ to:

(i) one place of decimal

(ii) two places of decimal

(iii) three places of decimal

a $0.\dot{1}6\dot{5} = 0.165165165\ldots$

first place of decimal — 5 or more?

$.0\dot{1}6\dot{5} \simeq 0.2000\ldots = 0.2$

b $0.\dot{1}6\dot{5} = 0.165165165\ldots$

second place of decimal — 5 or more?

$0.\dot{1}6\dot{5} \simeq 0.17000\ldots = 0.17$

c $0.\dot{1}6\dot{5} = 0.165165165\ldots$

third place of decimal — 5 or more?

$0.\dot{1}6\dot{5} \simeq 0.165000\ldots = 0.165$

5. Round off the following to one place of decimal:

a 0.142	**b** $0.\dot{8}\dot{7}$	**c** 12.298	**d** 1.046
e 0.854	**f** $0.\dot{3}$	**g** 11.994	**h** $0.\dot{1}\dot{5}$
i 2.095	**j** 1.9534	**k** 0.0816	**l** 11.952
m 12.675	**n** 13.8549	**o** 0.155	**p** $0.\dot{6}$

6. Round off the following to two decimal places:

a 11.9824	**b** 13.6519	**c** 0.4667	**d** $0.\dot{6}$
e 0.0954	**f** 5.0096	**g** 0.0014	**h** $0.\dot{3}$
i $0.\dot{1}\dot{8}$	**j** $0.\dot{7}\dot{9}$	**k** 1.4054	**l** $2.\dot{7}2\dot{3}$
m 0.00499	**n** 0.9987	**o** 3.9568	**p** 0.8956

7. Round off the following to three places of decimal:

a 4.34143	**b** 0.08162	**c** $0.\dot{3}$	**d** $0.\dot{6}$
e 0.81462	**f** 3.0255	**g** 14.61667	**h** 9.85809
i 3.99985	**j** 0.09872	**k** $.\dot{1}41\dot{8}$	**l** 0.00083
m 0.000412	**n** 0.00009	**o** 0.090946	**p** $0.\dot{6}2\dot{3}$

8 Find the product of .879 and .062. Round off the two factors and estimate the products. Check to see if the answer is a sensible one.

```
  .879    (three places of decimal)
  .062    (three places of decimal)
  ----
  1758
 52740
 -----
.054498   (six places of decimal)
```

$.879 \simeq .9$

$.062 \simeq .06$

$.879 \times .062 \simeq .054$

$.9 \times .06 = .054$

1 place 2 places 3 places

.054498 is a sensible answer. Our product is not likely to be seriously wrong.

9. Find the following products. Check to see if your answer is a sensible one by rounding off and estimating the product.

a 0.88×0.902 **b** $1.9 \times .09$

c 3.61×2.95 **d** $0.049 \times .0512$

e 0.85×0.106 **f** 0.324×0.213

g $9.09 \times .078$ **h** $2.68 \times .268$

i $.0784 \times .311$ **j** $.0986 \times .075$

10 Compute $.064 \div .75$. Check your answer by estimating.

$$.064 \div .75 = \frac{.064}{.75} = \frac{6.4}{75}$$

$$\begin{array}{r} .08533 \\ 75\overline{)6.4000} \\ \underline{6\ 00} \\ 400 \\ \underline{375} \\ 250 \\ \underline{225} \\ 250 \end{array}$$

(How do we know the '3' will repeat?)

$$\frac{6.4}{75} \simeq \frac{6}{80}$$

$$\begin{array}{r} .075 \\ 80\overline{)6.00} \\ \underline{5\ 60} \\ 40 \end{array}$$

$.075 \simeq .08$ $.085\dot{3} \simeq .09$

Our answer is likely to be correct.

11. Compute the following and check by estimating:

a $\frac{2.37}{.079}$ **b** $\frac{.02666}{.86}$ **c** $\frac{18.4}{.23}$ **d** $\frac{.03002}{.79}$

e $\frac{.3136}{.98}$ **f** $\frac{7.03}{1.9}$ **g** $\frac{.4688}{.08}$ **h** $\frac{.264}{.2}$

12. Round off to the nearest tenth:

a 1 nautical mile = 1.1515 land miles

b 1 metre = 1.0936 yards

c 1 kilogram = 2.2046 pounds

d 1 land mile = 0.8684 nautical miles

Now find x in the following equations, to the nearest 1 place of decimal.

(i) 6 nautical miles $= x$ land miles
(ii) 100 metres $= x$ yards
(iii) 8 kilograms $= x$ pounds
(iv) 10 land miles $= x$ nautical miles

CHAPTER TEST

1. Read the following in two ways. Write each numeral as an expanded numeral and as a mixed number: (The first one is done for you.)

a 3.06 (i) Three point zero six
(ii) Three and six hundredths

$3.06 = 3 + \frac{0}{10} + \frac{6}{100} = 3\frac{6}{100}$

b 17.38 **c** 6.409 **d** 100.001 **e** 7.094
f 9.0060 **g** 2.3016 **h** 1.9204 **i** 8.4825

2. Write the following in the form $\frac{a}{b}$, the form for fractions:

a 1.6 **b** 3.04 **c** .019 **d** 5.15
e .001 **f** 0.0145 **g** 7.605 **h** 13.13

3. Express the following numerals as decimal-fraction numerals:

a $\frac{2}{5}$ **b** $\frac{7}{10}$ **c** $\frac{1}{4}$ **d** $\frac{1}{2}$
e $\frac{7}{8}$ **f** $\frac{1}{8}$ **g** $\frac{3}{4}$ **h** $\frac{3}{5}$
i $\frac{17}{80}$ **j** $\frac{29}{40}$ **k** $\frac{11}{16}$ **l** $\frac{13}{50}$

4. Add:

a	b	c	d
3.016	14.806	6.8049	7.5626
.985	6.7	4.7687	5.2890
14.27	29.6149	1.5213	9.1003
9.6	7.358	8.6890	17.7893
27.849	208.49	9.1003	15.3477

e	f	g	h
5.016	28.069	32.158	206.815
.9887	.98	1.675	97.38
17.685	117.854	6.7152	546.876
2.3023	18.965	5.01746	95.082
29.9879	7.958	2.584	133.764

5. Subtract:

a 14.98 7.89	b 3.06 2.97	c 18.45 2.57	d 14 3.82
e 15.46 9.772	f 6.405 3.718	g 22.13 5.081	h 37.04 19.006
i 5.023 1.148	j 5.137 2.809	k 16.4 7.208	l 8.032 4.0329

6. Find the following products:

a 1.2×3.9	b 8.3×14.4	c 7.2×2.09
d 5.16×4.78	e 93×2.064	f $18.3 \times .008$
g $.6 \times .6$	h $.09 \times .086$	i $.87 \times .435$
j $2.08 \times .016$	k $950 \times .095$	l 73.8×2.059

7. Divide:

a $2.4 \div .6$	b $\dfrac{7.164}{.08}$	c $1.5\overline{).75}$
d $39.844 \div 2.8$	e $\dfrac{250.92}{8.2}$	f $.79\overline{).50165}$
g $116.48 \div 11.2$	h $\dfrac{89.585}{43.7}$	i $7.62\overline{)617.22}$
j $8186.4 \div 21.6$	k $\dfrac{4.9464}{.458}$	l $236\overline{)2308.08}$

8. Write the following as decimal-fraction numerals:

a $\frac{4}{9}$	b $\frac{5}{6}$	c $\frac{8}{7}$	d $\frac{13}{12}$	e $1\frac{1}{11}$
f $\frac{4}{13}$	g $\frac{11}{24}$	h $\frac{23}{18}$	i $\frac{19}{36}$	j $\frac{1}{48}$

9. In one week, the following amounts of rain fell:

Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
0″	0″	.46″	.06″	1.30″	0″	0″

What was the average daily rainfall?

10. Multiply each of the following by:
(i) 10 (ii) 100 (iii) 1,000

a 1.45	**b** .045	**c** 3.4	**d** 2.986	**e** .0006
f 2.049	**g** .773	**h** 39.27	**i** 14.1	**j** 13.02

11. Divide each of the following by:
(i) 10 (ii) 100 (iii) 1,000

a 46.9	**b** 2.04	**c** .069	**d** 3.3	**e** 214.85
f 9.01	**g** 3	**h** 2.754	**i** 13.6	**j** .04

12. Draw number lines and graph the following sets:

a $\{.1, .6, .9\}$	**b** $\{.04, .05, .06\}$	**c** $\{1, 1.1, 1.5\}$
d $\{2.0, 2.5, 3.0\}$	**e** $\{.001, .002\}$	**f** $\{.004, .006, .009\}$
g $\{1.24, 1.25, 1.30\}$	**h** $\{.36, .39, .45\}$	**i** $\{.001, .004, .007\}$
j $\{2, 2.001, 2.01\}$	**k** $\{.09, .10, .11\}$	**l** $\{.090, .100, .110\}$

13. Express the following in scientific notation:
a 10,000,000 **b** 46,000 **c** 217,900 **d** 5,354,000 **e** 6,900,000

14. Express the following in the usual decimal notation:
a 6.5×10^3 **b** 3.09×10^4 **c** 6×10^6 **d** 2.09×10^5 **e** 1.215×10^7

15. In some countries, liquids are measured in litres. If 1 litre is about the same quantity as 1.76 pints, about how many gallons are the same as 12 litres?

16. Mercury is about 13.6 times as heavy as water. A bottle holds .65 pounds of water. About what weight of mercury would this bottle hold?

17. What must be added to 1.035 to make 1.05?

18. An engineer was using metal plates that were $\frac{17}{100}''$ thick. How many plates of this thickness did he need to bolt together to give a total thickness of 2.55″?

19. At $.55 a dozen, how many oranges can be bought for $9.35?

20. An aeroplane used 53.25 gallons of fuel an hour. How many hours could it fly on 191.7 gallons of fuel?

21. A metal rod is 30 inches long. How many bolts can be made from this rod if each bolt needs 1.4 inches of the rod, and .1 inch of metal is lost for each cut?

22. Using a special instrument, an engineer measured the length of a steel bar and found it to be 0.99986 inches long. If the real length of the bar was 1.00012 inches, what was the amount of the error the engineer made in his measurement?

7 Ratio and Proportion

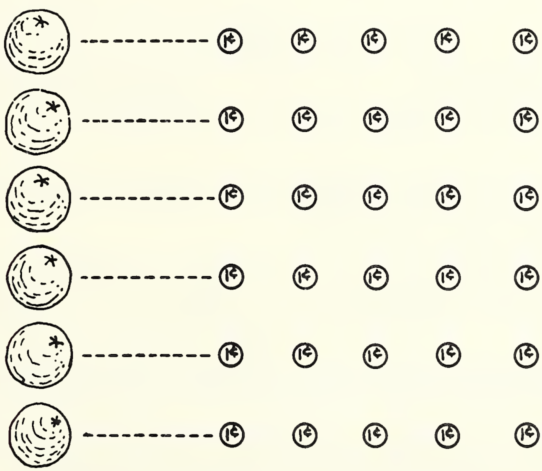
RATES AND EQUIVALENT ORDERED PAIRS

1. A notice at the store said: "Oranges, 6 for 30¢." Express this statement as a fraction rate.

In Grade 6 we learned that we could express statements such as "6 oranges for 30¢" as fraction numerals. We can write $\frac{6}{30}$. In this fraction numeral, '6' represents the number of oranges; '30' represents the number of cents that the 6 oranges cost.

$\frac{6}{30}$ $\begin{array}{l}\leftarrow \text{number of oranges}\\ \leftarrow \text{number of cents.}\end{array}$

2. Consider the 6 oranges and the 30 cents in the diagram below:



a At the rate given (i) what will 1 orange cost? (ii) 3 oranges? (iii) 4 oranges?

b Can we use the fraction numerals $\frac{1}{5}$, $\frac{3}{15}$, $\frac{4}{20}$, etc., to express the same rate as 6 for 30¢? Why?

3. In the table below, we have listed the cost of 1, 2, 3, . . . oranges

Number of Oranges	*Cost of Oranges*	*Fraction numeral*
1	5	$\frac{1}{5}$
2	10	$\frac{2}{10}$
3	15	$\frac{3}{15}$
4	20	$\frac{4}{20}$
5	25	$\frac{5}{25}$
6	30	$\frac{6}{30}$

a Is a rate of 1 orange for 5¢ the same rate as 4 oranges for 20¢? Is a rate of 5 oranges for 25¢ the same rate as 6 oranges for 30¢?

b What type of fractions are $\frac{1}{5}$, $\frac{2}{10}$, $\frac{3}{15}$, $\frac{4}{20}$, $\frac{5}{25}$, $\frac{6}{30}$, said to be?

4 In mathematics we often use another way to represent a rate. For a rate of 6 oranges for 30¢ or $\frac{6}{30}$ we write (6, 30). We say that we are representing the rate by an **ordered pair of numbers** *or* an **ordered number pair.** Other ordered pairs of numbers that we could use for this rate are:

(1, 5), (2, 10), (3, 15), (4, 20), . . .

(1, 5)

represents the number of oranges

represents the cost of 1 orange

5 We call each of the numbers in an ordered pair of numbers a **component** of the ordered pair.

(5, 25)

first component

second component

(3, 15)

first component

second component

6. If oranges are 6 for 30¢, what is the cost of 10 oranges? 12 oranges? 8 oranges?

We see that 10 oranges cost 50¢. For this rate we can write $\frac{10}{50}$. $\frac{10}{50}$ is written as (10, 50) as an ordered pair of numbers. 12 oranges cost 60¢. 8 oranges cost 40¢. For these we write $\frac{12}{60}$, $\frac{8}{40}$.

a Do the numerals $\frac{10}{50}$, $\frac{12}{60}$, $\frac{8}{40}$, represent the rate at which the oranges are sold?

b Write five more fraction numerals that express the rate at which the oranges are sold.

7 A set of fractions such as $(\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \ldots)$ is called a **set of equivalent fractions.**

8 We have seen that we can use ordered number pairs to represent rates. For the rate that we can represent by the set of fractional numerals $\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \ldots\}$, we can write the set of ordered number pairs as follows: $\{(1, 5), (2, 10), (3, 15), (4, 20), \ldots\}$ In place of the fraction numeral $\frac{1}{5}$, we have the ordered number pair (1, 5). What ordered number pairs take the place of the fraction numerals $\frac{4}{20}, \frac{6}{30}, \frac{8}{40}, \frac{15}{75}, \frac{100}{500}$?

When ordered number pairs represent the same rate, we say that they are equivalent number pairs.

(1, 5) and (4, 20) are equivalent number pairs. We can write

$(1, 5) \sim (4, 20)$.

The sign, $\sim$, means *is equivalent to*. Often in mathematics we use an equals sign for *is equivalent to*. When we write $\frac{1}{5} = \frac{2}{10}$ we really mean $\frac{1}{5}$ *is equivalent to* $\frac{2}{10}$.

9. Write sets of equivalent-fraction numerals to represent the following rates:
   - **a** A boy walks 4 blocks in 15 min.
   - **b** A recipe calls for 4 cups of flour to 1 cup of water.
   - **c** A school can accommodate 45 pupils per room.
   - **d** Soup is 2 cans for 37¢.
   - **e** An aeroplane flies 650 miles per hour.

10. Write sets of ordered number pairs to represent the rates in Exercise 9 above.

11 Write the set of ordered number pairs of which (4, 10) is one element. (4, 10) can be written as $\frac{4}{10}$. Can the set of ordered number pairs, of which (4, 10) is a member, be written as a set of equivalent fractions of which $\frac{4}{10}$ is a member? Why?

If we multiply the numerator of a fraction by a number, what must we do with the denominator of the fraction in order to have an equivalent fraction? $\frac{4}{10} = \frac{2}{2} \times \frac{4}{10} = \frac{8}{20}$

If we divide the numerator of a fraction by a number, what must we do with the denominator of the fraction in order to have an equivalent fraction?

$$\frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$$

The set of fraction numerals representing the rate $\frac{4}{10}$ is $\{\frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \ldots\}$ so the set of ordered number pairs representing the rate (4, 10) is $\{(2, 5), (4, 10), (6, 15), (8, 20), \ldots\}$.

12 We see that:

a if we multiply one component of an ordered number pair by a number, we must multiply the other component by the same number, in order to keep the ordered number pairs equivalent;

b if we divide one component of an ordered number pair by a number, we must divide the other component by the same number in order to keep the ordered number pairs equivalent.

13 We have seen that
$\{(1, 5), (2, 10), (3, 15), (4, 20), \ldots\}$ is a set of equivalent ordered number pairs. Take any two elements of this set.

Suppose we take (2, 10) and (6, 30).

Multiply the *first* component of (2, 10) and the *second* component of (6, 30).

We have $2 \times 30 = 60$

first component of (2, 10) — *second* component of (6, 30)

Multiply the second component of (2, 10) and the first component of (6, 30).

We have $10 \times 6 = 60$

second component of (2, 10) — *first* component of (6, 30)

What do you notice?

Now select any other two elements of the set. Suppose we select (1, 5) and (3, 15).

5×3

second component — *first* component

(1, 5) — (3, 15)

first component — *second* component

1×15

Does $5 \times 3 = 1 \times 15$?

14. Choose any two elements from the set of ordered number pairs below: $\{(3, 4), (6, 8), (9, 12), (12, 16), \ldots\}$.

Name 1 pair 'A' and the other pair 'B'.

a Multiply the first component of pair A and the second component of pair B.

b Multiply the second component of pair A and the first component of pair B.

What do you notice?

Select two more elements and multiply as in (a) and (b) above. What do you notice?

15 If (a, b) and (c, d) are equivalent ordered number pairs, then $a \times d = b \times c$.

16. Are (5, 8) and (20, 32) equivalent ordered pairs?
What is 5×32? What is 8×20? Does $5 \times 32 = 8 \times 20$? We see that $5 \times 32 = 160$ and that $8 \times 20 = 160$; so we know that (5, 8) and (20, 32) are equivalent ordered pairs of numbers.

17. Each example below has two ordered number pairs. Which examples have equivalent ordered number pairs?

a (3, 8), (9, 24)
b (4, 15), (6, 15)
c (12, 18), (20, 30)
d (1, 6), (6, 36)
e (108, 48), (36, 16)
f (10, 20), (100, 200)
g (5, 19), (15, 57)
h (18, 12), (12, 9)

18 $\frac{6}{10}$ and $\frac{15}{25}$ are two fraction numerals used to represent rates. Are the rates equivalent?

For $\frac{6}{10}$ and $\frac{15}{25}$ we can think of (6, 10) and (15, 25).

What is 6×25? What is 10×15?

Does $6 \times 25 = 10 \times 15$?

If (6, 10) and (15, 25) are equivalent ordered pairs of numbers, are $\frac{6}{10}$ and $\frac{15}{25}$ equivalent-fraction numerals?

We see that $\frac{6}{10} = \frac{15}{25}$.

Now multiply 6 and 25. We have 150.

Multiply 10 and 15. We have 150.

6 is the numerator of $\frac{6}{10}$. 25 is the denominator of $\frac{15}{25}$

$\frac{6}{10} \longrightarrow \frac{15}{25}$ $\qquad 6 \times 25 = 150$

10 is the denominator of $\frac{6}{10}$. 15 is the numerator of $\frac{15}{25}$.

$\frac{6}{10} \longrightarrow \frac{15}{25}$ $\qquad 10 \times 15 = 150$

19. Consider the following set of equivalent fractions:

$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$

Select any two of them. Suppose we select $\frac{2}{4}$ and $\frac{5}{10}$.

We have $\frac{2}{4} = \frac{5}{10}$.

$\frac{2}{4} \quad \frac{5}{10}$ What is 2×10?

$\frac{2}{4} \quad \frac{5}{10}$ What is 4×5?

Does $2 \times 10 = 4 \times 5$?

Choose any other pair of fractions from the set.

Multiply the numerator of the first fraction and the denominator of the second fraction.

Now multiply the denominator of the first fraction and the numerator of the second fraction. What do you notice?

20. Make a rule for equivalent fraction numerals.

21. Study the following:

a $\frac{4}{9} \quad \frac{12}{27}$ $\quad 4 \times 27 = 108$

$\frac{4}{9} \quad \frac{12}{27}$ $\quad 9 \times 12 = 108$

$\frac{4}{9} = \frac{12}{27}$

b $\frac{3}{5} \quad \frac{7}{8}$ $\quad 3 \times 8 = 24$

$\frac{3}{5} \quad \frac{7}{8}$ $\quad 5 \times 7 = 35$

$\frac{3}{5} \neq \frac{7}{8}$ (Remember that $\neq$ means *is not equal to*)

Now say which of the following pairs of fractions are equivalent:

c $\frac{15}{30}, \frac{2}{4}$ **d** $\frac{6}{9}, \frac{10}{15}$ **e** $\frac{3}{8}, \frac{4}{6}$ **f** $\frac{9}{10}, \frac{90}{100}$

g $\frac{1}{2}, \frac{2}{1}$ **h** $\frac{21}{35}, \frac{30}{50}$ **i** $\frac{30}{48}, \frac{45}{72}$ **j** $\frac{125}{350}, \frac{45}{126}$

22. (3, 4) and (x, 24) are equivalent ordered number pairs. What number does x represent?

a We have seen that, if we multiply both components of an ordered number pair by the same number, we have an equivalent ordered number pair. By what do we multiply 4 to obtain 24 as the product? If we multiply 4 by 6 to obtain 24, by what must we multiply 3 to obtain x as the product?

$24 = 6 \times 4$;

so $x = 6 \times 3 = 18$

The equivalent ordered number pairs are (3, 4) and (18, 24).

b (3, 4) (x, 24)

$4 \times x$

3×24

If the ordered number pairs are equivalent, then

$$4 \times x = 3 \times 24$$
$$4 \times x = 72$$
$$x = 18$$

The equivalent ordered number pairs are (3, 4) and (18, 24).

23. What numbers are represented by the letters in the ordered number pairs below, if in each example the two ordered number pairs are equivalent?

a (1, 2), (x, 10) **b** (6, 15), (3, y) **c** (4, f), (3, 45)
d (1, d), (2, 38) **e** (5, x), (2, 8) **f** (24, 5), (6, n)
g (y, 16), (15, 20) **h** (15, 100), (3, z) **i** (36, 42), (30, p)

24 $\frac{3}{15}$ and $\frac{4}{x}$ are equivalent fractions. What number is represented by x?

a $\frac{3}{15} = \frac{3 \div 3}{15 \div 3} = \frac{1}{5}$

So $\frac{1}{5} = \frac{4}{x}$. By what must we multiply 1 to have 4 as the product?

$4 \times 1 = 4.$

By what must we multiply 5 to obtain x?

$4 \times 5 = x.$ So $x = 20.$

The equivalent fractions are $\frac{3}{15}$ and $\frac{4}{20}$.

b $\frac{3}{15} = \frac{4}{x}$

$\frac{3}{15} = \frac{4}{x}$

By rule, $3 \times x = 15 \times 4 = 60.$

$3 \times x = 60,\ 3 \times 20 = 60.$

So $x = 20.$

The equivalent fractions are $\frac{3}{15}$ and $\frac{4}{20}$.

25. What number is represented by x in each example below?

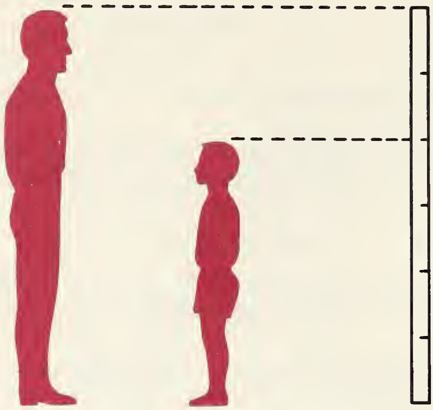
a $\frac{1}{5} = \frac{x}{30}$ **b** $\frac{3}{4} = \frac{27}{x}$ **c** $\frac{12}{15} = \frac{x}{20}$
d $\frac{x}{18} = \frac{12}{18}$ **e** $\frac{14}{21} = \frac{20}{x}$ **f** $\frac{16}{x} = \frac{12}{18}$
g $\frac{x}{63} = \frac{42}{54}$ **h** $\frac{128}{x} = \frac{2}{4}$ **i** $\frac{125}{750} = \frac{7}{x}$

RATIO

1 A man is 6 feet tall and his son is 4 feet tall. Compare their heights.

a We can compare by subtracting: The man is 6 ft. − 4 ft. or 2 ft. taller than his son.

b We can compare by dividing: The man is $\frac{6}{4}$ times as tall as his son. The boy is $\frac{4}{6}$ times as tall as his father.



From the diagram we can see that the father is also $\frac{3}{2}$ times as tall as his son and the boy is $\frac{2}{3}$ times as tall as his father. When we use a quotient to compare, we call the quotient a RATIO.

2 **a** We can also say:

"The father's height is to the son's height as 6 is to 4." A ratio of 6 to 4 can be written as 6:4. We say, "a ratio of 6 to 4".

b "The boy's height is to the father's height as 2 is to 3." This ratio can be written as 2:3. We say, "a ratio of 2 to 3".

3 We can use ordered pairs of numbers to express ratios. For $\frac{6}{4}$ or 6:4 we can write (6, 4). For $\frac{2}{3}$ or 2:3 we can write (2, 3).

4 What equivalent ratios can we write for a ratio of 6 to 4?

a As a fraction numeral, the ratio can be written as $\frac{6}{4}$.

A set of equivalent fraction numerals for $\frac{6}{4}$ is $\{\frac{3}{2}, \frac{6}{4}, \frac{9}{6}, \frac{12}{8}, \ldots\}$.

b A ratio of 6 to 4 can be written as 6:4. A set of equivalent ratio numerals is $\{(3{:}2), (6{:}4), (9{:}6), (12{:}8), \ldots\}$.

c Using ordered number pairs, we can write

$\{(3, 2), (6, 4), (9, 6), (12, 8), \ldots\}$.

5. Write the set of equivalent ratios for '2 to 3' in three different ways.

6 When we are writing ratios, we must remember that each numeral in the ratio represents the same unit of measure.

$\frac{6 \longrightarrow \text{number of feet.}}{4 \longrightarrow \text{number of feet.}}$

7. One book cost \$2.00 and another, 50¢. Compare the price of the \$2.00-book with the 50¢-book, using a fraction numeral. We see that one price is in dollars and that the other is in cents. Into what unit of money can we change both dollars and cents?

The ratio is $\frac{200}{50}$ or $\frac{4}{1}$.

8. In three different ways, write sets of equivalent ratios for the answer to exercise No. 7.

We can write; $\frac{4}{1}$ or 4:1 or (4, 1)

9. A rectangle is 4 yards long and 6 feet wide. Find:

a the ratio of the length to the width.

b the ratio of the width to the length.

a The length: 4 yards $= 4 \times 3$ feet $= 12$ feet.

The width is 6 feet.

The ratio of length to width is $\frac{12}{6}$ or $\frac{2}{1}$.

For $\frac{2}{1}$ we can write 2:1 or (2, 1).

b The width is 6 feet.

The length is 4 yards or 12 feet.

The ratio of the width to the length is $\frac{6}{12}$ or $\frac{1}{2}$.

For $\frac{1}{2}$ we can write 1:2 or (1, 2).

10. **a** Find the ratio of the measure in column A to the measure in column B.

b Find the ratio of the measure in column B to the measure in column A.

Write each answer in three different ways.

A	B
3 inches	15 inches
1 pint	1 gallon
500 pounds	1 ton
6 square inches	42 square inches
\$1.25	\$2.50
880 yards	2 miles
6 pints	10 quarts
1500 pounds	5 tons

11. We can also have ratios with more than two terms. Three trees are 10 ft., 20 ft. and 30 ft. tall respectively. What is the ratio of the heights of these trees? We can write 10:20:30. An equivalent ratio to this would be 1:2:3.

Can you find two more equivalent ratios to 10:20:30?

We name ratios like 1:2:3; *three* term ratios.

12 An alloy contains 12 lb. of zinc and 8 lb. of lead. What is the ratio of the amount of zinc in the alloy to the amount of lead in the alloy? Put your ratio in its lowest terms.

We have 12 lb. of zinc to 8 lb. of lead.

The required ratio is 12:8 or $\frac{12}{8}$.

For $\frac{12}{8}$ we have as an equivalent fraction in its lowest terms $\frac{3}{2}$. So we can write the ratio in its lowest terms as 3:2 or $\frac{3}{2}$.

13. In liquid measure 150 Canadian pints are equivalent to 180 U.S. pints. Find the ratio of a U.S. pint to a Canadian pint. Put your answer in its simplest form.

14. A picture frame is 4′ 8″ long and 2′ 11″ wide. What is the ratio of the length of this picture frame to its width? Put your answer in its lowest terms.

15. Write each of the ratios below in their simplest terms:

a 15:45 **b** 3:3 **c** 72:18 **d** 9:6 **e** 125:175

f $1\frac{1}{2}$:3 **g** $\frac{3}{4}$:$1\frac{1}{2}$ **h** $\frac{5}{8}$:$\frac{9}{8}$ **i** .25:1.75 **j** .65:.91

16. In a batch of concrete mix there were 2 bags of cement, 3 bags of sand and 6 bags of gravel. Find the ratio of the amounts of:

a cement to sand **b** cement to gravel

c sand to gravel **d** cement to gravel to sand.

PROBLEMS ON RATES AND RATIOS

1 We have seen that we can use fraction numerals and ordered number pairs to represent both rates and ratios.

For a rate of 6 cans for 1 dollar, we can write $\frac{6}{1}$ or (6, 1).

For a ratio of 12 pints to 5 pints, we can write $\frac{12}{5}$ or (12, 5).

We have noted that with ratios the two numbers represent the same kind of things, such as feet, pints, and so on. With rates, we have noted that

the two numbers may represent different kinds of things. In either case, the rules that we have learned for fraction numerals and ordered number pairs apply.

2 If 5 pounds of apples cost 47¢, how much must be paid for 15 pounds of apples at this rate?

We can represent the rate for the cost of apples as $\frac{5}{47}$.

$$\begin{array}{l} 5 \leftarrow \text{number of pounds} \\ \overline{47} \leftarrow \text{number of cents} \end{array}$$

We want to find the cost of 15 pounds of apples. Let x cents represent the cost. What fraction numeral will represent a rate of 15 pounds of apples for x cents?

Will this rate be equivalent to a rate of 5 lb. for 47¢? Why?

5 lb. for 47¢ gives a rate of $\frac{5}{47}$.

15 lb. for x¢ gives a rate of $\frac{15}{x}$.

Since the two rates are equivalent, we can write $\frac{5}{47} = \frac{15}{x}$.

If $\frac{5}{47}$ and $\frac{15}{x}$ are equivalent rates, what two products will be equal?

We see from $\frac{5}{47} = \frac{15}{x}$ that $5 \times x = 47 \times 15$.

Does $1 \times x = x$?

If 5 times $x = 47 \times 15$, how can we find 1 times x?

$$x = \frac{47 \times 15}{5}$$

How can we simplify the computation?

By what number can we divide both the denominator and a factor in the numerator?

$$x = \frac{47 \times \overset{3}{\cancel{15}}}{\underset{1}{\cancel{5}}} = 47 \times 3 = 141$$

If $x = 141$, what is the cost of 15 lbs. of apples?

Is a rate expressed as $\frac{5}{47}$ equivalent to a rate expressed as $\frac{15}{141}$? How do you know?

3 Here is the way in which we may have set our problem down:

Rate, if '5 lbs. of apples cost 47¢', is $\frac{5}{47}$.

Let 15 lbs. of apples cost x¢. The rate is $\frac{15}{x}$.

Now, $\frac{5}{47} = \frac{15}{x}$

$$5 \times x = 47 \times 15$$

$$x = \frac{47 \times 15}{5}$$

$x = 141$ 15 lbs. of apples cost 141¢ or $1.41.

4 A tree 30 ft. high casts a shadow 40 ft. long. What will be the height of a tree that casts a shadow 20 ft. long? Consider the tree that is 30 ft. high. What is the ratio of its height to the length of its shadow?

We have $\dfrac{30}{40}$ $\begin{array}{l}\leftarrow \text{number of feet in height of tree} \\ \leftarrow \text{number of feet in length of shadow}\end{array}$

We do not know the height of the tree that casts a 20-foot shadow. Let the height of this tree be x ft. What is the ratio of the height of this tree to the length of its shadow?

We have $\dfrac{x}{20}$ $\begin{array}{l}\leftarrow \text{number of feet in height of tree} \\ \leftarrow \text{number of feet in length of shadow}\end{array}$

Will the two ratios $\frac{30}{40}$ and $\frac{x}{20}$ be equivalent? Why?

We have $\frac{30}{40} = \frac{x}{20}$

Which two products in this sentence will be equal?

We have $30 \times 20 = 40 \times x$;

so $600 = 40x$.

Are equations reversible? If $600 = 40x$, what does $40x$ equal?

$40x = 600$

How can we find x if we know that $40x = 600$?

$40x = 600$; so $x = \frac{600}{40} = 15$

The height of the tree is 15 feet.

Are $\frac{30}{40}$ and $\frac{15}{20}$ equivalent ratios? How do you know?

5 The problem may be set down this way:

Ratio of height of tree to length of shadow is $\frac{30}{40}$.

Let x ft. be the height of the tree that casts a shadow 20 ft. long. Then the ratio of the height of this tree to the length of its shadow is $\frac{x}{20}$.

Now, $\frac{30}{40} = \frac{x}{20}$

So $30 \times 20 = 40 \times x$

$$40x = 30 \times 20$$

and $$x = \frac{30 \times 20}{40}$$

$$x = 15$$

The height of the tree is 15 ft.

6 In an election for head boy, John received 3 votes for every 2 votes that Peter received. If John received 291 votes, how many votes did Peter receive?

Let x represent the number of votes that Peter received.

Ratio of the votes for John to the votes for Peter is $\frac{3}{2}$.

Ratio of the total votes for John to the total votes for Peter is $\frac{291}{x}$.

Then, $\frac{3}{2} = \frac{291}{x}$

$$3 \times x = 2 \times 291$$
$$3x = 582$$
$$x = \frac{582}{3} = 194$$

Peter received 194 votes.

7. A map has a scale of 5 miles to an inch. What length on the map will represent a distance of 12 miles?

Let x inches represent the required length

5 miles to an inch can be expressed as $\frac{5}{1}$.

12 miles to x inches can be expressed as $\frac{12}{x}$

Then, $\frac{5}{1} = \frac{12}{x}$

So, $5x = 12$

and $x = \frac{12}{5} = 2\frac{2}{5}$

12 miles will be represented on the map by a length of $2\frac{2}{5}$ inches.

8. A store sells 5 cans of soup for every 2 cans of peas that it sells. In one day the store sold 45 cans of soup. How many cans of peas did it sell that day?

9. A boy has 3 Canadian stamps for each stamp that he has from England. He has 72 Canadian stamps. How many English stamps does he have?

10. Potatoes cost 50¢ for 10 lbs. How many pounds of potatoes can be bought for \$2.95?

11. For every 5 touchdowns that a football team scored, it scored 2 field goals. In one season the team scored 20 touchdowns. How many field goals were scored?

12. A ship travelled 1745 miles in 4 days. How far would it travel in 12 days at this rate?

13. The distance between two cities on a map is 7 inches. If 4 inches on this map represent an actual distance of 16 miles, what is the actual distance between these two cities?

14. A licence office issued 11 car licences for every 2 truck licences that it issued. One day the office issued 121 car licenses. How many truck licenses did it issue?

15. The ratio of the height of a certain apartment building to a house is 9:2. If the house is 30 feet high, how high is the apartment building?

16. A store reduced the price of all its goods by $8.00 for each $100.00 of the regular price. A rug was reduced in price by $12.00. What was the regular price of the rug?

17. Sales tax of $3.00 for every $100.00 of the cost of the article was charged in one province. What sales tax should be paid on a record player that cost $50.00?

18. The ratio of the length of a room to its width is 3 to 2. The room is 15 feet wide. How long is the room?

19. A bank charges $6.00 interest per year for each $100.00 that it lends. What interest will it charge for one year on a loan of $250.00?

20. At a sale, a store reduced all its prices by $10.00 for every $100.00 of the regular price. What discount would be given on a television set if its regular price was $250.00? (Discount is the amount by which the regular price is reduced.)

21. When a builder was making concrete, he used 1 part of cement to each 3 parts of sand. In one batch of cement, he used 60 shovelfuls of sand. How many shovelfuls of cement did he use?

22. Some artists maintain that the most pleasing picture size is a rectangle, the length of which has a ratio of 8 to 5 with its width. What should the length of a picture be if its width is 2 ft. 6 in.?

23. The gear ratio on a bicycle is 44:18. This means that when the cyclist turns the pedal 18 times the back wheel turns 44 times. How many times will the wheel of this bicycle turn if the pedals are turned 63 times?

24. Two business partners shared their profits in the ratio of 2:3. If the partner with the larger part received $914.00, how much did the other partner receive?

25. A man received $14.00 for working 8 hours. How much would he receive for working 44 hours at the same rate of pay?

PER CENT

1. A boy received the following scores in his term examinations: English: 23 out of 25; Mathematics: 46 out of 50; Literature: 44 out of 50; Science: 54 out of 60. In which subject did he do best?

We can think of his marks as a ratio between two numbers:

23 out of 25 can be thought of as $\frac{23}{25}$ or (23, 25). If we express all of the scores as ratios we can put them as: English, $\frac{23}{25}$; Mathematics, $\frac{46}{50}$; Literature, $\frac{44}{50}$; Science, $\frac{54}{60}$. How can we compare the size of the ratios? Did the boy do better in mathematics or in literature? Why is it easy to see that he did better in mathematics than in literature?

2. Consider the score in English. Suppose the mark possible had been 100, how many marks would he have obtained in English at a ratio of 23 out of 25? Let x represent the score he would have obtained had the possible score been 100. Then if he had obtained marks out of 100 at the ratio of 23 out of 25 we would have two equivalent ratios: $\frac{23}{25}$ and $\frac{x}{100}$. What is x?

   By computation we have:

   $$23 \times 100 = 25 \times x;$$

   So $$25x = 23 \times 100$$

   $$x = \frac{23 \times \overset{4}{\cancel{100}}}{\underset{1}{\cancel{25}}} = 92$$

   Is a ratio of $\frac{92}{100}$ an equivalent ratio to $\frac{23}{25}$? How do you know?

3. Now let us change the fraction numerals for the rates representing his scores in Mathematics, Literature, and Science into fraction numerals with 100 as the denominator.

   Mathematics:

   $$\frac{46}{50} = \frac{x}{100} \therefore 46 \times 100 = 50 \times x$$

   $$50 \times x = 56 \times 100 \quad \text{So } x = \frac{46 \times \overset{2}{\cancel{100}}}{\underset{1}{\cancel{50}}} = 92$$

   Literature:

   $$\frac{44}{50} = \frac{x}{100} \therefore 44 \times 100 = 50 \times x$$

   $$50 \times x = 44 \times 100; \text{ So } x = \frac{44 \times \overset{2}{\cancel{100}}}{\underset{1}{\cancel{50}}} = 88$$

   Science:

   $$\frac{54}{60} = \frac{x}{100}; \therefore 54 \times 100 = 60 \times x$$

   $$60 \times x = 54 \times 100; \therefore x = \frac{\overset{9}{\cancel{54}} \times \overset{10}{\cancel{100}}}{\underset{1\ \cancel{10}}{\cancel{60}}} = 90$$

For the four examinations we now have:

English: $\frac{92}{100}$, Mathematics: $\frac{92}{100}$, Literature: $\frac{88}{100}$, Science: $\frac{90}{100}$. Is each of these ratios equivalent to the ratios representing the scores the boy obtained in his examination?

Is $\frac{92}{100}$ equivalent to $\frac{23}{25}$? Why?

Is $\frac{92}{100}$ equivalent to $\frac{46}{50}$? Why?

Is $\frac{88}{100}$ equivalent to $\frac{44}{50}$? Why?

Is $\frac{90}{100}$ equivalent to $\frac{54}{60}$? Why?

Why is it easy now to compare the scores in the different tests? In which examination did the boy obtain his best results?

He did equally well in English and Mathematics. These were his best scores.

4 We have a special name for fraction numerals with a denominator of 100. We call them **per cents.**

$\frac{92}{100}$ is 92 per cent; $\frac{88}{100}$ is 88 per cent; $\frac{90}{100}$ is 90 per cent.
(*per cent* means *per hundred.*)

5 We use a symbol for per cent. It is %.
So for $\frac{92}{100}$ or 92 per cent we write 92%, and for $\frac{88}{100}$ or 88 per cent, we write 88%.

6 Study the columns below:

English	$\frac{23}{25}$	$\frac{92}{100}$	92 per cent	92%
Mathematics	$\frac{46}{50}$	$\frac{92}{100}$	92 per cent	92%
Literature	$\frac{44}{50}$	$\frac{88}{100}$	88 per cent	88%
Science	$\frac{54}{60}$	$\frac{90}{100}$	90 per cent	90%

7. Write the following using the per cent symbol:

$\frac{44}{100}$ $\quad$ $\frac{79}{100}$ $\quad$ $\frac{10}{100}$ $\quad$ $\frac{1}{100}$ $\quad$ $\frac{99}{100}$ $\quad$ $\frac{68}{100}$

8. Write the following as fraction numerals:

5% $\quad$ 75% $\quad$ 6% $\quad$ 84% $\quad$ 95% $\quad$ 18%

MORE ABOUT PER CENT

1 In an arithmetic test, a boy scored 50 marks out of a possible total of 50. What per cent of the possible score did he obtain in the test? Let $x\%$ be the per cent he obtained. The ratio of marks scored to marks possible is given by $\frac{50}{50}$. We need to find an equivalent rate pair, expressed as a fraction numeral, with 100 as the denominator.

$\frac{50}{50} = \frac{x}{100}$; so $50 \times 100 = 50 \times x$, and $50 \times x = 50 \times 100$.

$$x = \frac{\overset{1}{\cancel{50}} \times 100}{\underset{1}{\cancel{50}}} = 100$$

$\frac{50}{50} = \frac{100}{100}$. The boy obtained 100% of the marks.

2. Below is a list of the possible scores in a term examination. How many marks would one need to obtain in each examination in order to score 100% of the possible marks?

   Literature: 100; Composition: 60; Spelling: 40; Science: 100; Geography: 75; History: 75.

3. A Grade 7 class was collecting for Red Cross. The goal of the class was \$50.00. The class collected \$60.00. What per cent of the goal was collected? Let x% be the per cent of the goal which was collected.

   What ratio will express the number of dollars actually collected compared with the number of dollars in the goal?

   We can write: $\frac{60}{50}$ $\begin{matrix} \leftarrow \text{number of dollars collected} \\ \leftarrow \text{number of dollars aimed for} \end{matrix}$

   We need to express this ratio as a per cent. What per cent will it be equivalent to?

   $\frac{60}{50} = x\%$. For x% we can write $\frac{x}{100}$. $\frac{60}{50} = \frac{x}{100}$

   $\therefore\ 60 \times 100 = 50 \times x$ or $50x = 60 \times 100$

   $$\therefore \qquad x = \frac{60 \times \overset{2}{\cancel{100}}}{\underset{1}{\cancel{50}}} = 120$$

   The class collected an amount equal to 120% of its goal.

   Our answer is a per cent greater than 100%. $120\% = \frac{120}{100}$

   Some equivalent ratios to $\frac{120}{100}$ are $\frac{12}{10}$, $\frac{6}{5}$, $\frac{30}{25}$, etc.

4. Write the following ratios as per cents:

   $\frac{100}{100}$, $\frac{135}{100}$, $\frac{197}{100}$, $\frac{150}{100}$, $\frac{250}{100}$, $\frac{200}{100}$, $\frac{640}{100}$, $\frac{750}{100}$, $\frac{30}{20}$, $\frac{3}{1}$, $\frac{7}{4}$, $\frac{35}{20}$, $\frac{48}{40}$, $\frac{96}{48}$, $\frac{8}{5}$, $\frac{15}{2}$.

5. Write the following per cents as fraction numerals in their lowest terms:

   115%, 1000%, 600%, 725%, 175%, 540%, 760%, 555%, 110%, 475%, 2500%, 800%.

6. A science club went on a rock-hunting trip. Each boy was asked to find 25 different types of rock. On page 230 is a list showing how many different types of rock six of the boys collected:

John: 20; Tom: 28; William: 34; Al: 25; Robert: 18; Roy: 50. What per cent of the target set did each boy obtain?

Let $x\%$ be the per cent of the target obtained by John. For John, the ratio between the number of types of rocks collected and the target set was $\frac{20}{25}$. $\frac{20}{25}$ is equivalent to a ratio of $x\%$.

So $\frac{20}{25} = \frac{x}{100}$ $\qquad\qquad 20 \times 100 = 25 \times x$

$25 \times x = 20 \times 100,$ $\qquad\qquad x = \dfrac{20 \times \overset{4}{\cancel{100}}}{\underset{1}{\cancel{25}}} = 80$

John collected 80% of the number of different types of rock set as a target.

(Now find the per cent of the target obtained by the other five boys.)

7

Room	*Number Present*	*Enrolment*
1	32	35
2	30	32
3	32	34
4	28	30

What per cent of the enrolment was present in each of the rooms in the table above?

a Consider Room 1. Let $x\%$ be the required per cent.

Will $\frac{32}{35}$ be equivalent to $\frac{x}{100}$? Why?

$\frac{32}{35} = \frac{x}{100}$

$32 \times 100 = 35 \times x$

$35 \times x \quad = 32 \times 100$

$$x = \frac{32 \times \overset{20}{\cancel{100}}}{\underset{7}{\cancel{35}}} = \frac{640}{7} = 91\frac{3}{7}$$

Since $x = 91\frac{3}{7}$ we see that $91\frac{3}{7}\%$ of the pupils were present.

Here we have an example of a number of per cent which is not a whole number.

How else could we express $91\frac{3}{7}$?

What is $91\frac{3}{7}$ as a mixed decimal fraction numeral?

For $91\frac{3}{7}\%$ we can write 91.43% since $\frac{3}{7} = .43$(correct to 2 decimal places.)

b Now find the required per cents for rooms 2, 3, and 4.

Write the per cents in two different ways.

8. In a drive to raise money for charity, the students for various rooms in a school collected the amounts shown below. By each amount, the target for that room is written.

Room	*Amount Collected*	*Target*
10	\$22	\$25
11	\$40	\$30
12	\$20	\$20
13	\$45	\$35
14	\$27	\$30
15	\$18	\$20

What per cent of the target was collected by each room?

FRACTION NUMERALS, DECIMAL NUMERALS, AND PER CENTS

1. When we were studying fraction numerals, we learned that we could express whole number numerals and mixed decimal numerals as fraction numerals.

For whole number numerals such as 4, we can write $\frac{4}{1}$, $\frac{8}{2}$, $\frac{12}{3}$, . . .
For mixed decimal numerals such as 3.56, we can write $3\frac{56}{100}$, $\frac{356}{100}$, $\frac{3560}{1000}$, . . .
Can we write whole number numerals and decimal numerals as per cents?

2 **a** Consider the whole number numeral 5. $5 = \frac{5}{1} = \frac{10}{2}$ . . .
To express a number as a per cent, we need to express the number as a fraction numeral with a denominator of 100. $5 = \frac{500}{100}$.
What is $\frac{500}{100}$ as a per cent? We see that $5 = 500\%$.

b Consider 5. $5 = \frac{5}{1}$ . . .
Let $x\% = \frac{5}{1}$; then $\frac{x}{100} = \frac{5}{1}$;
Then $x \times 1 = 100 \times 5$; $x = 100 \times 5 = 500$.
So $5 = 500\%$.

c Now express the following as per cents:
6, 10 , 8, 1, 4, 7, 23, 115, 96, 13.

3 **a** Consider the mixed decimal numeral 2.81.
$2.81 = 2\frac{81}{100} = \frac{281}{100}$
What is $\frac{281}{100}$ expressed as a per cent?
We see that $2.81 = 281\%$.

b $2.81 = \dfrac{2.81}{1}$. Why?

Let $x\% = \dfrac{2.81}{1}$; then $\dfrac{x}{100} = \dfrac{2.81}{1}$

and $x \times 1 = 100 \times 2.81$; so $x = 281$.

$2.81 = 281\%$.

4 **a** Express .685 as a decimal per cent.

$$.685 = \frac{6}{10} + \frac{8}{100} + \frac{5}{1000} = \frac{685}{1000}$$

Let $x\% = \dfrac{685}{1000}$; then $\dfrac{x}{100} = \dfrac{685}{1000}$

and $x \times 1000 = 100 \times 685$; so $1000x = 100 \times 685$

$$x = \frac{\overset{1}{\cancel{100}} \times \overset{137}{\cancel{685}}}{\underset{\underset{2}{\cancel{10}}}{\cancel{1000}}} = \frac{137}{2} = 68\tfrac{1}{2}$$

$\therefore .685 = 68\frac{1}{2}\%$

b $.685 = \dfrac{685}{1000} = \dfrac{685 \div 10}{1000 \div 10}$ (Why do we divide numerator and denominator by 10?)

$\dfrac{685 \div 10}{1000 \div 10} = \dfrac{68.5}{100} = 68.5\%$ Why?

For 68.5% we can write $68\frac{1}{2}\%$. Why?

5 Express .0575 as a per cent.

$$.0575 = \frac{575}{10000} = \frac{575 \div 100}{10000 \div 100} = \frac{5.75}{100}$$

$.0575 = 5.75\%$. Since $5.75 = 5\frac{3}{4}$, for 5.75% we can write $5\frac{3}{4}\%$.

6. Express each of the following decimal numerals as a per cent:

.01	.04	.05	.28
7.44	.6	.8	.1
1.7	1.3	4.6	.245
8.6925	7.7350	5.0815	9.1005
.59	.93	1.39	1.50
.9	.3	1.2	1.8
3.75	.7275	.9625	.6625
7.6645	12.2095	7.6105	0.0005

7. Express each of the following per cents as decimal numerals: (The first is done for you.)

5% $\left(5\% = \frac{5}{100} = .05\right)$ 84% 30% 125% 110% 37.5% 8.34%

53% 119% 40.5% 12.25% 352.875% 16% 4.82%

8 Express $16\frac{2}{3}\%$ as a fraction numeral.

$$16\tfrac{2}{3}\% = \frac{16\frac{2}{3}}{100} = \frac{\frac{50}{3}}{100} = \frac{\frac{50}{3} \times 3}{100 \times 3} = \frac{\overset{1}{\cancel{50}}}{\underset{6}{\cancel{300}}} = \frac{1}{6}$$

Study the example above. Why did we multiply $\frac{50}{3}$ by 3 and 100 by 3?

9. Express the following per cents as fraction numerals:

$33\frac{1}{3}\%$ $37\frac{1}{2}\%$ $66\frac{2}{3}\%$ $87\frac{1}{2}\%$ $83\frac{1}{3}\%$

$6\frac{1}{4}\%$ $8\frac{1}{3}\%$ $12\frac{1}{2}\%$ $62\frac{1}{2}\%$ $81\frac{1}{4}\%$

10 Express $\frac{17}{25}$ as a per cent.

Let $\frac{17}{25} = x\%$. Then $\frac{17}{25} = \frac{x}{100}$

So $17 \times 100 = 25 \times x$ and $25x = 17 \times 100$

$$x = \frac{17 \times \overset{4}{\cancel{100}}}{\underset{1}{\cancel{25}}} = 68$$

$$\frac{17}{25} = 68\%$$

11 Write $1\frac{2}{3}$ as a per cent.

Let $1\frac{2}{3} = x\%$. Now $1\frac{2}{3} = \frac{5}{3}$. So $\frac{5}{3} = \frac{x}{100}$

$5 \times 100 = 3 \times x$ and $3x = 5 \times 100$

$$x = \frac{5 \times 100}{3} = \frac{500}{3} = 166\tfrac{2}{3}$$

$\therefore 1\frac{2}{3} = 166\frac{2}{3}\%$. For $166\frac{2}{3}\%$ we can write 166.67%, since $\frac{2}{3} = .67$ to 2 decimal places.

12. Write the following fraction numerals as per cents:

$\frac{1}{4}$ $\frac{2}{5}$ $\frac{5}{6}$ $\frac{3}{8}$ $\frac{1}{6}$ $\frac{18}{200}$ $\frac{60}{500}$ $\frac{135}{900}$

$\frac{1}{3}$ $\frac{4}{7}$ $\frac{33}{88}$ $\frac{5}{18}$ $\frac{4}{9}$ $\frac{8}{11}$ $\frac{7}{15}$ $\frac{42}{49}$

$1\frac{1}{5}$ $2\frac{2}{3}$ $3\frac{1}{6}$ $4\frac{1}{9}$ $5\frac{7}{8}$ $2\frac{11}{12}$ $14\frac{5}{12}$ $19\frac{11}{15}$

$\frac{4}{5}$ $\frac{7}{10}$ $\frac{7}{25}$ $\frac{11}{20}$ $\frac{125}{1000}$ $\frac{75}{1500}$ $7\frac{3}{4}$ $\frac{91}{27}$

1 A boy answered 95% of the questions correctly in a test that contained 40 questions. How many questions did he answer correctly?

Let x represent the number he answered correctly.

The ratio of the number of questions answered correctly to the number of questions in the test is $x:40$ or $\frac{x}{40}$. This is the same ratio as 95% $= \frac{95}{100}$.

So $\frac{x}{40} = \frac{95}{100}$ $\therefore$ $x \times 100 = 40 \times 95$ and $100 \times x = 40 \times 95$

$$x = \frac{\overset{2}{\cancel{40}} \times \overset{19}{\cancel{95}}}{\underset{\underset{1}{\cancel{5}}}{\cancel{100}}} = 38$$

He answered 38 questions correctly.

Is $\frac{38}{40}$ equivalent to $\frac{95}{100}$? Is $\frac{38}{40}$ equivalent to 95%?

2 In an election, one candidate received 540 votes out of a total of 720 votes cast. What per cent of the votes cast did he receive?

Let x% be the per cent of votes that he received.

He received 540 out of 720 votes. As a ratio, this equals $\frac{540}{720}$. To what per cent is this ratio equivalent?

$\frac{540}{720} = \frac{x}{100}$ $\therefore$ $540 \times 100 = 720 \times x$

$720x = 540 \times 100$

$$x = \frac{\overset{\overset{15}{\cancel{60}}}{\cancel{540}} \times \overset{\overset{5}{\cancel{10}}}{\cancel{100}}}{\underset{\underset{\underset{\underset{1}{\cancel{2}}}{\cancel{8}}}{\cancel{72}}}{\cancel{720}}} = 75$$

He received 75% of the votes.

Is $\frac{540}{720}$ equivalent to $\frac{75}{100}$?

Is $\frac{540}{720}$ equivalent to 75%?

3 A girl saved 55% of the money she earned during the holidays. She saved \$33.00. How much did she earn in the holidays?

Let x be the number of dollars that she earned.

Then the ratio of the number of dollars that she saved to the number of dollars that she earned is $33:x$.

We can write this as $\frac{33 \leftarrow \text{number of dollars saved}}{x \leftarrow \text{number of dollars earned.}}$

This ratio is equivalent to 55%.

Is $\frac{33}{x}$ equivalent to $\frac{55}{100}$? Why?

We can write $\frac{33}{x} = \frac{55}{100}$; so $33 \times 100 = x \times 55$

$$55 \times x = 33 \times 100;\ x = \frac{\overset{3}{\cancel{33}} \times \overset{20}{\cancel{100}}}{\underset{5}{\cancel{55}}} = 60$$

She earned $60.00

Is $\frac{33}{60}$ equivalent to $\frac{55}{100}$? Is $\frac{33}{60}$ equivalent to 55%?

4 What number is $37\frac{1}{2}\%$ of 64?

Let x be the number. Then the ratio of x to 64 is $x{:}64$ or $\frac{x}{64}$.

This ratio is equivalent to $37\frac{1}{2}\%$.

$$37\tfrac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{\frac{75}{2}}{100} = \frac{\frac{75}{2} \times 2}{100 \times 2} = \frac{\overset{3}{\cancel{75}}}{\underset{8}{\cancel{200}}} = \frac{3}{8}.$$

So $$\frac{x}{64} = 37\tfrac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{3}{8}$$

$$\frac{x}{64} = \frac{3}{8};\ \text{and } x \times 8 = 64 \times 3$$

$$x = \frac{64 \times 3}{8} = 24$$

24 is $37\frac{1}{2}\%$ of 64.

5. How many problems did a boy do correctly in a test of 20 problems if he did 85% of them correctly?

6. A man earns $6,800 a year. He plans to follow a budget of: Food, 25%; rent, 20%; clothing, 15%; savings, 10%; other items, 30%. How much does he plan to spend on each part of his budget?

7. A boy saved $7.00 out of each $10.00 that he earned. What per cent of his earnings did he save? If he saved $35.00 in all, what did he earn?

8. The attendance at school one day was 720. This represented 96% of the total enrolment. What was the total enrolment of the school?

9. The price of a house was $15,900.00. What was paid as a down payment if $16\frac{2}{3}\%$ of the cost price was required as down payment?

10. One week a salesman sold \$900.00 worth of goods. The next week he sold 10% more goods. What was the value of the goods he sold in the second week of sales?

11. Find n.

    a $35 = n\%$ of 70

    b $n\%$ of $60 = 190$

    c 110% of $n = 66$

    d $12\frac{1}{2}\%$ of $72 = n$

12. A certain alloy contained 56% copper, 36% zinc, and the rest was tin. How many pounds of copper, zinc, and tin, respectively, were there in each ton of the alloy?

13. A T.V. set was bought for \$240. The down payment was 25% of the cost. The balance was paid in 12 monthly payments.

    a What was the amount of the down payment?

    b What was the amount of each monthly payment?

14. A boy sold his bicycle for $83\frac{1}{3}\%$ of what he gave for it. He sold it for \$45. What did he pay for the bicycle?

15. In a school, $13\frac{1}{3}\%$ of the pupils are in Grade 7. If 86 pupils are in Grade 7, how many pupils are there in the school in all?

16. In one basketball season, a player scored $93\frac{3}{4}\%$ of the free throws that he made. If he scored 15 free throws, how many free throws did he attempt that season?

17. The population of Canada in 1956 was about 16 million. Now the population is about $112\frac{1}{2}\%$ of that number. What is the present approximate population of Canada?

18. A man earns \$415. a month. After payments for savings, income tax, etc., his take-home pay is \$332. What per cent of his salary is his take-home pay?

19. 2100 is what per cent of 700?

20. 168 is $37\frac{1}{2}\%$ of what number?

21. The price of a house was \$15,600. The mortgage covered $66\frac{2}{3}\%$ of this amount. What was the sum of money needed as a down payment on this house?

22. Find n if $n\%$ of 191844 is 31974.

1 Some Provinces levy a tax on certain goods that are sold. This tax is called a **sales tax.** This tax is usually expressed as a rate per cent. A sales tax of 3% means that 3% of the selling price is added on to the cost of each article that is sold. If the cost of an article is \$100. and the sales tax is 3%, then a buyer would have to pay \$100. for the article plus 3% of \$100.

$$3\% \text{ of } \$100. = \frac{3}{100} \text{ of } \$100. = \frac{3}{100} \times 100 = \$3.$$

The article would cost \$100. + \$3. or \$103.

2. **a** Find the sales tax to be paid on the following articles at the rates indicated: (you may have to work to the nearest cent.)

   b Find the amount a purchaser would have to pay for each of these articles:

Price of Article	\$200.00	\$56.98	\$2.00	\$17.98	\$18.80
Rate of Tax	2%	1%	3%	$2\frac{1}{2}$%	$1\frac{1}{2}$%
Price of Article	\$15.36	\$1099.00	\$0.99	\$4.00	\$758.00
Rate of Tax	$3\frac{1}{2}$%	3%	2%	$2\frac{3}{4}$%	$1\frac{1}{4}$%

3 Articles are often sold at a reduction in price. The amount by which the regular price of an article is reduced is called a discount. If a pair of shoes has a regular price of \$10. and is sold for \$9., we say that the discount is \$1. The price paid after discount is called the net price. We often express the discount rate as a per cent of the regular price. In the case of the shoes that had a regular price of \$10. and sold at a discount of \$1., we see that the rate of discount is \$1. for each \$10. of the regular price.

The discount rate is expressed as $\frac{1}{10}$ ← number of dollars in discount. / ← number of dollars in regular price.

$\frac{1}{10}$ is equivalent to what per cent?

A discount at the rate of $\frac{1}{10}$ is a rate of discount of 10%.

4 A camera that sold regularly for \$32. was sold at 15% discount. What was the discount allowed? What was the net price of the camera?

a Let $\$x.$ be the amount of the discount.

Then the ratio of the discount to the regular price is given by $\frac{x}{32}$. Why?

Is $\frac{x}{32}$ equivalent to a rate of 15%? Why?

We see that $\frac{x}{32}=\frac{15}{100}$ $\therefore$ $x\times 100=32\times 15$

$$x=\frac{\overset{8}{\cancel{32}}\times\overset{3}{\cancel{15}}}{\underset{\underset{5}{\cancel{20}}}{\cancel{100}}}=\frac{24}{5}=4\frac{4}{5}\text{ or }4.8$$

$\$4\frac{4}{5}=\$4.8=\$4.80$. The discount is \$4.80.

b We can think of the discount as being 15% of \$32.

$15\%=\frac{15}{100}$; so 15% of \$32. $=\frac{15}{100}$ of \$32.

$$\frac{15}{100}\text{ of }\$32.=\frac{15}{100}\times\$32.=\frac{\overset{3}{\cancel{15}}\times\overset{8}{\cancel{\$32}}}{\underset{\underset{5}{\cancel{20}}}{\cancel{100}}}=\frac{\$24}{5}=\$4.80$$

c The net price = cost price − discount

$= \$32. - \$4.80$

$= \$27.20$

5 The net price of a television set after a discount of 20% was \$180. What was the regular price of the television set?

Let $\$x.$ be the regular price.

Since there was a discount of 20%, the net price paid for the television set must have been 80% of the regular price. (We can think of the regular price as being 100% of the regular price; 20% of the regular price from 100% of the regular price is 80% of the regular price.)

The ratio of the net price to the regular price was $180:x$. We can write this as $\frac{180}{x}$. Is $\frac{180}{x}$ equivalent to 80%? Why?

But $80\%=\frac{80}{100}$; so

$\frac{180}{x}=\frac{80}{100}$ $\therefore$ $180\times 100=x\times 80$

$$80\times x=180\times 100,\text{ and }x=\frac{\overset{45}{\cancel{180}}\times\overset{\overset{5}{\cancel{10}}}{\cancel{100}}}{\underset{\underset{\underset{1}{\cancel{2}}}{\cancel{8}}}{\cancel{80}}}$$

$x=45\times 5=225$

The regular price was \$225.

20% is equivalent to $\frac{20}{100}=\frac{1}{5}$

Discount $=20\%$ of \$225. $=\frac{1}{5}$ of \$225. $=$ \$45.

Net price $=$ \$225. $-$ \$45. $=$ \$180.

6. A table that cost \$43. regular price was on sale at 18% discount. What was the net price of the table?

7. Study the table below and find (**a**) the discount, (**b**) the net price, of each of the articles listed.

Article	**Regular Price**	**Discount Rate**
Refrigerator	\$245.00	30%
Stove	\$175.00	25%
Floor Polisher	\$39.95	10%
Electric Mixer	\$24.85	15%

8. The net price of a record player after a discount of 15% was \$42.50. What was the regular price?

9. At a sale, all regular prices were reduced by $33\frac{1}{3}\%$. What was the regular price of a hockey sweater that sold for \$7.60?

10. A pair of shoes was regularly priced at \$12.00. A discount of \$2.00 was given. What per cent of the regular price was the discount?

11. What is the rate of discount if a rug regularly costing \$180. is sold for \$150.?

12. A man bought a suit for \$64. The regular price was \$72. What was the rate of discount?

13. A boy receives a discount of 30¢ on a hockey stick. The regular price of the hockey stick is \$1.80. What is the rate of discount?

14. After receiving a discount of 25%, a boy paid \$7.50 for a baseball mitt. What was the regular price of the mitt?

15. A store received a discount of 3% from the wholesale supplier if it paid within 30 days. The store paid immediately for goods it received. The cash discount was \$174. What was the cost of the goods before discount?

16 Agents are people who buy or sell goods for other people. For this service, they are paid a sum of money called a **commission**. The com-

mission is usually a per cent of the value of the item bought or sold. The per cent of the value of the item bought or sold is called the **rate of commission**.

17. A real estate agent sold a house for \$13,000. His rate of commission was $3\frac{1}{2}\%$. What commission did he receive?
Let \$**x**, represent the commission he received.
What ratio is \$**x**, to the value of the house? What will this ratio be when expressed as a per cent?

$$\frac{x}{13{,}000}=\frac{3\frac{1}{2}}{100}; \text{ so } x\times 100=13{,}000\times 3\tfrac{1}{2}$$

$$x=\frac{\overset{\overset{65}{\cancel{6500}}}{\cancel{13000}}\times\frac{7}{\cancel{2}_{1}}}{\underset{1}{\cancel{100}}}=455$$

He received \$455. commission.

What ratio is the commission to the price of the house?

Is $\frac{455}{13{,}000}$ equivalent to $3\frac{1}{2}\%$? Let $\frac{455}{13{,}000}=n\%$

$$\frac{455}{13{,}000}=\frac{n}{100}, \therefore\ 455\times 100=13{,}000\times n$$

$$\text{and } n=\frac{\overset{91}{\cancel{455}}\times\overset{1}{\cancel{100}}}{\underset{\underset{26}{\cancel{130}}}{\cancel{13{,}000}}}=\frac{91}{26}=3\tfrac{13}{26}=3\tfrac{1}{2}$$

$n\%=3\frac{1}{2}\%$. Our computation is correct.

18. An agent sold some goods for \$5,000. His rate of commission was 4%. What commission did he receive?

19. An agent who sold a car for \$2,000 received a commission of \$150. What was his rate of commission?

20. A real estate agent received 5% commission on all houses that he sold. He sold one house and received a commission of \$775. What was the price for which the house was sold?

21. An agent sold a farm for \$36,000. He received a commission of \$900. What was his rate of commission?

22. A man sold some property through an agent whose rate of commission was $4\frac{1}{2}\%$. If the man received \$9,045. for the property after the agent had taken his commission, what was the selling price of the property?

23. A saleswoman's rate of commission was 7%. One week her commission was \$56. How many dollars worth of goods did she sell that week?

24. Find the commissions on the following sales:

Amount of sale:	\$7,955.	\$2,700.	\$1,976.	\$6,015.
Rate of commission:	8%	$5\frac{1}{2}$%	$12\frac{1}{2}$%	$16\frac{2}{3}$%

25. A sales clerk received \$35. a week plus a 1% commission on all sales. One week he sold \$1,196 worth of merchandise. How much did he earn that week?

26. A commission agent charged $6\frac{1}{4}$% commission. What commission did he receive for selling 520 baskets of peaches at \$1.40 a basket?

27 When money is borrowed from a bank or some other business, the borrower is charged **interest** for the use of the money. The amount of money borrowed is called the **principal.** The interest is usually reckoned as a per cent of the principal. A rate of 5% per annum tells us that if we borrow \$100. we will have to pay \$5. interest for each year that the money is borrowed. The sum of the interest and the principal is called the **amount**. Therefore, if we borrow \$100. for 1 year at 5% interest, the amount due in 1 year would be

$$\underset{\text{principal}}{\underset{\uparrow}{\$100}} \quad + \quad \underset{\text{interest}}{\underset{\uparrow}{\$5.}} \quad = \quad \underset{\text{amount}}{\underset{\uparrow}{\$105.}}$$

28 A man borrowed \$400. at 6% interest for 1 year. How much interest did he owe? What was the amount due?

a Let $\$x$. be the interest due.

The ratio of the number of dollars in the interest to the number of dollars in the principal is $x:400$ or $\frac{x}{400}$. But the interest is to be paid at the rate of 6% which is $\frac{6}{100}$. Are $\frac{x}{400}$ and $\frac{6}{100}$ equivalent ratios? Why?

$$\frac{x}{400}=\frac{6}{100} \quad \therefore \; x\times 100=400\times 6$$

$$x=\frac{\overset{4}{\cancel{400}}\times 6}{\underset{1}{\cancel{100}}}=24$$

The interest owed is \$24.

$$\text{The amount due} = \quad \underset{\text{principal}}{\underset{\uparrow}{\$400.}} \quad + \quad \underset{\text{interest}}{\underset{\uparrow}{\$24.}} \quad = \quad \underset{\text{amount}}{\underset{\uparrow}{\$424.}}$$

b The interest owed was 6% of the principal

$$\frac{6}{100} \text{ of } \$400. = \frac{6}{\cancel{100}_{1}} \times \overset{4}{\cancel{400}} = \$24.$$

Amount = principal + interest = $400. + $24. = $424.

29 How much interest was owed by a man who borrowed $300. at an interest rate of $4\frac{1}{2}\%$ per annum if he repaid the loan in 6 months?

Let $\$x$. be the interest to be paid for 1 year.

The ratio of the interest to the principal is $x:300$ or $\frac{x}{300}$. This is equivalent to a rate of $4\frac{1}{2}\%$ or $\frac{4\frac{1}{2}}{100}$. Why?

$$\frac{x}{300} = \frac{4\frac{1}{2}}{100} \qquad \therefore x \times 100 = 300 \times 4\frac{1}{2};$$

$$\text{so} \quad x = \frac{\overset{3}{\cancel{300}} \times 4\frac{1}{2}}{\underset{1}{\cancel{100}}} = 13\frac{1}{2}$$

The interest to be paid in 1 year $= \$13\frac{1}{2}$ or $13.50.

6 months is $\frac{1}{2}$ of a year. So in 6 months the interest owed would be $\frac{1}{2}$ of the interest owed for 1 year; $\frac{1}{2}$ of $\$13.50 = \frac{\$13.50}{2} = \$6.75$.

30 In the example above, what interest would be owed if the money were borrowed for 6 years?

In 6 years the interest would be 6 times the interest for 1 year, or 6×$13.50, which is $81.00.

31. Calculate (**a**) the interest, (**b**) the amount, for each of the loans below:

Principal	$400.	$300.	$750.	$1000.	$200.	$450.
Rate	5%	$6\frac{1}{4}\%$	4%	$3\frac{3}{4}\%$	$2\frac{1}{2}\%$	6%
Time of loan	1 year	1 year	5 yrs.	3 yrs.	3 mos.	1 yr. 6 mos.

32. When you put money into the bank, the bank pays interest on the money you deposit. If a boy deposits $50., how much interest will he receive after 1 year if the bank pays 3% interest on all moneys deposited?

33. What interest will be earned by the deposits below?

Deposit	$75.	$200.	$100.	$650.	$25.	$1500.
Rate of Interest	4%	$2\frac{1}{2}\%$	$3\frac{3}{4}\%$	$4\frac{1}{2}\%$	3%	3%
Time of deposit	1 yr.	3 yrs.	6 mos.	4 yrs.	3 mos.	5 yrs.

34 A boy deposited money in a bank that paid interest at a rate of 4%. If he received \$3. interest after 1 year, how much money did he deposit?

Let \$$x$. be the deposit.

The ratio of the interest paid to the deposit is $3:x$ or $\frac{3}{x}$.

Interest is paid at the rate of 4% or $\frac{4}{100}$.

Why are $\frac{3}{x}$ and $\frac{4}{100}$ equivalent rates?

$$\frac{3}{x}=\frac{4}{100}; \therefore 3\times 100=x\times 4 \text{ or } 4\times x=3\times 100$$

$$x=\frac{3\times \overset{25}{\cancel{100}}}{\underset{1}{\cancel{4}}}=75$$

The boy deposited \$75.

Find 4% of \$75. Why should this equal \$3.?

35. After depositing his savings in the bank for 3 years, a boy received \$15. interest. The rate of interest was 5%. What sum did he deposit?

Let \$$x$. be the sum deposited.

After 3 years the interest was ▒ ?

What is the ratio of the interest for 1 year to the deposit? With what ratio is this equivalent? Now find the sum he deposited.

36. What sum of money was deposited in each of the examples below?

Interest	\$12.	\$35.	\$11.	\$15.	\$250.	\$225.
Rate of Interest	6%	$2\frac{1}{2}$%	4%	$3\frac{3}{4}$%	5%	$2\frac{1}{4}$%
Time of deposit	1 yr.	2 yrs.	6 mos.	1 yr.	10 yrs.	3 yrs.

37. A girl deposited \$75. in a bank that paid interest at the rate of 4%. How long must she leave it in the bank before she receives \$12. interest?

Let \$$x$. be the interest after 1 year.

Then $\frac{x}{75}=\frac{4}{100}$ (Why?)

So $x\times 100=75\times 4$ and $x=\frac{\overset{3}{\cancel{75}}\times \overset{1}{\cancel{4}}}{\underset{\underset{1}{\cancel{25}}}{\cancel{100}}}=3$

In 1 year she will receive \$3.

At a rate of \$3. interest per year, how many years will be needed to gain \$12. interest?

38. For what time will the deposits below have to be left in the bank to give the interests stated at the rates stated?

Deposit	\$100.	\$200.	\$50.	\$1000.	\$350.	\$600.
Rate of Interest	4%	$6\frac{1}{4}$%	5%	3%	4%	6%
Interest	\$10.	\$25.	\$1.25	\$45.	\$42.	\$48.

CHAPTER TEST

1. Use (i) fraction numerals
   (ii) ordered number pairs
   to express the following rates:
   a 6 cans for \$1.00
   b 4 miles in 1 hour
   c eggs at 49¢ a dozen
   d a rocket travelling at 18,000 miles an hour
   e 1 cup of sugar to $\frac{1}{2}$ pound of flour
   f bananas at 29¢ a pound.

2. Each example below has two ordered number pairs. Which examples have equivalent ordered number pairs?
   a (1, 2); (3, 6) **b** (3, 4); (21, 28) **c** (3, 5); (5, 3)
   d (3, 2); (3, 4) **e** (7, 9); (84, 108) **f** (20, 2); (10, 1)
   g (4, 9); (2, 9) **h** (1, 4); (1, 4) **i** (5, 21); (65, 273)
   j (36, 15); (12, 5)

3. What numbers are represented by the letters in the ordered number pairs below, if each example contains two ordered number pairs that are equivalent?
   a $(x, 4)$; (9, 12) **b** $(5, y)$; (15, 6) **c** (7, 9); $(z, 45)$
   d (19, 13); $(38, n)$ **e** $(m, 12)$; (2, 6) **f** $(p, 42)$; (9, 6)

4. What number is represented by x in the examples below?
   a $\frac{1}{4}=\frac{x}{72}$ **b** $\frac{x}{9}=\frac{15}{27}$ **c** $\frac{14}{13}=\frac{20}{x}$ **d** $\frac{11}{x}=\frac{121}{132}$
   e $\frac{x}{15}=\frac{14}{21}$ **f** $\frac{9}{12}=\frac{15}{x}$ **g** $\frac{18}{144}=\frac{7}{x}$ **h** $\frac{x}{81}=\frac{4}{12}$

5. A car travels 240 miles in $5\frac{1}{2}$ hrs. How long will it take to travel 720 miles at this rate?

6. 35 gallons of gasoline cost \$14.50. At this rate, how much should be paid for 28 gallons of gasoline?

7. **a** Find the ratio of the measure in column A to the measure in column B.
   b Find the ratio of the measure in column B to the measure in column A. (Write each answer in three different ways.)

A	B
1 yard	9 inches
2 quarts	1 gallon
250 pounds	3 tons
45¢	\$1.35
660 yards	1 mile
$6\frac{1}{4}$ inches	2 feet, 1 inch
3 quarters	3 dimes
16 pounds	1 ounce
4 ounces	$1\frac{1}{2}$ pounds
6 pints	3 gallons

8. A store sells 5 pounds of apples for every 3 pounds of bananas that it sells. In one day, the store sold 243 pounds of bananas. How many pounds of apples were sold?

9. Soup was selling at 2 cans for 35 cents. How many cans of soup could be bought for \$6.65?

10. The actual distance between two towns is 37 miles. How far apart on a map would these two towns be placed if the scale of the map is 2 inches to represent 5 miles?

11. An aeroplane travelled at an average speed of 714 miles per hour. How far would it travel in 3 hours 20 minutes at this rate?

12. The ratio of the heights of two buildings is 5:4. If the taller building is 75 feet high, how tall is the other building?

13. Two partners shared their profits in the ratio of 4:5. If the partner with the smaller share received \$4840.00, how much did the other partner receive?

14. In $3\frac{3}{4}$ hours, a man earned \$7.48. How much would he receive for $11\frac{1}{4}$ hours at this rate?

15. Write the following as per cents:

$\frac{16}{100}$	$\frac{27}{100}$	$\frac{350}{100}$	$\frac{790}{100}$	$\frac{81}{100}$	$\frac{1}{100}$
$\frac{3}{4}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{7}{10}$	$\frac{1}{2}$	$\frac{3}{10}$
$1\frac{1}{2}$	$3\frac{4}{5}$	$2\frac{9}{10}$	4	$3\frac{17}{100}$	$6\frac{16}{25}$

16. Write the following as fraction numerals in their lowest terms:

48%	95%	15%	10%	50%	80%
135%	600%	725%	228%	1000%	100%

17. Express each of the following as per cents:

.04	1.62	3.355	14.2	.38	.50
4.5	7.926	.001	1.56	.325	.875

18. Write the following as decimal numerals:

10%	35%	115%	275%	1%	8%
24.5%	3.75%	6.1%	9.216%	4,001%	.875%

19. Express the following as per cents:

$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{1}{16}$
$1\frac{3}{16}$	$2\frac{4}{5}$	$1\frac{2}{3}$	$4\frac{7}{8}$	$5\frac{3}{10}$	$1\frac{1}{12}$
$4\frac{5}{12}$	$2\frac{3}{15}$	$5\frac{8}{9}$	$12\frac{5}{6}$	$10\frac{13}{15}$	$4\frac{11}{18}$

20. A boy had 72 hockey cards. 27 of these cards were pictures of defence men. What per cent of the total of hockey cards was made up of pictures of defence men?

21. A boy earned \$65. one summer vacation. He saved \$39. of this amount. What per cent of his earnings did the boy save?

22. A girl went 95% of the distance to school by bus. If she travelled 1900 yards by bus, how many yards did she live from the school?

23. What is $27\frac{1}{2}\%$ of 1000?

24. $37\frac{1}{2}\%$ of an amount of money is \$61.41. What is the amount of money?

25. A man bought a house costing \$17,500. He made a down payment of $12\frac{1}{2}\%$ of the cost price. What was the amount of his down payment?

26. Find x.

a $16\frac{2}{3}\%$ of $x = 315$ **b** $x\%$ of $150 = 50$

c 10% of \$3. $= x$ cents   **d** $66\frac{2}{3}$ of $x = 1$ mile

27. A boy deposited a sum of money in the bank at a rate of $2\frac{3}{4}\%$ interest. If he received \$15.00 interest after 4 years, what sum of money did he put in the bank originally?

28. At a sale, all goods were reduced in price by 30%. What was the pre-sale price of an article that was on sale for \$32.48?

29. An agent sold some goods for \$6,950. His rate of commission was $3\frac{1}{2}\%$. How much commission did he receive?

30. What sales tax should be paid on an article selling for \$49.95 if the rate of sales tax is $1\frac{3}{4}\%$?

31. A hockey team played 48 games and won 42. What per cent of its games did it win?

32. In a class of 36 pupils, 27 were present. What per cent of the class was absent?

33. At a sale a camera sold for \$18.00. If the regular price of the camera was \$22.50, what per cent of the regular price was the reduction?

34. Last week a boy had 12 problems correct in a test. If he had 80% of the test correct, how many problems were there in the test?

35. A girl worked weekends in a store. She received a raise of $12\frac{1}{2}\%$ of what she was earning. If the raise amounted to 10¢ an hour, how much an hour had she been earning before?

36. A girl spent 60% of her allowance on lunches and 10% of her allowance on school supplies. If she spent \$3.00 on lunches, how much did she spend on school supplies?

37. A man borrowed \$350.00 from the bank. At the end of 1 year he paid the bank \$371.00. What rate per cent was the interest charged by the bank?

38. The regular price of a car was \$3,600. The car was sold for \$3,240. What rate per cent was the discount?

39. A car salesman received $7\frac{1}{2}\%$ commission on old cars and 5% commission on new cars. In one month he sold 4 old cars whose total value was \$6,400 and 5 new cars whose total value was \$13,600. What was the amount of commission he received for that month's sales?

40. N is a set of pairs of numbers. Each of these pairs of numbers consists of two numbers which are in the ratio of 5:2. Which of the following pairs of numbers are members of N?

a (1305; 391)	**b** (6000; 2400)	**c** (50; 2)	**d** (95; 38)
e (1704; 680)	**f** (35; 16)	**g** (860; 344)	**h** (10+5; 4+2)

41. In a test John had 7 correct answers for every 5 answers that Bob had correct. If John had 84 correct answers in a test of 100 questions, what per cent of the possible score did Bob have?

42. A commission agent received a commission of \$490 on a sale that was made for \$14,000. At what rate per cent was the commission paid to the agent?

43. In a school 85% of the boys played hockey. If 799 boys played hockey, what was the school enrolment?

44. A boy put \$25 in the bank and left it there for two years. At the end of these two years he found that he had \$27 in his account. What was the rate per cent of the interest the boy received?

45. A bank paid $3\frac{1}{2}\%$ interest per year on deposits. What number of years must a girl leave a deposit of \$20 in the bank for her to receive \$3.50 in interest?

46. At one store a television set was sold at a regular price of \$280. At a sale the discount was 10%. At another store the regular price of a similar television set was \$300. This store had a discount of 15% at its sale. At which store was the sale price least, and by how much?

47. At an election Tom received 6 votes to every 4 votes that Harry received. If the number of votes cast for Tom was 750, how many votes did Harry receive?

48. One number is $37\frac{1}{2}\%$ of another number. If the smaller number is 242, what is the larger number?

49. A salesman sold radios for \$40 each. His commission was 8% of the selling price of each radio. One week he earned \$60.80 commission. How many radios did he sell that week?

50. An agent sold books at \$3.50 each. He received a commission on each book he sold. One week he sold 50 books and earned \$63 commission. At what rate per cent of the selling price of each book was the commission he received?

8 Geometry

POINTS AND LINES

1 The word *geometry* comes to us from two Greek words meaning *earth* and *measure*. The branch of mathematics that we call geometry started when man began to think of the shapes of things in the world around him, and when he began to think of ways of measuring these things. In geometry, we shall study ideas that help us to describe the shapes of things around us and that help us to find ways of measuring the sizes of these things.

2 In geometry, we study sets of *points*. *Points* are ideas; they are things that we have in our mind. To represent points, we use dots.

Is this a point? Why not?

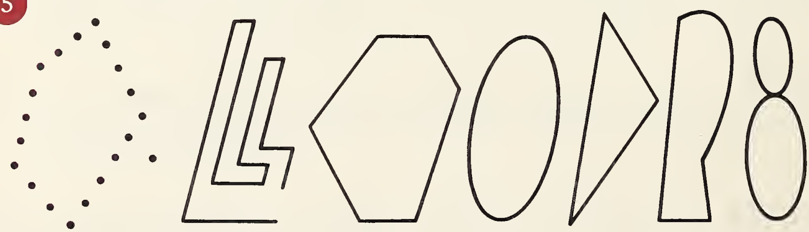
Is this a point? Why not?

Does this *represent* a point?

3 We cannot draw points. They are so small that they have no thickness, no width; in fact, they have no size. When we put a dot on the paper, no matter how small we make the dot with our pencil, the dot is a set of millions and millions of points. We let dots *represent* points, just as in arithmetic we let numerals represent numbers.

4 When we say, "Draw a point," what we really mean is, "Draw a picture of a point," or "Draw a model of a point." As long as we remember that we are only drawing *pictures* or *models* of points when we put a dot on the paper, it is all right for us to use the expression, "Draw a point."

5



Above, we have drawings of different sets of points. In geometry, we call any set of points a **geometric figure**. The set can consist of only one point, but it is still a geometric figure.

6 We usually name points by capital letters. We name the three points we have represented by the dots shown below, as *point A*, *point B*, and *point C*. More simply, we could just say A, B, and C. When we say *A*, we mean point *A*.

What is meant by B? C?

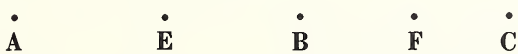
7. Consider the points below:

Is point B between points A and C?

Is point D between points A and C?

Can we put another point between points A and B? between points B and C?

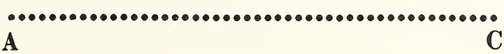
Suppose we label these two points E and F



Can we now put points between A and E? between E and B? between B and F? F and C? Let us call the new points G, H, I, and J respectively.

Could we continue to put points between these given points? Could we always put a point between two other points. Why?

Here is a picture that we might draw to represent all the points between A and C:



Why is this not a good picture of all the points between A and C?

8. The set of all the points between A and C, together with the points A and C is called **line segment AC.** For line segment AC we usually write $\overline{AC}$. We have seen that it is difficult to represent all these points by drawing dots; so we usually draw a line segment like this:

Remember that this drawing is not line segment AC; it is only a picture of a model of line segment AC. Since points have no width and no thickness, what can you say about the width and thickness of line segment AC?

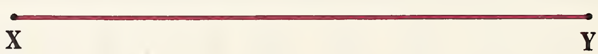
The points A and C are called the **endpoints** of $\overline{AC}$.

9. A geometric figure that consists of (a) two points, called **endpoints**, and (b) all the points between these two points is called a **line segment**. For line segment, we often say, *segment.*

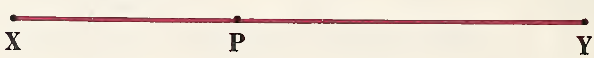
10. Look at your pencil point. Think of all the points between your eye and the pencil point. Together with this set of points, consider the point at your eye and the point at the tip of your pencil point. This entire set of points is a line segment.

11. The things that we see around us and call line segments are really models of line segments. For example, the edge of the page of this book is a model of a line segment. Think of the edge of your ruler. Is this a line segment or a model of a line segment? Now list 10 models of line segments.

12. Consider the line segment XY.



Take any point between X and Y. Label it P.



This picture shows two new segments: they are $\overline{XP}$ and $\overline{PY}$. Do all of the points of $\overline{XP}$ belong to $\overline{XY}$? We say, "$\overline{XY}$ contains $\overline{XP}$". Do all of the points on $\overline{PY}$ belong to $\overline{XY}$? We say, "$\overline{XY}$ contains $\overline{PY}$".

13. **a** Draw two points and label them A and B.

b Draw a point X between A and B.

c Draw *all* the points between A and B.

d What name can we give to this set of points?

e Copy and complete the following:
$\overline{AB}$ contains ▒▒▒ and ▒▒▒ .

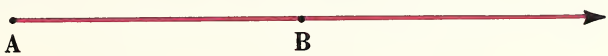
f Is X on $\overline{AB}$?

g Is A on $\overline{AB}$?

h Is A on $\overline{XB}$?

i Is B on $\overline{AX}$?

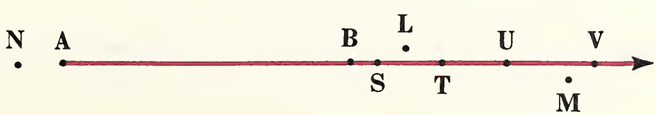
14 Another idea that we can consider in geometry is that of a set of points termed a **ray**. We represent a ray thus:



On this diagram, we have labelled one point A. Is A an endpoint? Another point has been labelled B. Is B an endpoint? The arrow in the diagram shows that the ray goes on for ever. We write the symbol $\overrightarrow{AB}$ to represent the ray in the diagram above. Notice that we start with the letter for the endpoint when we are naming the ray. Now, what points does this ray contain?

a It contains all the points in the set of points that make up line segment AB.

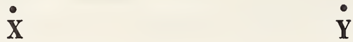
b It contains all the points that are such that point B is *between* these points and the point A.



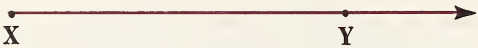
Notice: B is between S and A; so S is on $\overrightarrow{AB}$.
B is between T and A; so T is on $\overrightarrow{AB}$.

Now which of the points N, L, V, M, U are on ray AB? Explain your answers.

15 Draw two points and label them X and Y.

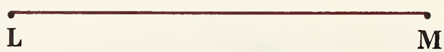


Now draw the ray XY.
Ray XY begins with the point X and is represented thus:



Is this the same ray as $\overrightarrow{YX}$?
Ray YX begins with point Y and is represented thus:

16 Consider two points L and M. Draw the line segment LM.

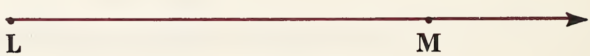


Let T be any point such that M is between T and L.

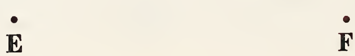
Ray LM contains:

a line segment LM and

b <u>All</u> points such that M is between L and these points. We represent $\overrightarrow{LM}$ thus:



17. Consider any two points E and F.



Using these two points, draw $\overrightarrow{EF}$ and $\overrightarrow{FE}$.

Does this geometric figure contain $\overline{EF}$? $\overrightarrow{EF}$? $\overrightarrow{FE}$?

Does this geometric figure have any end?

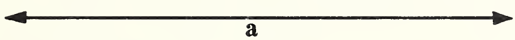
We call the set of points containing, (a) line segment EF,
(b) ray EF,
(c) ray FE,

the line EF. Sometimes we use the symbol $\overleftrightarrow{EF}$ to represent the line EF.

18. Can we name the line above as $\overleftrightarrow{FE}$ or $\overleftrightarrow{EF}$? Why? We sometimes call lines such as $\overleftrightarrow{EF}$ (which is the same as $\overleftrightarrow{FE}$) **straight** lines.

In future, when we say *line* or *lines* we shall think of a straight line or of straight lines.

19 With respect to a line, we often use just one letter to name it. When we use one letter, we often use a small letter.



We can name the line above as *line* **a** or just **a**.

20. **a** Draw two points X and Y. Now draw $\overline{XY}$. What are its endpoints?

b Draw two points R and S. Draw $\overrightarrow{RS}$. How many endpoints does it have?

What are they? Now draw $\overrightarrow{SR}$ and colour it red. What are its endpoints?

Which points are common to $\overrightarrow{RS}$ and $\overrightarrow{SR}$?

c Draw two points L and M. Draw a point P between them. Now draw $\overline{LM}$. Name three segments that contain P.

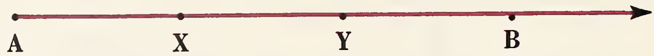
d Draw two points A and B. Draw $\overleftrightarrow{AB}$. What are its endpoints?

e In the statements below, the symbol = means that the symbol on the left names the same thing as the symbol on the right.

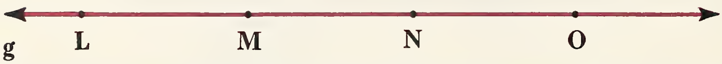
Which statements are true?

$\overleftrightarrow{XY} = \overleftrightarrow{YX}$	$\overrightarrow{RS} = \overrightarrow{SR}$	$\overline{LW} = \overline{WL}$
$\overrightarrow{AB} = \overleftrightarrow{BA}$	$\overline{MN} = \overleftrightarrow{MN}$	$\overrightarrow{MN} = \overrightarrow{MN}$
$\overrightarrow{TV} = \overrightarrow{VT}$	$\overline{OP} = \overrightarrow{OP}$	$\overleftrightarrow{CD} = \overrightarrow{CD}$

f Below, is ray AB. On this ray, some points have been named.



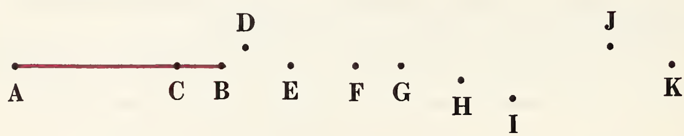
Name all the rays shown on the diagram.
Name all the segments shown on the diagram.



g Give six names for the line above using the letters L, M, N and O.
Name all the segments shown.
Name all the rays shown.

h Draw $\overrightarrow{PX}$. Draw $\overrightarrow{XP}$. What geometric figure does the diagram represent now? Name it, using the letters X and P.

i



Use a ruler to find which of the points named above are on $\overline{AB}$. Which points are on $\overrightarrow{AB}$?

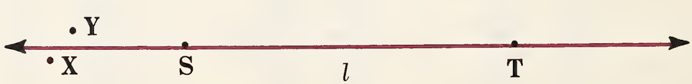
j Draw a diagram containing both $\overrightarrow{ST}$ and $\overrightarrow{TS}$. Name the points common to $\overrightarrow{ST}$ and $\overrightarrow{TS}$.

k Draw two points X and Y.
How many lines can you draw that contain both of the points X and Y?

l

Does $\overline{SV}$ contain $\overline{SU}$? Does $\overline{UT}$ contain $\overline{SV}$?
Does $\overline{ST}$ contain $\overline{UV}$? Does $\overline{TU}$ contain $\overline{VS}$?
Explain your answers. What name do we give to the points common to both $\overline{SV}$ and $\overline{UT}$?

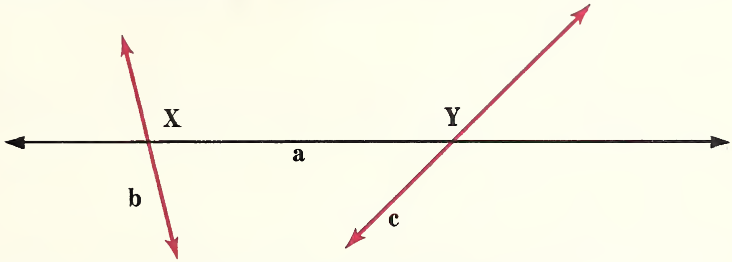
21



The figure $\overleftrightarrow{ST}$ is a line. One name for it is line l. The line l contains the points S and T. We can say, "Line l lies on point S" or "Line l lies on point T." We can also say, "Point S lies on line l" and "Point T lies on line l."

Is point X on line l?

Is line l on point Y?

22. 

a What points on line a are named in the drawing?

b What points on line b are named?

c What points on line c are named?

d Name all the lines on point X that are named in the drawing.

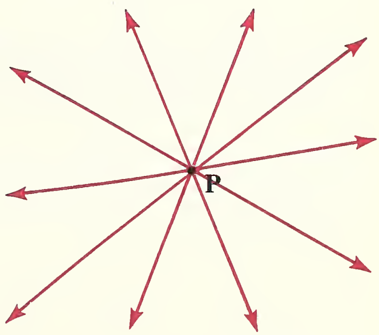
e Name all the lines on point Y that are named in the drawing.

23 Point X is on line a and on line b. We say that X is on the *point of intersection* of lines a and b.

24. What is the point of intersection of lines a and c?

25. Draw line MN. How many points are on this line? Can you count them?

26. Draw point P. How many lines can you draw on this point? Does this diagram help you?

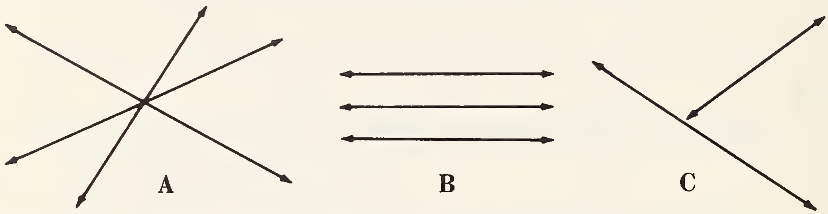


27. **There is an unlimited number of points on a line.**
There is an unlimited number of lines on a point.

28. When two or more lines lie on the same point, the lines are said to be **concurrent**.

29. Draw a point on your paper and label it X. Now draw three concurrent lines that lie on X.

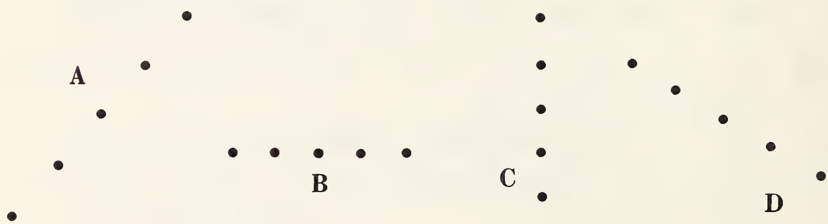
30. Which of the diagrams below illustrate concurrent lines?



31. On your paper draw two points and label them A and B. How many different lines can be drawn that pass through these two points?

32. Because one, and only one, line can be drawn through two different points, we say that any two different points *determine* a line.

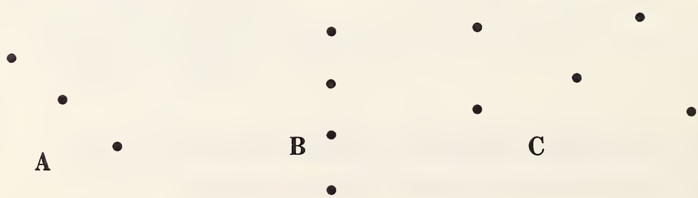
33. Study the sets of points below.



Can one line be drawn so that the set of points in diagram A lie on it? Can a line be drawn so that the points in diagram B lie on it?

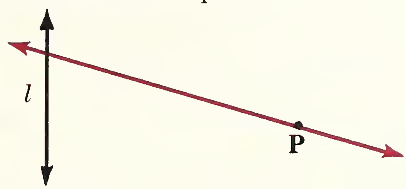
34. Points that lie on the same line are said to be **collinear**.

35. Which of the sets of lines below are collinear? (Use a straight edge to help you.)

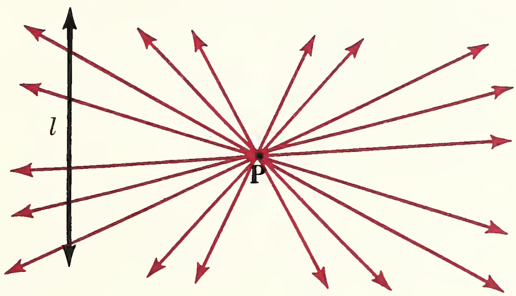


1 Take a line l and a point P not lying on line l.

Draw a line on P that contains a point on l.

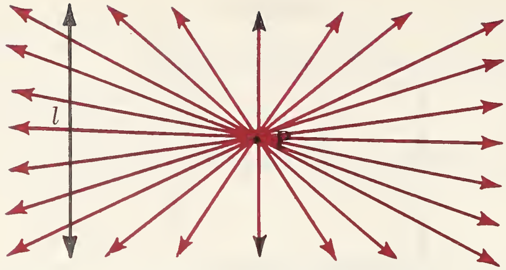


Think of the set of all lines lying on point P and containing a point on line l.



Now think of the set of all the points lying on these lines. This set of points will be flat and even. The set goes on forever. There is a line that we can draw that lies on P but contains no point on l. Here it is drawn.

This line does not intersect line l. We say that this line is parallel to line l. Line l is parallel to this line. Let us add it to the lines we were thinking of before. We now have:

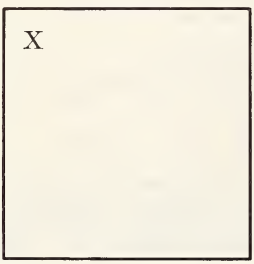


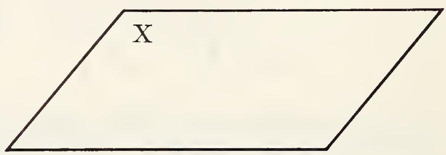
The set of all the points that lie on the lines that contain point P and also intersect l, together with the set of points lying on the line passing through P and parallel to l, forms a **plane.**

2 A plane is flat and has no end because the lines from which it is made up have no end.
How thick is a point? How thick is a plane?

3 Any flat surface such as a pane of glass is a model of a plane. It is not a plane because a plane has no end and no thickness.
List ten models of a plane.

4. Below are two ways in which we may represent a plane:





Each is a drawing to represent plane X. Why are these drawings inaccurate representations of a plane?

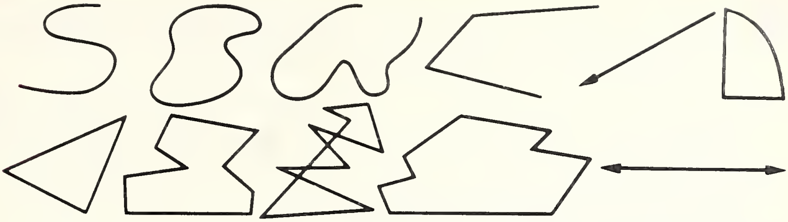
CURVES

1 Without lifting your pencil, draw a set of points.

2 Draw another set different from the first. Do not lift your pencil.

3 You have just drawn two models of **curves**.

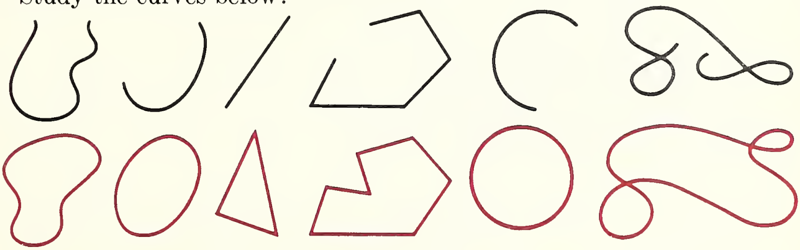
4 Below, there are pictures of different curves:



From the pictures, we can see that there are many different figures that are curves. We notice that some of the curves are what we usually think of as straight lines.

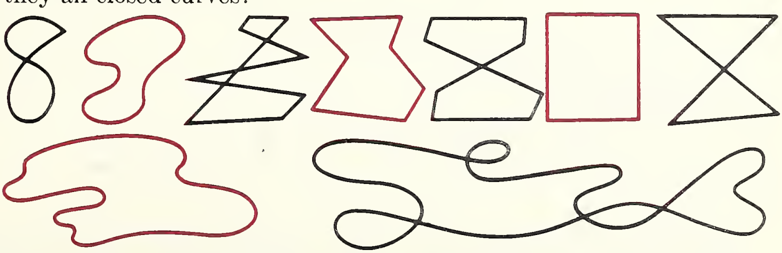
5 Because there are so many different kinds of curves, it is difficult to find a definition for a curve. We shall agree to think of a curve as being a figure, or set of points, which can be represented by a drawing that can be done without lifting the pencil.

6. Study the curves below:



What difference do you notice between the curves drawn in black and those drawn in colour? We call the curves that have been drawn in colour **closed curves.**

7. What differences do you notice between the curves drawn below? Are they all closed curves?



What do you notice about the curves drawn in colour? in black? The coloured curves do not cross themselves. We call them **simple** closed curves.

8. Simple closed curves do not cross themselves.

9. We have seen that a *straight* line is one type of curve. In future when we say **line**, we shall always think of a *straight* line.

10. The curves we have seen have all been drawn in one plane. Is it possible to have a curve that cannot be drawn in one plane? Can you think of one? Is a coil spring a model of a curve? Does the set of points it represents lie on one plane? Can you trace with your finger the model of a curve not lying in one plane?

11.

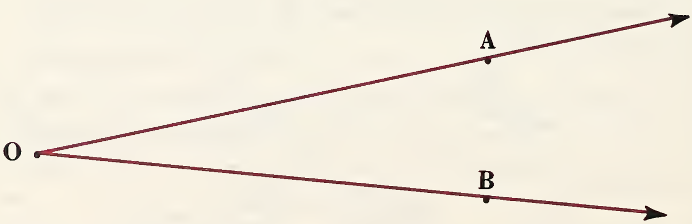
(i) Which of the above are drawings of curves?

(ii) Which are closed curves?

(iii) Which are simple closed curves?

ANGLES

1. Study the figure below:



What name do we give to $\overrightarrow{OA}$? to $\overrightarrow{OB}$? Is the endpoint of $\overrightarrow{OA}$ the same point as the endpoint of $\overrightarrow{OB}$? We see that the figure consists of two rays, $\overrightarrow{OA}$ and $\overrightarrow{OB}$, having the same endpoint O. We call figures like this **angles.**

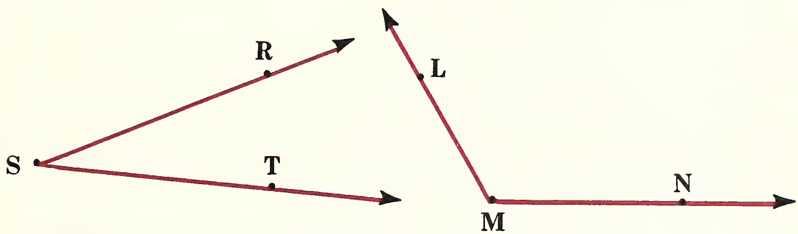
2 The angle in the diagram above is called *angle AOB* or *angle BOA*.
For *angle AOB* we may write $\angle$AOB.
For *angle BOA* we may write $\angle$BOA.
$\angle$AOB and $\angle$BOA are two different names for the same angle.

3 What is the endpoint of $\overrightarrow{OA}$?
What is the endpoint of $\overrightarrow{OB}$?

The endpoint of the two rays is O. We have a name for the common endpoint of the two rays that make up an angle. It is called the **vertex**. Sometimes we use the vertex to name the angle. We can call the angle in the diagram *angle O*. *Angle O* may be written as $\angle$O.

4 The angle in the diagram above may be represented as:
$\angle$AOB or $\angle$BOA or $\angle$O

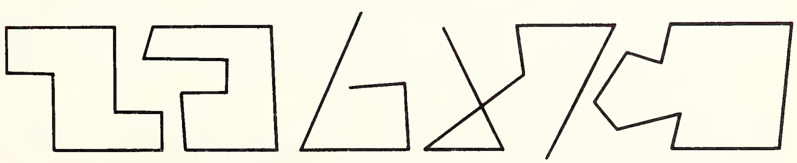
5.



a What two rays make up each of the angles above?
b What is the vertex of each of the angles?
c Name each of the angles in three different ways.

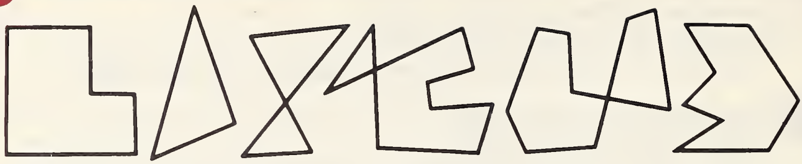
POLYGONS

1 What name can we give to the figures below?



We see that they are curves.
What makes up each of these curves?
We sometimes have a special name for curves that are made up of line segments. We call them 'broken lines'.

2 Study the curves below:



a What makes up each curve?

b Which of them are closed curves?

c Is each curve in one plane only?

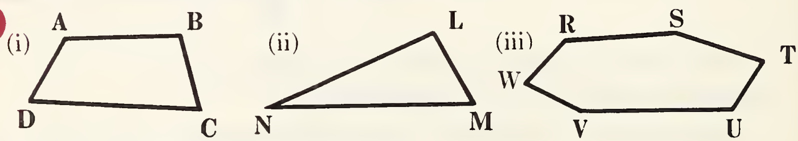
Each figure (i) is made up of line segments
(ii) is a closed curve
(iii) lies in one plane.

Such figures are called **polygons.**

3 A polygon is a closed curve made up of line segments.

4. Which of the closed curves in example 2 are *simple* closed curves?

5 A polygon that is a simple closed curve is called a **simple polygon**. We shall study more about the simple polygons.

6 

a Study the simple polygons above.
In diagram (i) the polygon is made up of the line segments AB, BC, CD, DA. We call these line segments the **sides** of the polygon. Each of the endpoints of the segments AB, BC, CD, DA, is called a **vertex**. So A is a vertex; so is B; so is C, and so is D. The plural of vertex is **vertices**. A, B, C, and D are vertices of polygon ABCD. If two sides have a common endpoint, we call them **adjacent** sides. In polygon ABCD, $\overline{AB}$ and $\overline{BC}$ are adjacent sides. Name two other pairs of adjacent sides in this polygon.

b Name the polygons in diagrams (ii) and (iii), using the letters given.

c Name the sides of the polygon in diagrams (ii) and (iii).

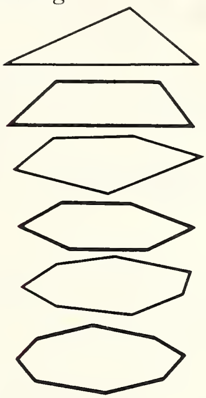
d Name the vertices of polygon LMN, of polygon RSTUVW.

e Is $\overline{LM}$ adjacent to $\overline{NM}$? Is $\overline{NM}$ adjacent to $\overline{LM}$? What sides are adjacent to $\overline{LN}$?

f Name four pairs of adjacent sides in RSTUVW.

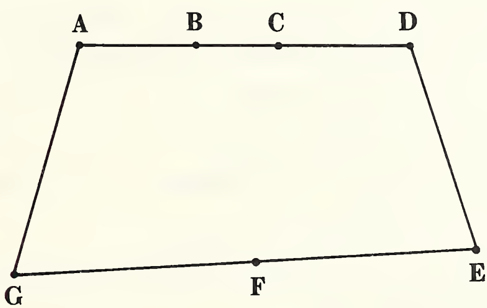
g What sides are adjacent to $\overline{TU}$? $\overline{RW}$? $\overline{ST}$?
h What side is adjacent to both $\overline{WV}$ and $\overline{RS}$?

7 Polygons are given special names according to the number of sides they have. Here are some examples:

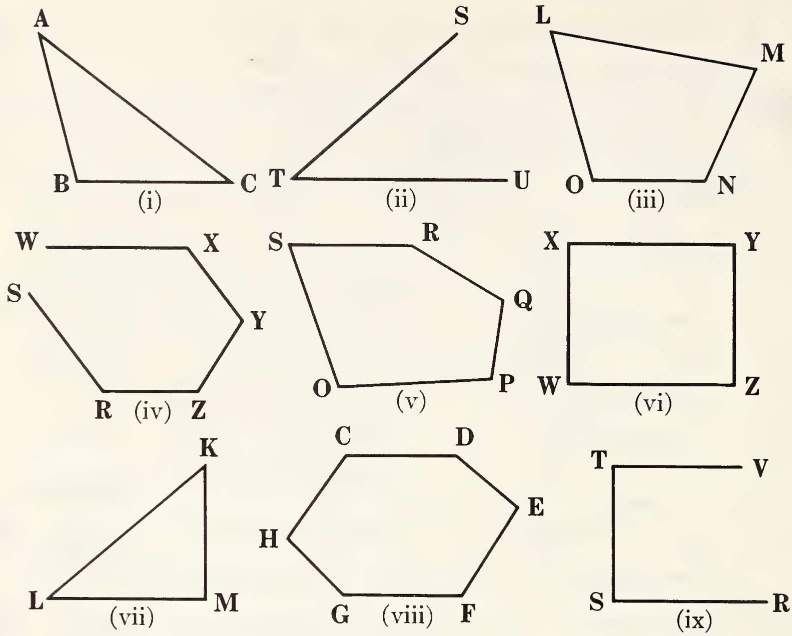


Drawing	Number of Sides	Name
	3	Triangle
	4	Quadrilateral
	5	Pentagon
	6	Hexagon
	7	Heptagon
	8	Octagon

8. a Using a ruler, draw points ABCD and GFE similar to those shown below.



b Join the points in the order AB . . . until you have a closed figure.
c Is this a polygon? Why?
d Name the sides of the figure you have drawn.
e What type of figure is this?



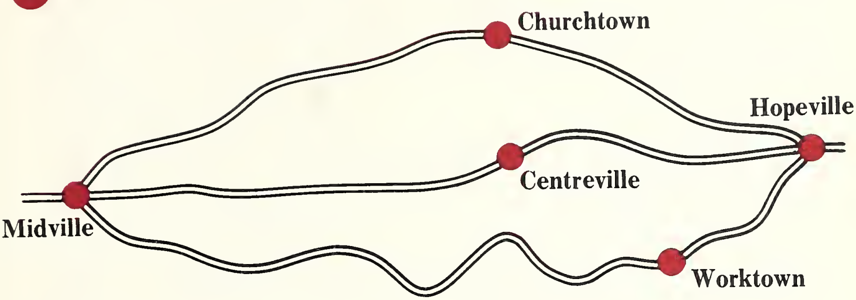
9. **a** Which of the diagrams above are polygons?
   b Name the vertices in diagram (viii).
   c Name the quadrilaterals.
   d Is $\overline{CD}$ in diagram (viii) adjacent to $\overline{GF}$? Explain your answer.
   e How many vertices has a triangle? How many sides?
   f How many vertices has a pentagon? How many sides?
   g Can you make a statement about the number of sides and the number of vertices that a polygon has?
   h In diagram (ix) how many line segments can you name? Why is this figure not a triangle?
   i Name a pair of adjacent sides in diagram (v).
   j How many sides does the polygon in diagram (viii) have? What type of polygon is it?

10. Draw a picture of (a) a triangle (b) a hexagon (c) an octagon

11. Can you draw a triangle that is not a simple polygon? Explain your answer.

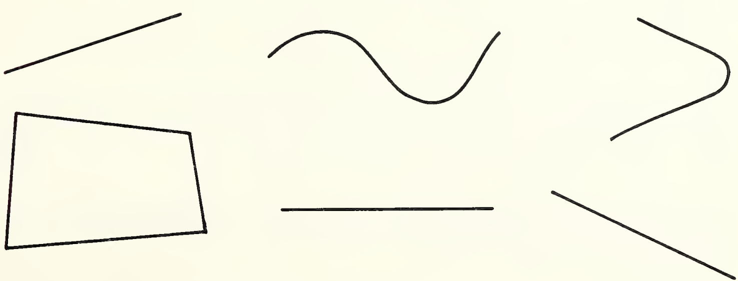
12. Draw a quadrilateral that is *not* a simple polygon.

1



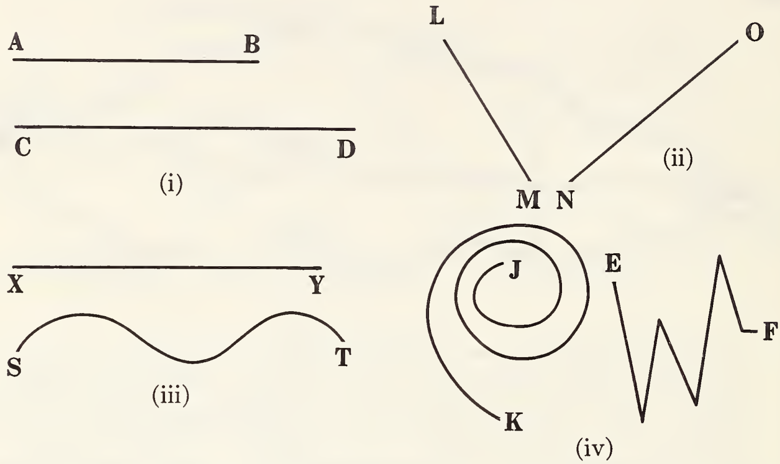
Above is a map showing different routes that may be taken from Midville to Hopeville. If you were in a hurry, which route would you take? Why? This is a problem in comparing distances. We often have to solve such problems in everyday life. We may have to compare the heights of boys; we sometimes need to know whether a piece of cloth we have is long enough to make a certain type of dress; we may want to know whether a piece of wood is of the right length to make a shelf that we need. These are problems involving 'measurement of length'. Often we can tell by merely looking. Sometimes we may have to use other means.

2 One set of geometric figures that we can compare in length is the set of 'one-dimensional figures'. Here are drawings of some one-dimensional figures.



In what way are these figures alike?
One-dimensional figures can all be straightened out to make a line or a line segment.

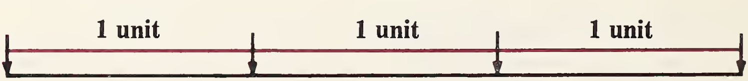
3. Below there are pictures of pairs of one-dimensional figures. Which one of each pair is larger?



How many decisions can you make by just looking? How could you use a piece of string to compare the lengths of the figures in examples (iii) and (iv)?

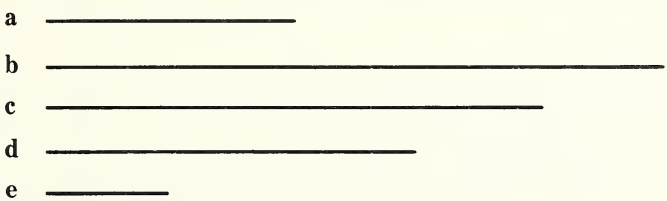
4 When we are measuring one-dimensional figures, we find it convenient to compare the length of the one-dimensional figure with the length of a line segment that we take as one **unit of length**. What is the length of line segment $\overline{AB}$? Use the length of line segment $\overline{PQ}$ as a unit segment.

We can lay unit segments end to end so that together they have the same length as AB.



We can see that we need 3 unit lengths to match the length of $\overline{AB}$. We say that $\overline{AB}$ has a **measure of 3.**
The measure of the length is a number. When we were finding the number for the segment AB, we were **measuring.**
When we put the measure of a segment with the unit of measure, we are giving the segment a **length.**
The measure of $\overline{AB}$ is 3.
The length of $\overline{AB}$ is 3 units.
We have used the words **measure**, **measuring** and **length** many times in our lives. In geometry, they have special meanings. We have now defined the meanings that these words have in geometry.

5. Take this segment as a unit segment:
Use it to find the measures of the following segments:

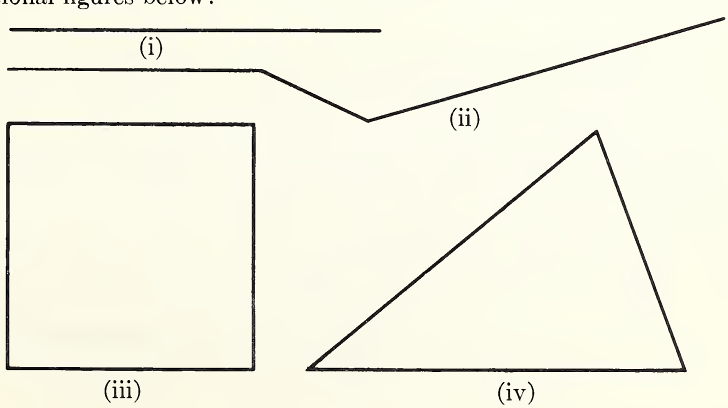


What is the length of each of these segments?

6. **a** Take the following segment as a unit segment:

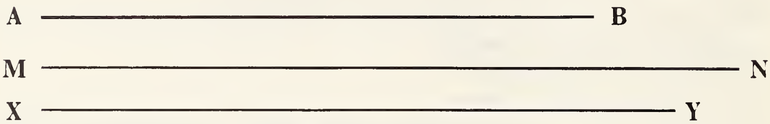
b Draw a line segment 4 units long and divide it into 4 equal parts. This is a 'ruler' with units equal in length to $\overline{AB}$.

c Use this ruler to find the measure and the length of the one-dimensional figures below:



7. Use the unit segment given to find the lengths of segments AB, MN and XY.

unit segment



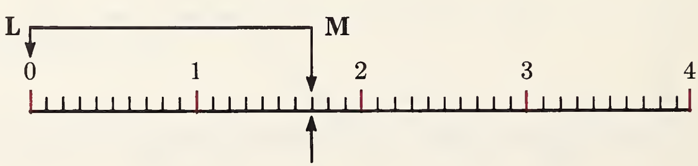
Can you find an exact measure for each of these segments using the unit given? Why not?

Now use a unit segment $\frac{1}{4}$ as long as the unit given. Find the measures of $\overline{AB}$, $\overline{MN}$ and $\overline{XY}$. What do you notice? Can you get an exact measure now?

8 We have seen that we sometimes need a unit segment smaller than the one given if we want a more exact measure of a one-dimensional figure. The unit we chose was $\frac{1}{4}$ as long as the original unit. These smaller units are called **subunits**, and they are usually simple fractional parts of the original unit. Here we show a unit segment that has been divided into 10 equal parts:

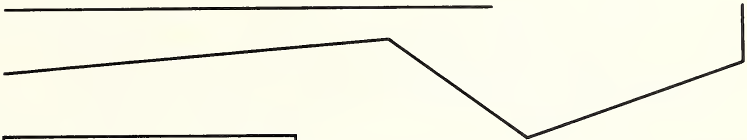
Make a 'ruler' containing 4 of these units divided into tenths. It will look like this:

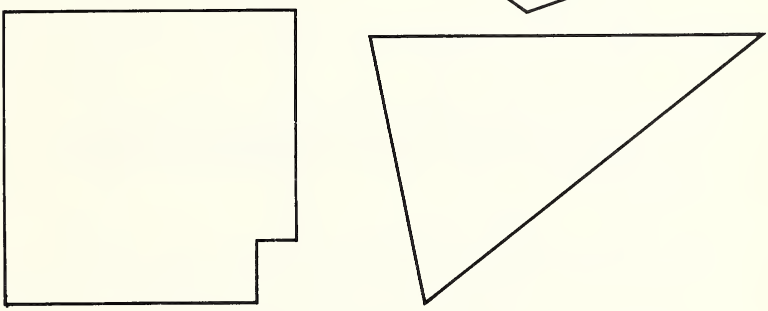
Here is the way in which we might use it to find the measure of $\overline{LM}$ below:



$\overline{LM}$ has a measure of 1 unit + 7 subunits. Since each subunit has a measure of $\frac{1}{10}$ of the unit segment, we can say that the measure of $\overline{LM}$ is $1 + \frac{7}{10}$, or $1\frac{7}{10}$. The length of $\overline{LM}$ is 1 unit + $\frac{7}{10}$ units or $1\frac{7}{10}$ units. Now use your 'ruler' to find the measure and the length of the one-dimensional figures below:

a

b 

c 

9 So far in our measuring we have been choosing our own units of length. Suppose we used the unit of length in example 6 and sent the following order to a store: "1 piece of gold cord 40 units long". What must the store know before it could deliver the order? Suppose the store took as its unit of length the distance from the storeowner's elbow to the end of his second finger, what would you know about the length of gold cord that he would send?

Because confusion results when everyone uses different units of length, an agreement is made to use the same units of length when we are measuring. Several units have been chosen and most of us use them when we are measuring. There is nothing special about these units. They have been in use for many years, and we are familiar with them. They are the **inch**, the **foot**, the **yard**, and the **mile**. Other units, such as the **fathom** and the **rod**, are used in special cases. Many countries use a unit of length called the **metre** as the basis for measuring. The different units we use are related as follows:

12 inches (in.) = 1 foot (ft.)
3 feet (ft.) = 1 yard (yd.)

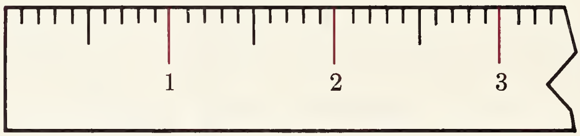
5,280 feet (ft.) or 1,760 yards (yd.) = 1 mile (mi.)

10 The instrument that we use most commonly for measuring is called a ruler. The ruler can be divided into unit lengths of 1 inch or into unit lengths of 1 inch and subunits that are fractional parts of 1 inch:

a

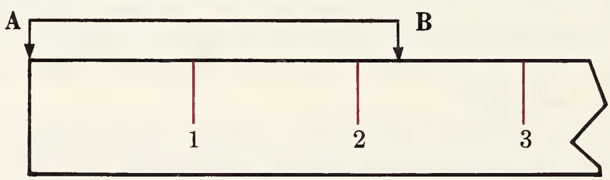
b

c



The diagrams of rulers above show rulers divided into units of 1 inch. Two of the rulers have subunits of 1 inch shown. What subunits are shown in ruler (b)? ruler (c)?

11. Use a copy of the ruler shown in example 10 (a) to find the length of segment AB.



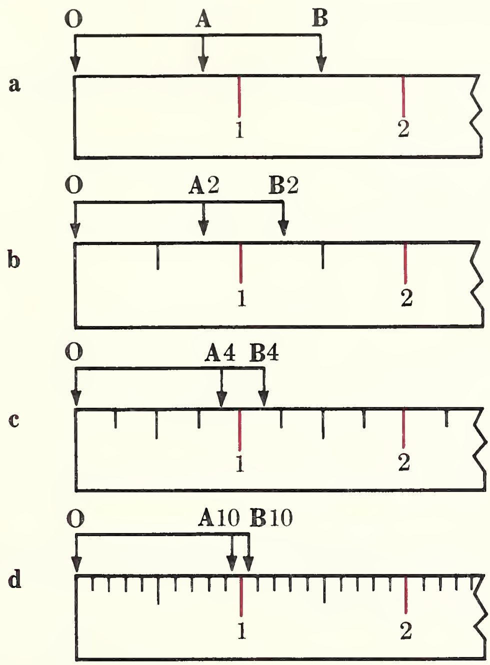
$\overline{AB}$ is more than 2 inches long.

the measure   the unit

$\overline{AB}$ is less than 3 inches long.
Is B nearer to the 2-inch mark or the 3-inch mark?
We say $\overline{AB}$ is 2 inches long (to the nearest inch).

12. Now use copies of the rulers shown in 10(b) and 10(c) to find the length of $\overline{AB}$ (i) to the nearest $\frac{1}{4}$ inch
(ii) to the nearest $\frac{1}{10}$ inch.

13 The accuracy with which we can measure a length depends on the size of the units and the subunits on the ruler that we use.
Study the diagrams below:



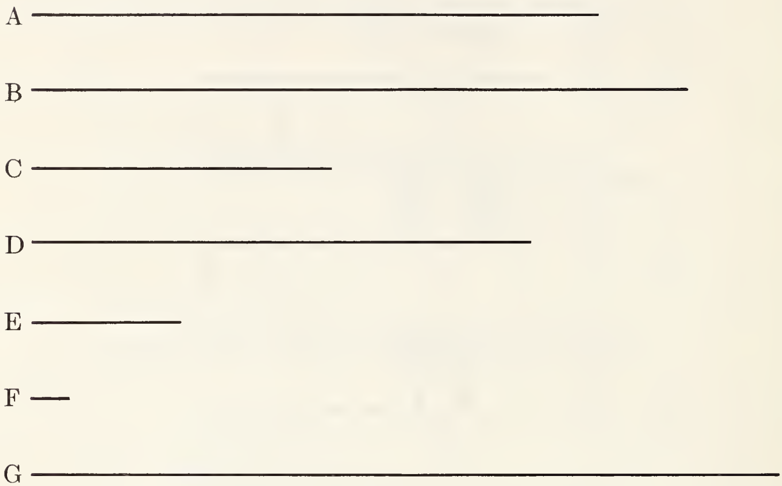
a If we use a ruler marked off in inch units, we would say that any line segment that is as long as $\overline{OA}$, or as long as $\overline{OB}$ would measure 1 inch to the nearest inch. Line segments greater in length than $\overline{OA}$ and also shorter in length than $\overline{OB}$, would also be said to measure 1 inch, to the nearest inch.

b Study the ruler in (**b**). What is the size of its units? its subunits? If we use this ruler and measure the length of a segment and say its length is 1 inch, to the nearest half inch, what is the greatest length the segment could be? the shortest length?

c Find the limits for the lengths of a segment measured by ruler (**c**) and said to be 1 inch long, to the nearest $\frac{1}{4}$ inch.

d Find the limits for a line said to be 1 inch long to its nearest $\frac{1}{10}$ inch.

14. Take a ruler divided into units of 1 inch and subunits of $\frac{1}{2}$ inch, $\frac{1}{4}$ inch and $\frac{1}{8}$ inch. Use it to measure the segments below, to the nearest $\frac{1}{2}$ inch, the nearest $\frac{1}{4}$ inch, and the nearest $\frac{1}{8}$ inch.



15. Draw 3 segments of different lengths. The length of each segment is to be 4 inches to the nearest inch.

16. Copy the ruler below and mark on it where segments might end if they measure 2 inches to

a the nearest inch

b the nearest $\frac{1}{2}$ inch

c the nearest $\frac{1}{4}$ inch

17. Make two marks 1 inch apart. Also make two marks 1 foot apart. Try to remember how long these segments are. Now estimate:

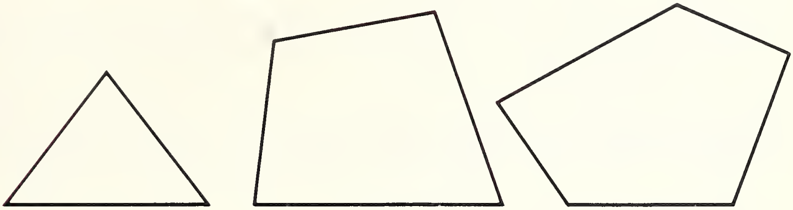
a The length and width of your desk top.

b The length, width and thickness of this book.

c Measure the desk and the book to see how good your estimates were.

PERIMETERS

1 What is the distance around each of the polygons below? The measure is shown in 1-inch units.



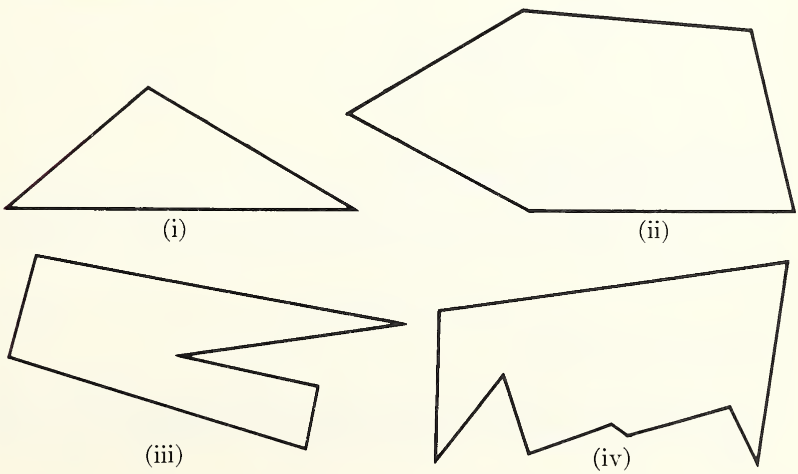
(i) (ii) (iii)

For (i) the measure is $1+1+1\frac{1}{4}=3\frac{1}{4}$.
The distance around is $3\frac{1}{4}$ inches.
The distance around is the sum of the length of its sides. Now find the distances around polygons (ii) and (iii).

2 You have just found the distance around three different polygons. You have found the **perimeter** of these polygons.

3 The perimeter of a polygon is the distance around the polygon. It is the sum of the length of its sides.

4. Find the perimeters of the polygons below. Use a ruler and measure to the nearest $\frac{1}{8}$ inch.



(i) (ii) (iii) (iv)

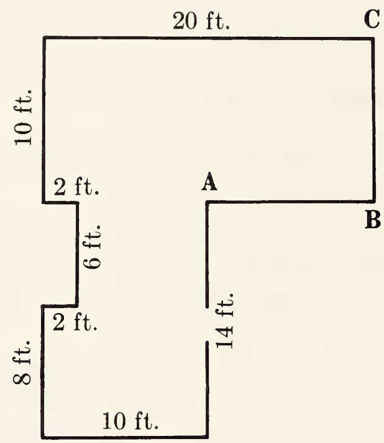
5. The sides of a quadrilateral have lengths of 5.6 in., 4.9 in., 3.3 in., and 5.8 in. What is its perimeter?

6. An octagon has all its sides of the same length. The sides have lengths of $5\frac{3}{16}$ in. What is the perimeter of the octagon?

7. One side of a triangle is 5 inches long. It is 4 inches shorter than one of its adjacent sides. If the perimeter of the triangle is 25 inches, what are the lengths of its sides?

8. The perimeter of a hexagon whose sides are of equal length is $122\frac{2}{5}$ inches. What is the length of each side?

9. Below is a diagram of a room.



a What is the length of $\overline{AB}$? $\overline{BC}$?

b Find the perimeter of the room.

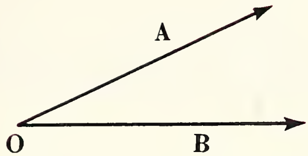
c What would it cost for footboard for this room, if footboard costs 19¢ a foot? (There are two doorways into the room each of which is 3 feet wide.)

10. The sides of a field measure 648 yd., 1,309 yd., 596 yd., and 1,583 yd. What is its perimeter?

11. A room is 18 ft. long and 13 ft. 6 in. wide. What is its perimeter? How much would it cost for a border around this room if border costs 9¢ a foot?

THE MEASURE OF AN ANGLE

1

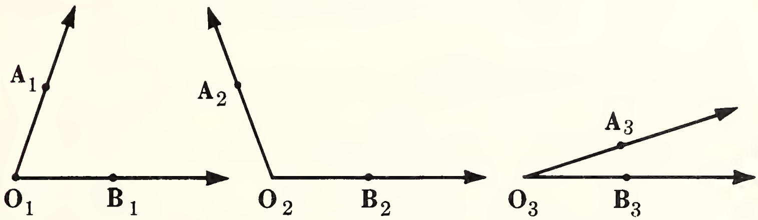


a What kind of geometric figure is $\overrightarrow{OA}$?

b What kind of geometric figure is $\overrightarrow{OB}$?

c Is it possible to find a length for $\angle AOB$? Why not?

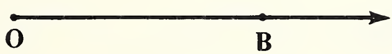
2 We can see that it does not make sense to try to measure the size of angles by finding their lengths. We need to find some way of measuring angles so that we can compare their sizes.
Consider the angles below:



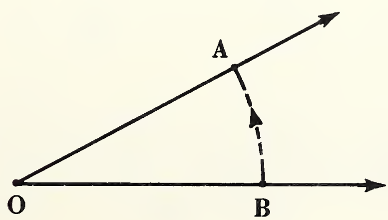
Which is the largest? Why do you think so?

3 One way in which we can think of angles is to take a ray and *rotate* it until we have the angle we want.

a Start with ray $\overrightarrow{OB}$

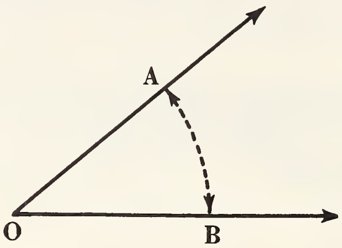


b Now rotate $\overrightarrow{OB}$ until it is in the same position as $\overrightarrow{OA}$. Keep O in a fixed position.



c If we can decide on some way of measuring the amount that $\overrightarrow{OB}$ must be rotated to get it into the same position as $\overrightarrow{OA}$, we shall have a measure of $\angle AOB$.

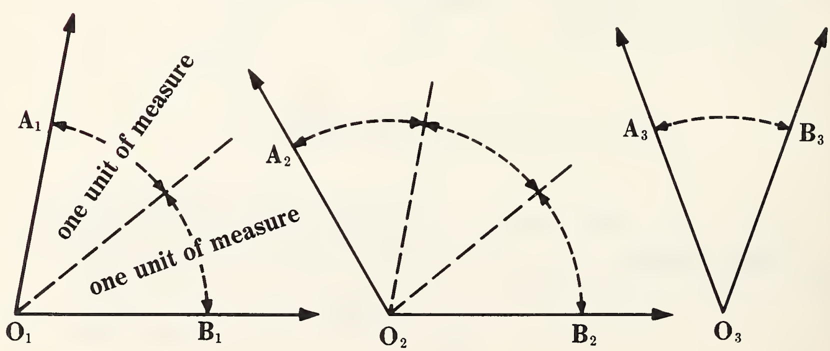
4 Let us fix a unit of measure. We shall call the amount that we rotate $\overrightarrow{OB}$ to get into the same position as $\overrightarrow{OA}$ *one unit of measurement for angles.*



This angle has a measure of 1.
Its size is 1 unit of measurement for angles. It is a *unit angle.*

5 Draw an angle like $\angle AOB$ above on a piece of paper. Cut it out. This will be our unit of measurement.

6 Now let us measure $\angle A_1O_1B_1$, $\angle A_2O_2B_2$, and $\angle A_3O_3B_3$ by seeing how many times we have to put the unit of measurement on them to "fill them up".

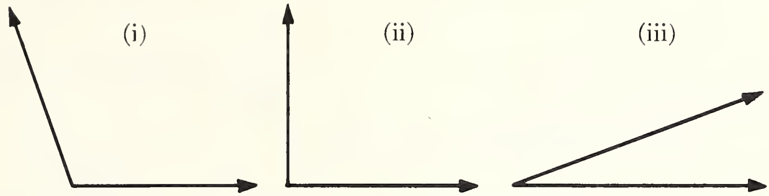


$\angle A_1O_1B_1$ is two units of measurement for angles. It is $2 \times$ the unit angle.

$\angle A_2O_2B_2$ is three units of measurement for angles. It is $3 \times$ the unit angle.

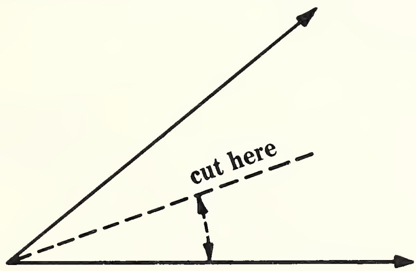
$\angle A_3O_3B_3$ is one unit of measurement for angles. It is $1 \times$ the unit angle.

7. Use the unit angle you have made to find the measure of the angles below. Measure the angles to the nearest whole unit.

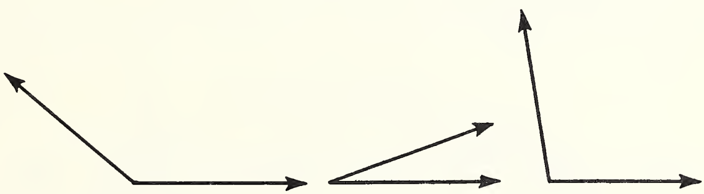


8. What did you notice when you were trying to find the measure of the angle in diagram (iii)?

9. Cut out another unit angle, and then cut it into two pieces to make half units, like this:

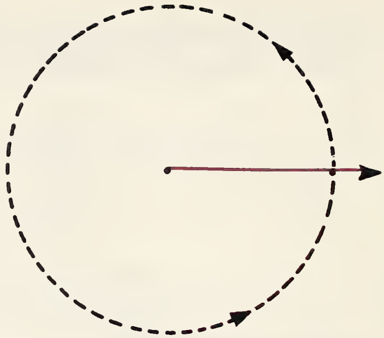


10. Use your unit angle and your half-unit angle, to find the measures of the angles below:



11 Suppose you wanted to tell someone the measure of these angles, what difficulty would you have? What would we have to agree on before anyone would know what you meant when you said, "An angle has a measure of 3; its size is 3 units of measure for angles"?
We need a **standard unit** of measure for angles. The standard unit is called a **degree**. People have agreed for many years to call $\frac{1}{360}$ of a complete revolution, 1 degree.

Start in this position.

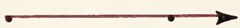


Rotate back to the starting position.

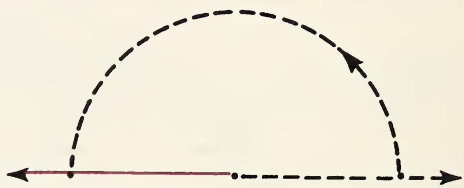
The ray has been rotated through an angle of 360 degrees.

12 Study the diagrams below:

a



Start in this position.

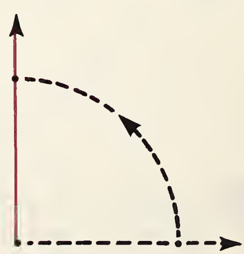


Rotate to this position.

The ray has made $\frac{1}{2}$ of a complete revolution. It has been rotated through 180 degrees.

b

Start in this position.



Rotate to this position.

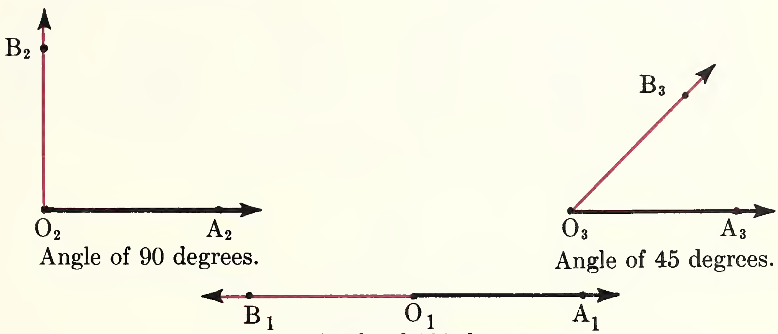
The ray has made $\frac{1}{4}$ of a complete rotation. It has been rotated through 90 degrees.

c

Start in this position. Rotate to this position.

The ray has made $\frac{1}{8}$ of a complete rotation. It has been rotated through 45 degrees.

Now see the reason for giving the angles below the measures that are written beneath them.

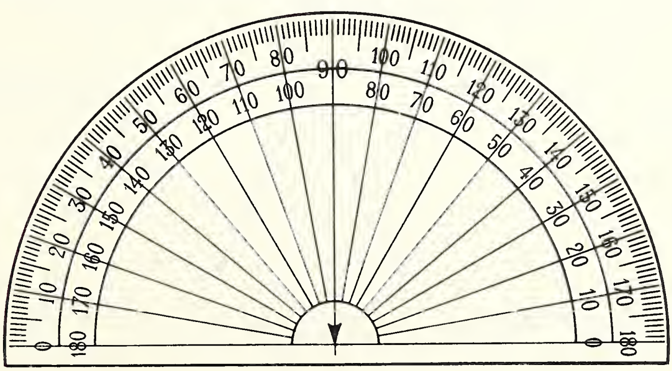


Angle of 90 degrees. Angle of 45 degrees.

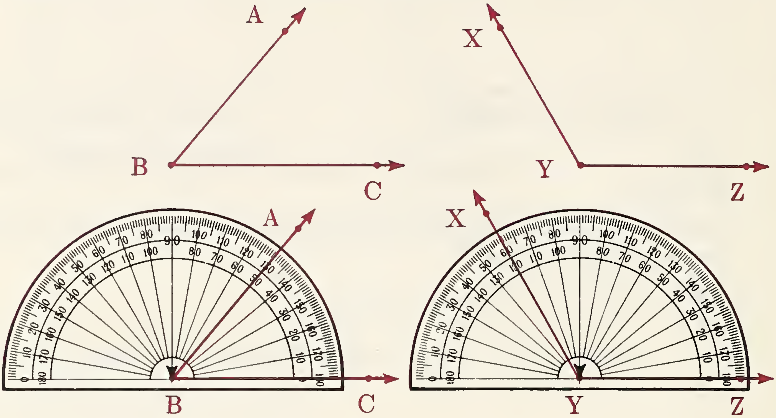
Angle of 180 degrees.

13 We have a symbol that we use to represent *degrees*. It is (°). For 45 degrees, we can write 45°. What can we write for 90 degrees? 180 degrees?

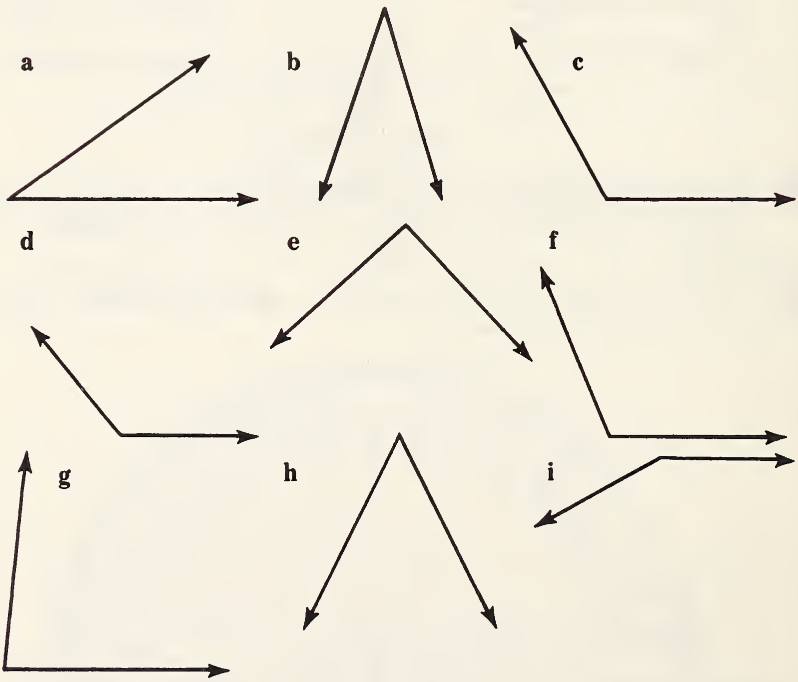
14 To measure angles, we use an instrument called a **protractor.** Here is an illustration of a protractor.



15 Study the diagrams below. They show us how to use a protractor to measure the sizes of $\angle ABC$ and $\angle XYZ$.



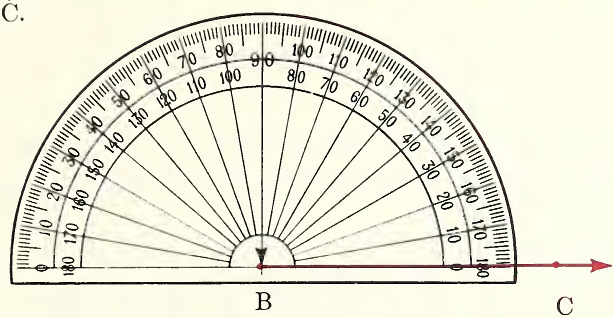
16. Use a protractor to measure the angles drawn below:



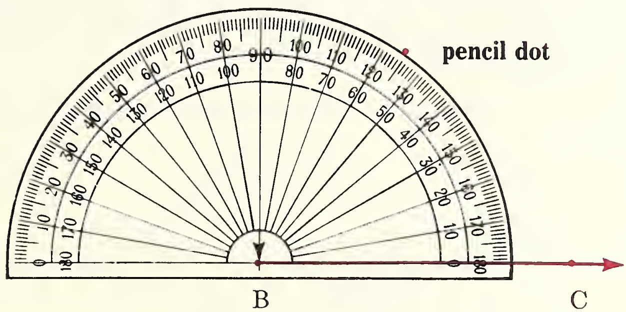
17. Use a protractor to construct an angle of 55°.

a Draw a ray BC.

b Put the arrow of the protractor on B and the base of the protractor on $\overrightarrow{BC}$.

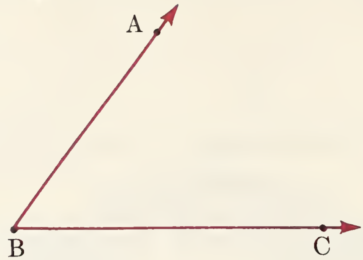


c Put a dot next to the 55° mark.



d Remove the protractor.

e Draw a ray joining the point B to the pencil dot. Name this ray $\overrightarrow{BA}$.



18 a Draw angles of 45°, 75°, 80°, 15°, 85°.

These angles are all less than 90°. We call angles that are less than 90° **acute angles.**

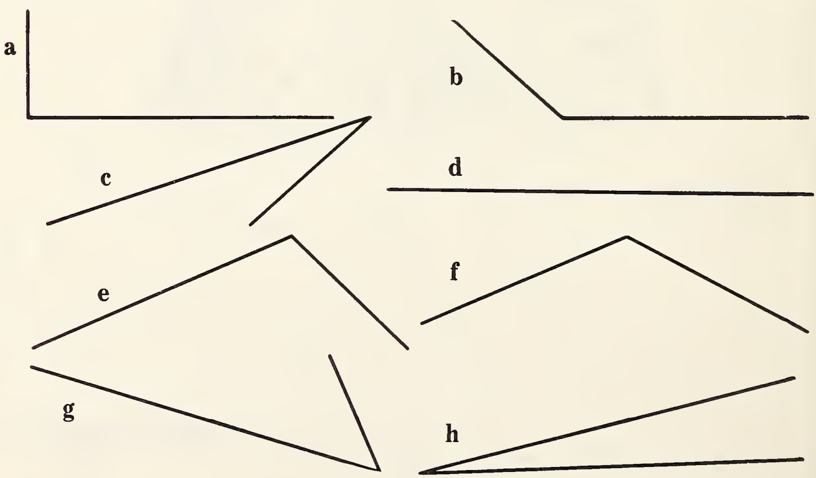
b Draw an angle of 90°.

We call an angle of 90° a **right angle.**

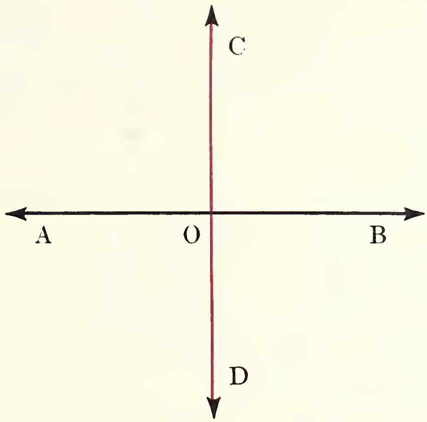
c Draw an angle twice as large as a right angle. What is the size of this angle? How many degrees does it contain? How else could you describe this angle? We call an angle of 180° a **straight angle.**

d Draw angles of 110°, 165°, 150°, 170°, 95°. These angles are all more than 90° but less than 180°. We call angles between 90° and 180° **obtuse angles.**

19. Measure the angles below. Give their size and state the type of angle each one is.



20 Consider the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. They intersect each other at O.



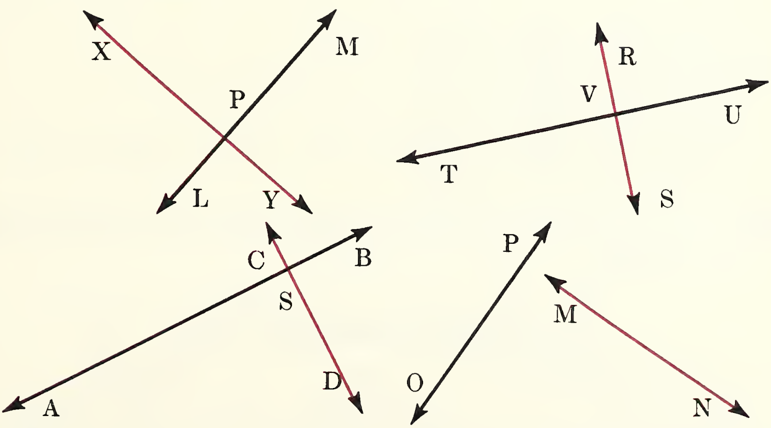
Measure angles COB, DOA, BOD, and AOC.
What do you notice?
We say:

a $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are perpendicular to each other;

or **b** $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$;

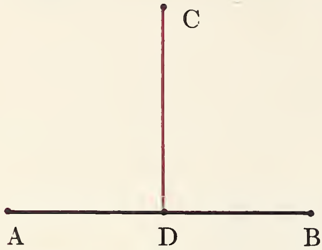
or **c** $\overleftrightarrow{CD}$ is perpendicular to $\overleftrightarrow{AB}$.

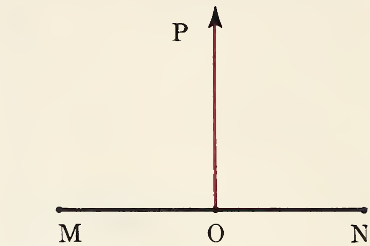
21. Study the diagrams below:



Which of these pairs of lines are perpendicular to each other? How do you know?

22 We can also have line segments and rays perpendicular to each other.



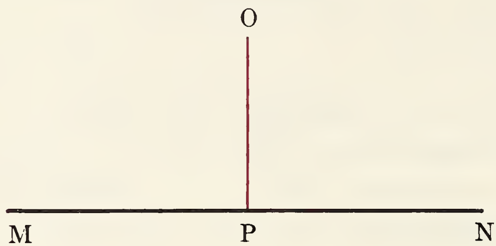


$\overline{CD}$ is perpendicular to $\overline{AB}$
$\overline{AB}$ is perpendicular to $\overline{CD}$

$\overrightarrow{OP}$ is perpendicular to $\overline{MN}$
$\overline{MN}$ is perpendicular to $\overrightarrow{OP}$

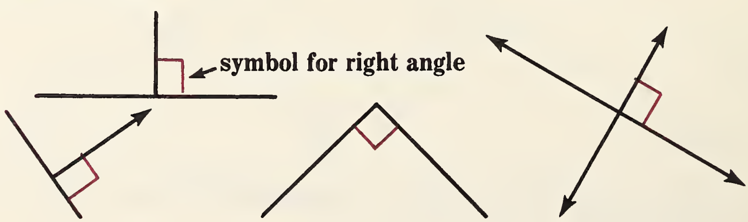
23 A symbol that stands for perpendicular is $\perp$.

24. Draw line segments $\overline{MN}$ and $\overline{OP}$ $\perp$ to each other at P.



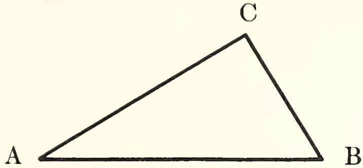
25. **a** Draw $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ $\perp$ at O.
    b Draw $\overline{EF}$ and $\overrightarrow{MN}$ $\perp$ at S.
    c Draw $\overleftrightarrow{XY}$ and $\overleftrightarrow{LM}$ $\perp$ at S.
    d Draw $\overline{UV}$ and $\overline{WV}$ $\perp$ at V.
    e Draw $\overleftrightarrow{AB}$ and $\overrightarrow{BC}$ $\perp$ at B.

26 We have a symbol that we use for a right angle when we make geometrical drawings.

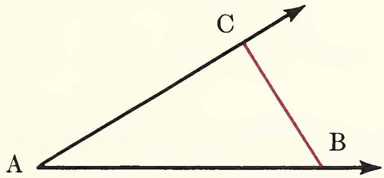


SOME SPECIAL POLYGONS

1 We have now learned how to measure the size of angles and how to draw angles of varying sizes. Let us consider a triangle. Does a triangle contain any angles? (Remember: an angle is formed by two rays.) With polygons, we consider that the sides are rays and that the corners are angles.

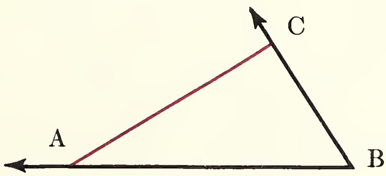


To make an angle at A, we think of the triangle as:

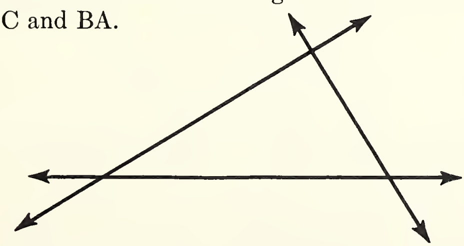


The sides AC and AB are thought of as rays AC and AB.

To make an angle at B we think of the triangle as:

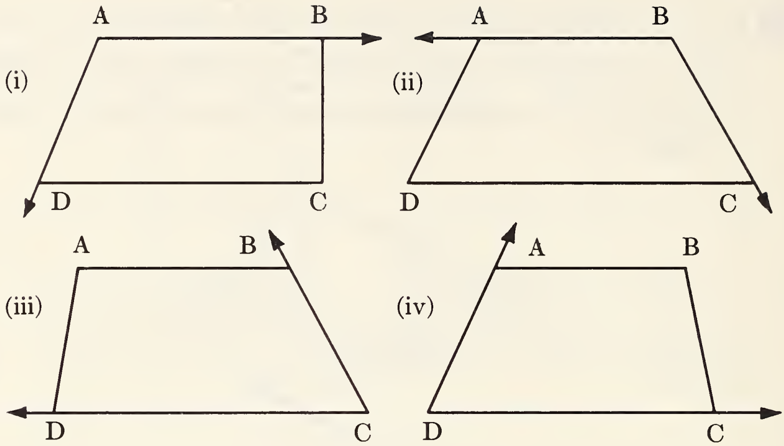


Now the sides BC and BA are thought of as rays BC and BA.



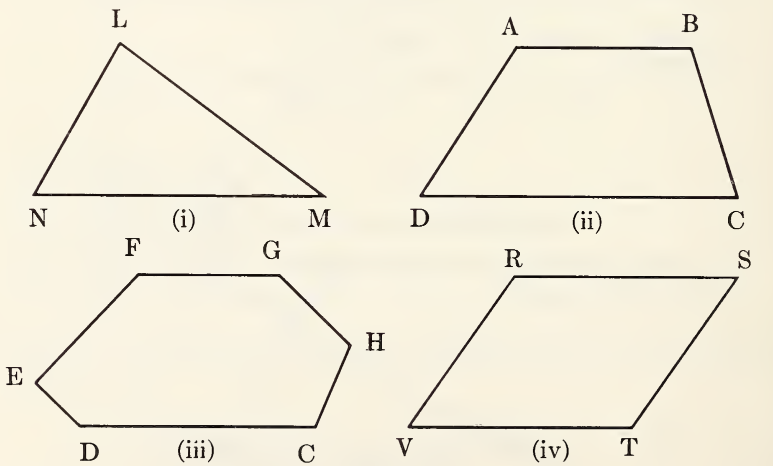
We make the following kind of agreement or convention about polygons. We think of each pair of adjacent sides as being rays so that each corner can be considered as an angle.

2. Study the figures:



Which *corner* do we consider as an angle in figure (i)? (ii)? (iv)?

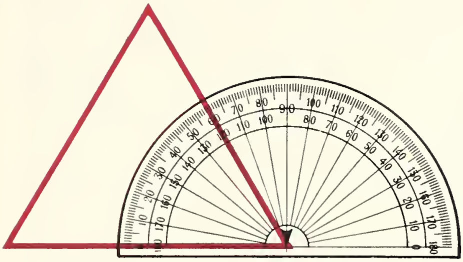
3. Copy the polygons below:

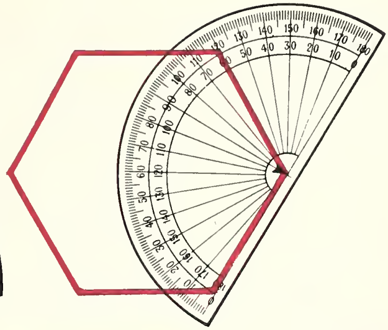


Indicate the adjacent sides that we must extend into rays to make the following angles:

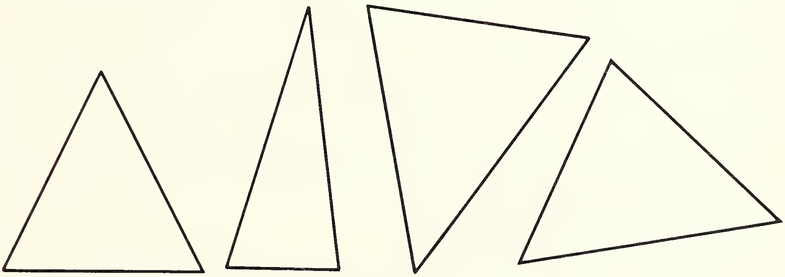
a	∠LNM	**b**	∠ABC	**c**	∠GHC
d	∠RST	**e**	∠EDC	**f**	∠DCH

4 Study the diagrams below: (They help us to see how to measure the size of the angles of polygons.)



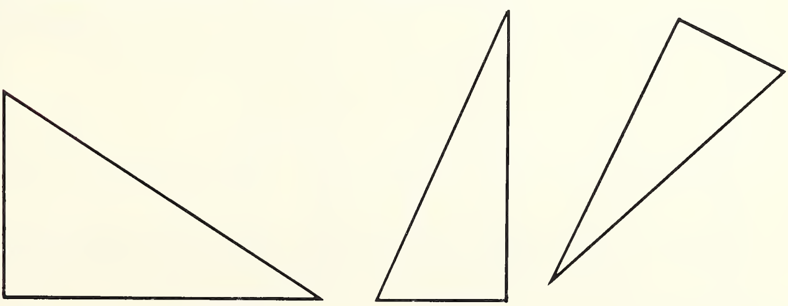


5. Measure the angles of each of the following triangles:



6 When a triangle has three acute angles, we call it an **acute triangle**.

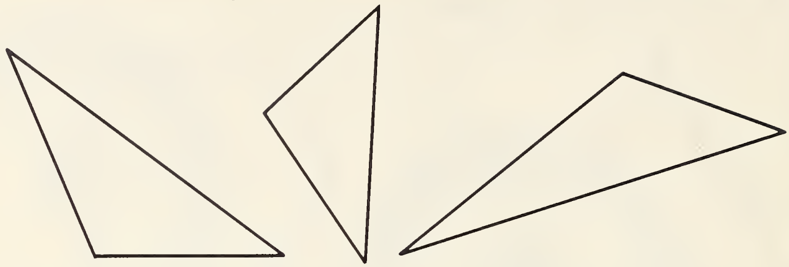
7. Measure the angles of each of these triangles:



One angle in the first triangle is the same size as one of the angles in each of the other two. What do we call an angle of 90°?

8 When a triangle has one right angle, we call it a **right triangle**.

9. Measure the largest angle in each of these triangles:



What do we call an angle that is greater than 90°?

10 A triangle having one obtuse angle is called an **obtuse triangle**.

11.

ABCD is a quadrilateral.

$\overline{AB}$ and $\overline{BC}$ are adjacent sides.

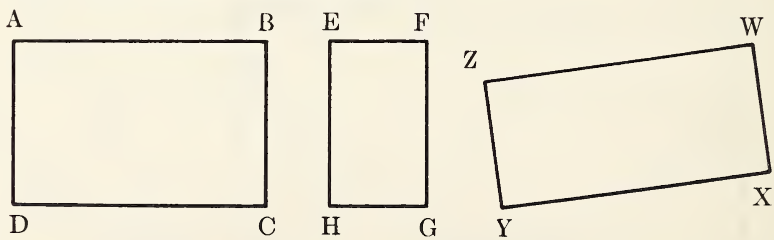
$\overline{AD}$ and $\overline{DC}$ are adjacent sides.

Name two other pairs of adjacent sides.

$\overline{AB}$ and $\overline{CD}$ are **opposite** sides.

Name another pair of opposite sides.

12.

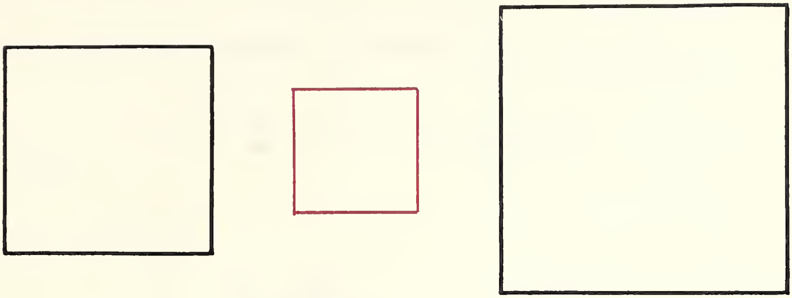


Measure all the angles in the quadrilaterals above. What do you notice? Measure the lengths of the sides of the quadrilaterals. What do you notice about the lengths of *opposite* sides?

13 If the four angles of a quadrilateral are all right angles, the quadrilateral is called a **rectangle**.

The opposite sides of a rectangle are equal in length.

14. Are the four angles of these quadrilaterals all right angles?

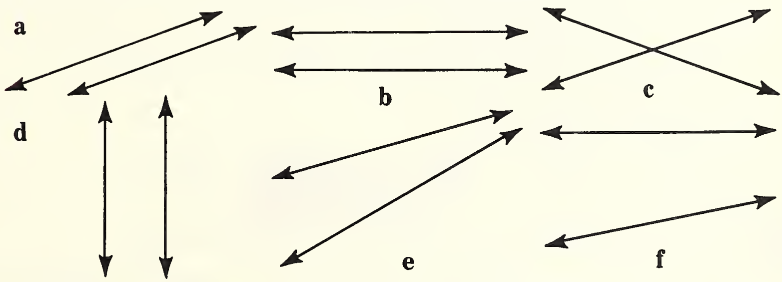


Are these figures rectangles?

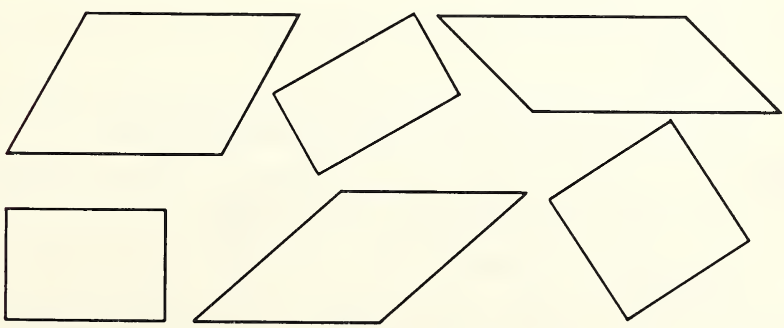
Measure the lengths of the sides of each of these rectangles. What do you notice?

15 Rectangles having all their sides equal in length are called **squares**.

16. Which of the following pairs of lines are parallel?



17. Study these quadrilaterals:



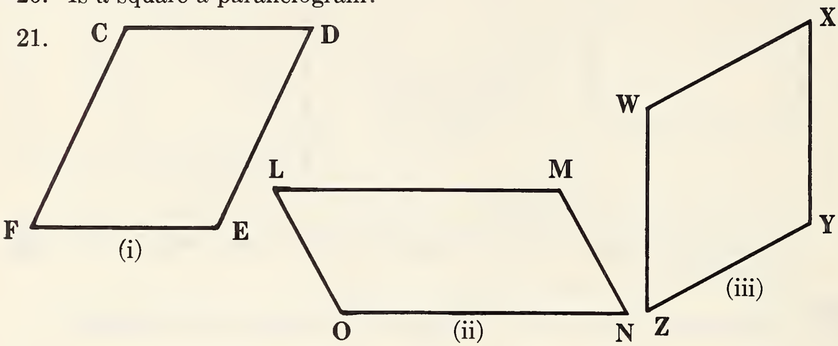
Are the *opposite* sides of each quadrilateral above parallel?

18 Quadrilaterals having both pairs of opposite sides parallel are called **parallelograms.**

19. Are both pairs of opposite sides of a rectangle parallel? Is a rectangle a parallelogram? Why?

20. Is a square a parallelogram?

21.



a Are the above polygons quadrilaterals?
Are they parallelograms?

b In fig. (i) measure $\overline{CF}$ and $\overline{DE}$. What do you notice?
Now measure $\overline{CD}$ and $\overline{FE}$.
What do you notice?

c $\overline{LM} = ?$ $\overline{ON} = ?$
$\overline{LO} = ?$ $\overline{MN} = ?$

d $\overline{WX} = ?$ $\overline{ZY} = ?$
$\overline{WZ} = ?$ $\overline{XY} = ?$

22 The opposite sides of parallelograms are equal in length.

23. Two sides of a parallelogram measure 16 in. and 12 in. respectively. What is its perimeter?

24. A square has a side of 3 ft. 9 in. What is the measure of its perimeter in yards?

25. One side of a rectangle measures 15 in. Its perimeter is 4 ft. 2 in. What are the lengths of the other sides of the rectangle?

26 In a rectangle, the length of each shorter side is called the **width** and the length of each longer side is called the **length.**

27. The length of a rectangle is 28 in. Its width is $\frac{1}{2}$ yd. What is its perimeter?

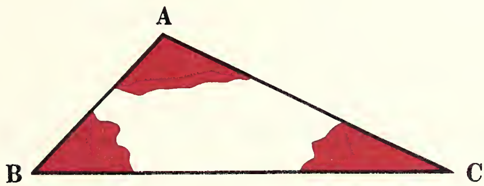
28. A square has a perimeter of 3 yd. 1 ft. 8 in. What is the length of its side?

THE ANGLES OF A TRIANGLE

1. On a sheet of stiff paper draw six triangles that are not alike.

2. Cut out the triangles.

3. Take each triangle in turn and tear off the corners.



4. Fit the three corners of each triangle together so that the vertices meet in a point.

5. What do you notice in each case?

6. Draw six more triangles that are not alike. Label each triangle ABC.

7. Use a protractor to measure the angles A, B and C.

8. What is the sum of the sizes of the angles for each triangle?

9 Our experiments and exercises lead us to believe that the sum of the measures of the angles of a triangle is 180°. This can be proved. We have not proved it; we have demonstrated that **In any triangle the sum of the measures of the angles is 180°.**

10. In triangle XYZ, $\angle ZXY = 45°$, $\angle YZX = 76°$. What is the size of $\angle XYZ$?

a Draw a rough sketch.

b Mark in the sizes of $\angle ZXY$ and $\angle YZX$.

c Let the size of $\angle XYZ$ be $y°$.

d What do we know of the sum of the sizes of the three angles of triangle XYZ?

e Make an equation showing this

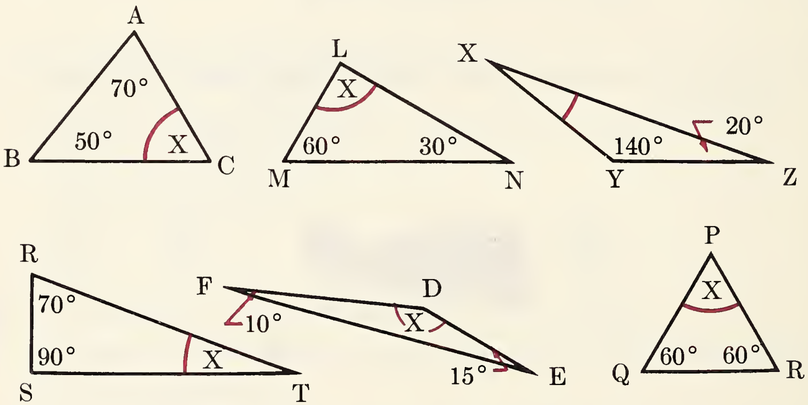
$45° + 76° + y° = 180°$

so $121° + y° = 180°$

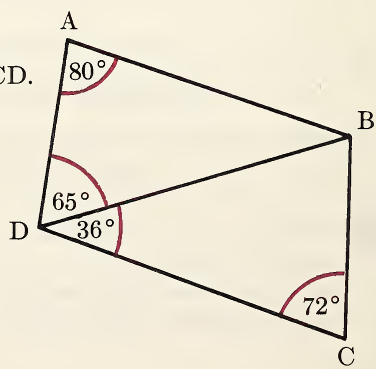
and $y° = 59°$ (Why?)

f $\angle XYZ$ contains $59°$.

11. Find the sizes of the angles marked 'X' in the triangles below: (Do not use a protractor.)



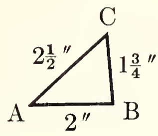
12. Without using a protractor, find the size of $\angle ABC$ in the figure ABCD.



CONSTRUCTING TRIANGLES

1 Draw triangle ABC in which $\overline{AB}=2''$, $\overline{BC}=1\frac{3}{4}''$ and $\overline{CA}=2\frac{1}{2}''$.

a Draw a rough diagram.

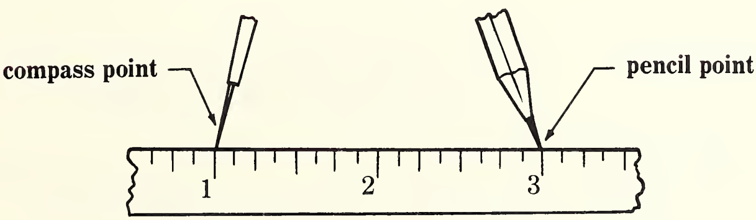


b Let us think of the side that is $2''$ long. Draw a line segment that is slightly more than $2''$ long:

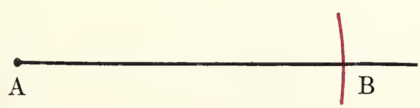
c Name the point at one end of the segment A.

d How can we find a point which we can name as B? (Remember $\overline{AB}=2''$) We can use a ruler or we can use a compass.

e Set the compass so that the pencil point and the compass point are two inches apart.

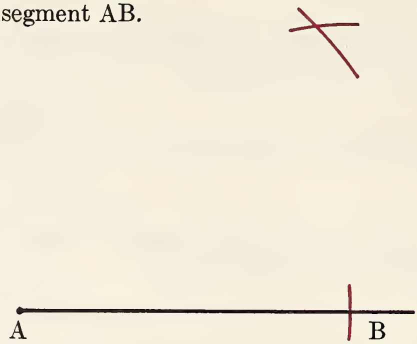


f With the compass point on A, make a mark cutting the line segment. Why is the point where the mark cuts the line segment $2''$ from A? Why can we label this point B? Will $\overline{AB}=2''$?



g We now have to locate point C. How far is point C from point A? (Remember $\overline{CA}=2\frac{1}{2}''$.)

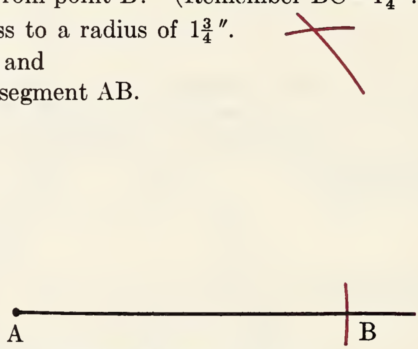
h Set the compass now to a radius of $2\frac{1}{2}''$. Put the point on A and draw a mark above segment AB.



How far is every point on this mark from point A?

i How far is point C from point B? (Remember $BC = 1\frac{3}{4}''$.)

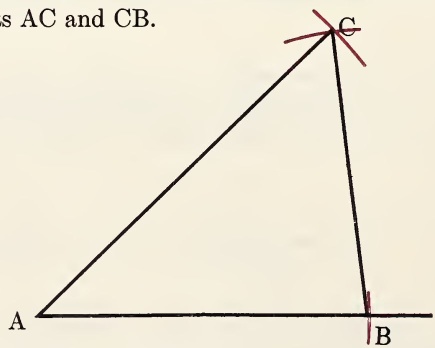
j Now set the compass to a radius of $1\frac{3}{4}''$.
Put the point on B and
draw a mark above segment AB.



How far is every point on this mark from point B?

k Which point will be $2\frac{1}{2}''$ from point A and also $1\frac{3}{4}''$ from point B? Name the point where these two marks cut each other, point C.

l Draw line segments AC and CB.



The required triangle is triangle ABC.

2. Make a drawing of a triangle with $\overline{AB}=2\frac{3}{4}''$, $\overline{AC}=3''$ and $\overline{BC}=2''$.

3. Construct triangle LMN so that $\overline{LM}=3''$, $\overline{LN}=3''$ and $\overline{MN}=2''$. Measure its angles. What do you notice?

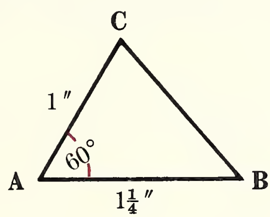
4. In triangle XYZ, $\overline{XY}=2''$, $\overline{YZ}=2\frac{1}{2}''$ and $\overline{XZ}=1\frac{1}{2}''$. Measure its angles. What kind of triangle is it?

5. Draw triangle RST so that $\overline{RS}=1.0''$, $\overline{ST}=1.2''$ and $\overline{TR}=1.3''$. What kind of triangle is it?

6. Is is possible to construct a triangle with sides $3''$, $2''$ and $\frac{3}{4}''$ long? Why?

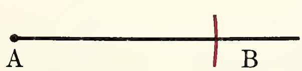
7. Draw triangle ABC in which $\overline{AB}=1\frac{1}{4}''$, $\angle BAC=60°$, $\overline{AC}=1''$.

a Make a rough sketch.

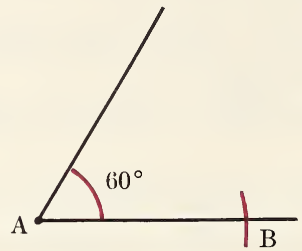


b Draw a line segment slightly more than $1\frac{1}{4}''$ long. Name the point at one end of the segment, point A.

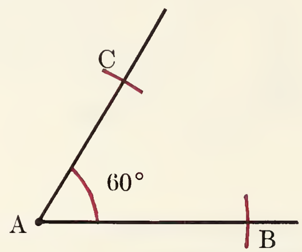
c Use a compass to mark off a length equal to $1\frac{1}{4}''$. Name the point where the mark cuts this segment, point B.



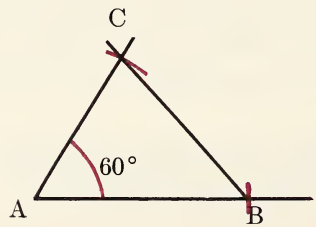
d At point A construct, using your protractor, an angle equal to 60°. Extend the side so that it is slightly more than 1″ in length.



e Use your compass to mark off a length equal to 1″ on the segment you have last drawn. Name the point where the mark cuts this segment, point C.



f Join CB.



ABC is the required triangle.

8. Construct triangle ABC given $\overline{AB} = 2''$, $\angle BAC = 45°$, $\overline{BC} = 3''$.

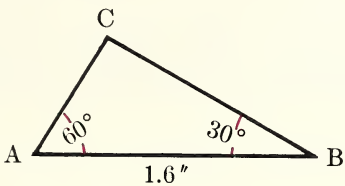
9. Construct triangle LMN given $\overline{LM} = 1\frac{1}{2}''$, $\angle MLN = 110°$, $\overline{MN} = 2\frac{1}{2}''$.

10. Draw triangle XYZ so that $\overline{XY}=3''$, $\angle ZXY=60°$, $\overline{YZ}=3''$. Measure all the angles and sides of this triangle. What do you notice?

11. Construct triangle PQR given:
$\overline{PQ}=2''$, $\angle QPR=90°$, $\overline{QR}=2''$.
Measure the angles of the triangle. What do you notice?

12 Draw triangle ABC in which $\overline{AB}=1.6''$, $\angle CAB=60°$, $\angle ABC=30°$.

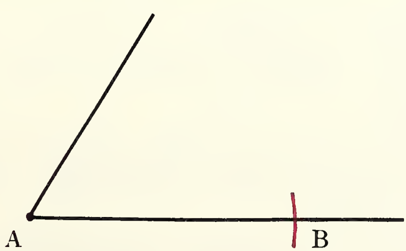
a Make a rough sketch.



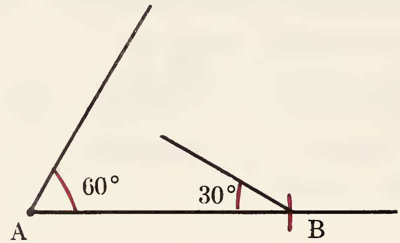
b Draw a line segment slightly longer than 1.6″. Name one of its endpoints, point A.

c Use your compass, set at 1.6″, to mark off a length of 1.6″. How does this help you to locate point B?

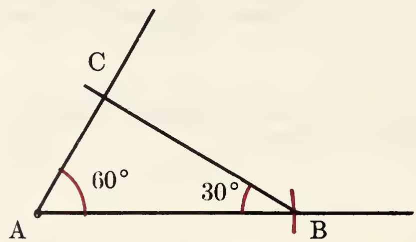
d Use your protractor to construct an angle equal to 60° at A.



e Use your protractor to construct an angle equal to 30° at B.



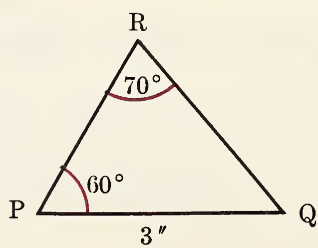
f Extend the sides of angles A and B until they intersect.
Why can we label the point of intersection of these two segments, point C?



ABC is the required triangle.

13. In triangle PQR, $\overline{PQ} = 3''$, $\angle RPQ = 60°$, $\angle QRP = 70°$. Draw the triangle.

a Make a rough sketch:



b What is the measure of the sum of the angles of a triangle? What is the measure of the sum of angles RPQ and PRQ? How can you find the measure of $\angle RQP$?

c Now that you have the measure of $\angle PQR$, construct the triangle.

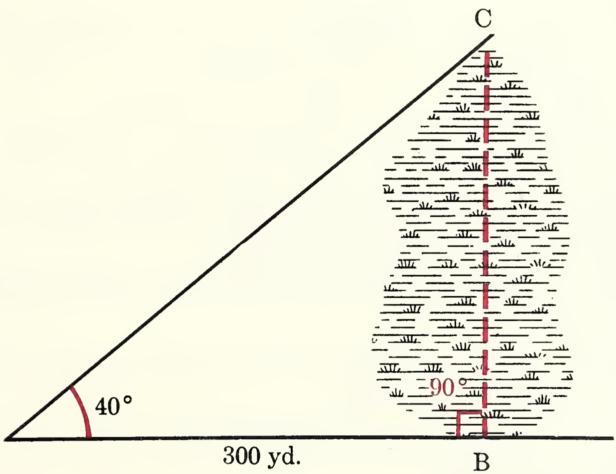
14. Make a drawing of triangle RST so that $\overline{RS} = 3''$, $\angle RST = 60°$, $\angle STR = 60°$. Measure all the angles and sides. What do you notice?

15. Construct triangle ABC so that $\overline{AB} = 2\frac{3}{4}''$, $\angle BAC = 60°$, $\angle ABC = 30°$.

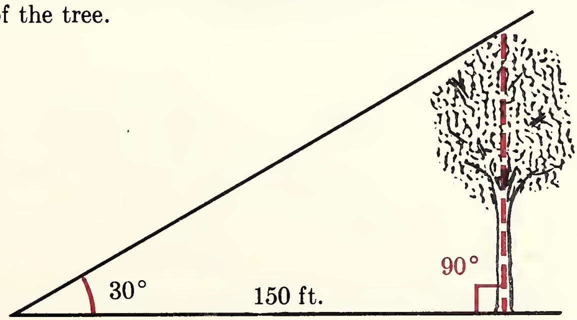
16. Draw triangle XYZ so that $\overline{XY} = 2\frac{1}{2}''$, $\angle ZXY = 45°$, $\angle XYZ = 45°$. Measure all the angles and sides. What kind of triangle is this?

17. Draw triangle PQR so that $\overline{PQ} = 1.8''$, $\angle RPQ = 40°$, $\angle QRP = 60°$.

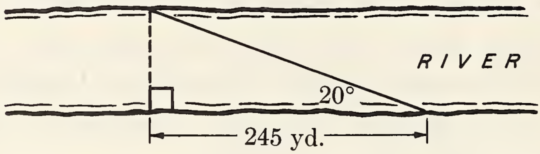
18. In the drawing below, you see the measurements made to find the distance across a swamp. Make a scale drawing, $1''$ to represent 80 yd., and find the distance $\overline{BC}$.



19. To find the height of a tall tree, the measurements shown below were made. Make a scale drawing, using $1''$ to represent 50 ft., to find the height of the tree.

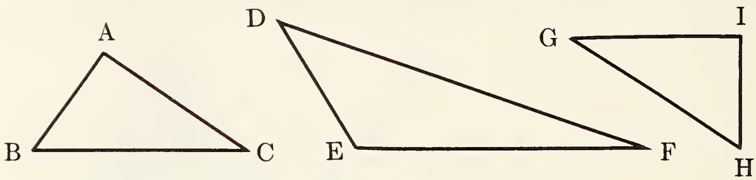


20. To find the distance across a river, the measurements shown below were made. Make a scale drawing, using a scale 1″ to represent 100 ft., to find the width of the river.



CLASSIFICATION OF TRIANGLES BY SIDES

1.



Measure the lengths of the sides of the triangles above.
What do you notice about the lengths of the sides of triangle ABC?
Are any two sides of the same length?
What do you notice about the lengths of the sides of triangle DEF? of triangle GHI?

2 When the sides of a triangle are all of different lengths, the triangle is said to be a **scalene** triangle.

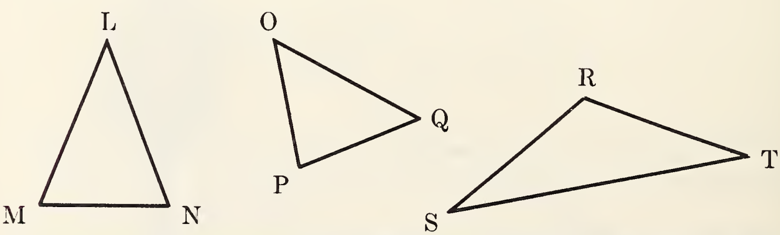
3. Which of the three scalene triangles above is:

a an acute triangle?

b an obtuse triangle?

c a right triangle?

4. Measure the sides of the triangles below.



What do you notice about the lengths of two of the sides in each triangle?

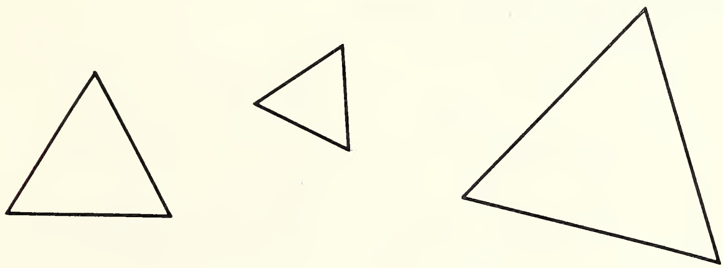
5 When two of the sides of a triangle are of equal length, the triangle is said to be an **isosceles** triangle. (The word 'isosceles' means "two-legged" and comes to us from the Greek language.)

6. In the triangles above, measure **a** $\angle$LMN and $\angle$LNM
   b $\angle$POQ and $\angle$PQO
   c $\angle$RST and $\angle$RTS

   What do you notice? What are you led to believe about the measures of two of the angles of an isosceles triangle?

7. Draw four isosceles triangles. Measure the angles of each triangle. Does the rule you have formulated apply to each of these triangles?

8. Measure the sides of the triangles below.



9 When all three sides of a triangle are equal in length, the triangle is said to be an **equilateral** triangle.

10. Measure the angles of the three equilateral triangles above. What do you notice?

11. We have now learned two ways of classifying triangles: by angles and by sides.

12. What is a right triangle?

13. What is an obtuse triangle?

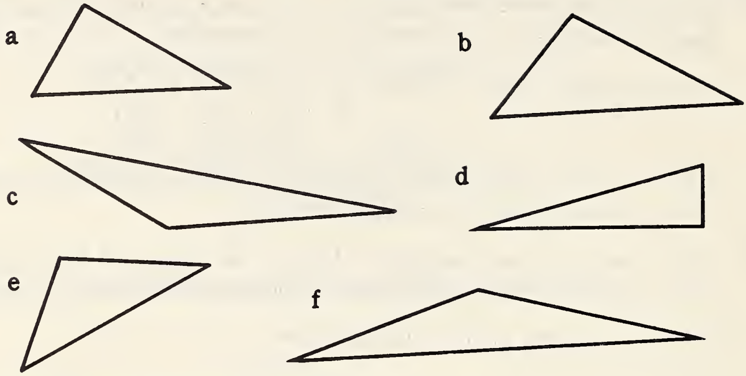
14. What is an acute triangle?

15. What is an isosceles triangle?

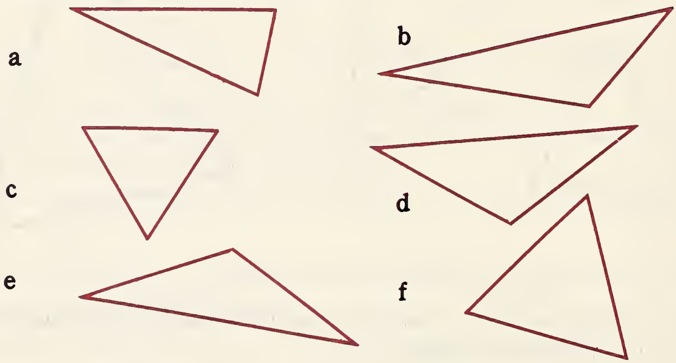
16. What is an scalene triangle?

17. What is an equilateral triangle?

18. Classify these triangles by angles.



19. Classify these triangles by sides.



20. Draw a right triangle.

21. Draw an isosceles triangle.

22. Draw an obtuse triangle.

23. Draw an equilateral triangle.

24. Can a right triangle be scalene?

25. Can a right triangle be equilateral?

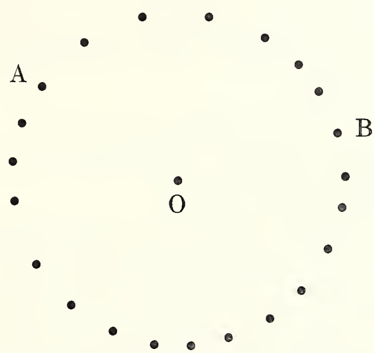
26. Can a right triangle be isosceles?

27. Draw an obtuse isosceles triangle.

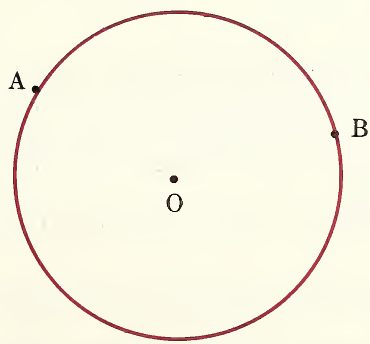
28. Draw an obtuse scalene triangle.

THE CIRCLE

1. a Draw a point, O, approximately in the middle of your paper.
   b Use your ruler to find another point that is 1″ from O. Label it A.
   c Now use your ruler to find a point, different from A, that is 1″ from O. Label it B.
   d Find another twenty points that are 1″ from O.
   e Here is how your figure may look:



   f How many points 1″ from O is it possible to find?
   g Can you count the number of points in the set of all the points 1″ from O?
   h What instrument can you use to draw a model of all the points 1″ from O?
   i If we use a compass, we can put the point of it on O and draw a figure like the one below.



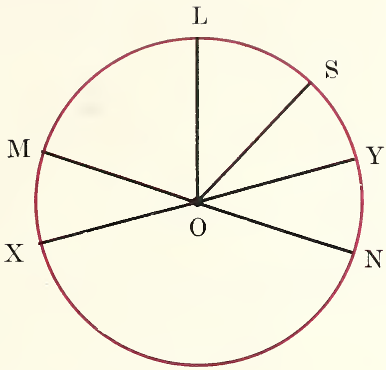
   j Does O belong to the set of points 1″ from O?

2. **a** Draw a point, Q, near the middle of your page.
   b Use your compass to draw a model of the figure made up of all the points that are 3″ from Q.
   c Draw a line segment with Q as one endpoint and any point in the figure you have just drawn as another endpoint. What is its length?
   d Draw another segment from Q to a point in the set you have drawn. What is its length?
   e Why would you expect the two segments that you have drawn to have the same length?
   f Can you state what the lengths will be of all the segments having Q as one endpoint and a point in the set drawn as another endpoint? Why should this be so?

3. **a** Draw a point V, near the middle of your page.
   b Use your compass to draw a model of the set of points made up of all the points that are 2″ from V.
   c Choose any two points in the set of points you have drawn. Use a ruler to join two of the points on the set with a segment that does not contain V. What is the length of this segment?
   d Now join three other pairs of points on the set with segments that do not contain V. Use a ruler to join these points with a segment. Measure the segments having these pairs of points as endpoints. What do you notice about these lengths?
   e Now draw a segment that contains V and has endpoints in the set you have drawn. What is its length?
   f Draw 4 more segments through V having endpoints in the set you have drawn. Measure them. How do these five segments compare with those that do not contain V?
   g Are the segments that contain V the longest ones you can draw which have points in the set as endpoints?

4 By now we should have discovered these things:
   a The set of points made up of all the points in that plane equidistant from a fixed point in that plane, is called a **circle**.
   b The fixed point is called the **centre** of the circle. The centre is *not* part of the circle.
   c The distance of each point on a circle from the centre of the circle is the same.
   d Each segment from the centre of a circle to a point on the circle is called a **radius**. (The plural of radius is **radii**.)
   e The longest segment that can be drawn having two points on the circle as endpoints passes through the centre of the circle.

f We call a segment containing the centre and having endpoints on the circle a **diameter.**



g Copy and complete:

(i) $\overline{OL}$ is a ______ of the circle.

(ii) $\overline{MN}$ is a ______ of the circle.

(iii) ______ names the centre of the circle.

(iv) ______ and ______ are two radii.

(v) $\overline{OY}$ is a ______.
$\overline{OX}$ is a ______.
$\overline{XY}$ is a ______.

(vi) Since the length of $\overline{OX}$ = the length of $\overline{OY}$ = the length of the radius, $\overline{XY}$ is twice the length of a ______.

(vii) The length of the diameter of a circle is twice the length of a ______ of a circle.

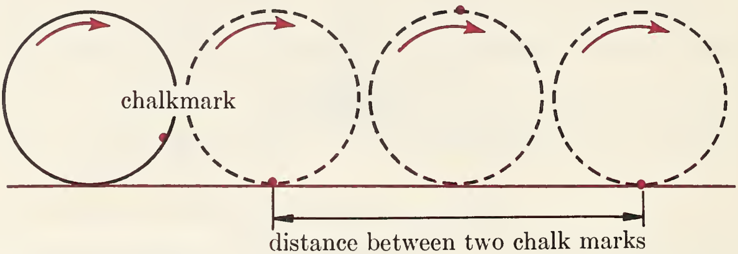
5 **a** We have learned that the perimeter of a polygon is the distance around the polygon. We found this by finding the sum of the lengths of its sides. What do we mean by the perimeter of a circle? Does a circle have line segments as its boundary?

b A circle does not have line segments as its boundary; so we cannot find the measure of its sides and add them to find the perimeter. We think of the "distance around a circle". We have a special name for *distance around a circle.* We call it the **circumference** of the circle.

6. **a** Take a piece of stiff cardboard and on it draw a circle with a diameter 6″ long.

b Cut out the circle. Place a string around the circle and then straighten the string out and measure its length.

c Check the distance around the cardboard circle by putting a chalk mark on the circle, and then rolling the circular piece of cardboard along a flat surface and measuring the distance between the two chalk marks on the flat surface. (Be careful that the circle does not slip when it is being rolled.)



d Do the same kind of measuring with circles cut from cardboard having diameters of 1″, 2″, 4″, and 5″ respectively.

e Take some circular objects such as a jar, a paint can, a hoop, etc. Measure the diameters of these objects and then measure the distance around the circular part.

f Draw up a table similar to the one below and do the computation in column (iii).

	(i) Diameter of circle	(ii) Distance around circle	(iii) $\frac{\text{Distance around circle}}{\text{Diameter of circle}}$
e.g	8″	25″	$\frac{25}{8}$

What is the value of the ratio between the distance around the circle and the diameter of the circle?

7 **a** If you have done your measuring accurately in Exercise 6, you will have found that:

$$\frac{\textbf{distance around circle}}{\textbf{diameter of circle}} = \textbf{a number which is a little more than 3.}$$

b From this we can see that the distance around a circle is a little more than 3 times the diameter of the circle:

c Although we have only demonstrated this for a few circles, it is true for *all* circles that **the distance around a circle is about 3 times the diameter of the circle.**

d It is hard to see what the size of this number is which is a *little more than* 3. Later in mathematics you will learn that this number cannot be written as a fractional number; it cannot be written as a terminating decimal; it cannot be written as a repeating decimal.

e We know that this number can be written as 3.14159 . . . without end. The decimal part does not repeat or terminate. A more accurate measure of it would be 3.14159265 . . . .

f Because we cannot write this number as a fractional numeral, or a terminating decimal, or a repeating decimal, we name it in a special way; we name it with the Greek letter π. This letter is pronounced as '**Pi**'.

g In mathematics, we use an approximation for the value of π. We usually use the values of $3\frac{1}{7}$ or $\frac{22}{7}$ or 3.14.

h Since $\pi = 3.14159265$ . . . . show that $3\frac{1}{7} > \pi$ and that $\pi > 3.14$. The value of π lies *between* $3\frac{1}{7}$ and 3.14. $\pi < 3\frac{1}{7}$ and $\pi > 3.14$.

8 **a** We often use a formula to help us remember the relationship between the circumference of a circle and its diameter. Let C represent the measure of the circumference of a circle, and d represent the measure of the diameter of this circle. Then $\frac{C}{d} = \pi$.

b $\frac{C}{d} = \pi \quad C = d \times \pi.$ (Why?)

$C = d \times \pi \quad C = \pi \times d$ (Why?)

$\pi \times d$ can be written as πd.

We remember: $C = \pi d$

number of units of length in circumference of circle.

a number which $< 3\frac{1}{7}$ but > 3.14

number of units of length in diameter of circle.

9 We have seen that the diameter of a circle is twice as long as the radius of a circle, which has a diameter of d.

Then $d = 2 \times r$. We know that $C = \pi d$. For d we can write $2 \times r$; so $C = \pi d = \pi \times 2 \times r = 2 \times \pi \times r$. (Why?)

For $2 \times \pi \times r$ we can write $2\pi r$.

So $C = \pi d$ or $C = 2\pi r$.

10 Find the circumference of a circle whose diameter measures 5″.

a We know that $C = \pi d$.

For π, let us take the approximate value of 3.14.

Then $C \approx 3.14 \times d$

The symbol "$\approx$" means *is approximately*.

So $C \approx 3.14 \times 5 \approx 15.70$.

The circumference measures 15.70″, approximately.

b We know that for all circles the ratio $\frac{C}{d} = \frac{\pi}{1}$

Take 3.14 as an approximate value of π.

We know that $d = 5$;

so $\frac{C}{5} \approx \frac{3.14}{1}$

and $C \times 1 \approx 5 \times 3.14 \quad C \approx 15.70$.

The circumference measures 15.70″, approximately.

11 Find the circumference of a circle if its radius measures $3\frac{1}{2}$″.

a We know that $C = 2\pi r$.

For π take the approximate value of $3\frac{1}{7}$.

$$\begin{aligned} \text{Then } C &\approx 2 \times 3\tfrac{1}{7} \times 3\tfrac{1}{2} \\ &\approx 2 \times \tfrac{22}{7} \times \tfrac{7}{2} \\ &\approx 22 \end{aligned}$$

The circumference measures 22″, approximately.

b We know that $\frac{C}{d} = \frac{\pi}{1}$

But $d = 2r$; so $\frac{C}{2r} = \frac{\pi}{1}$

So, $\frac{C}{2 \times 3\frac{1}{2}} \approx \frac{3\frac{1}{7}}{1}$

$C \times 1 \approx 2 \times 3\frac{1}{2} \times 3\frac{1}{7}$

$C \approx \overset{1}{\cancel{2}} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}}} \times \frac{22}{\underset{1}{\cancel{7}}}$

$C = 22$

The circumference of the circle measures 22″, approximately.

12 The circumference of a circle measures 9.42″. What is the length of its diameter?

a $C = \pi d$. For π use 3.14

Then $9.42 \approx 3.14 \times d$

$3.14 \times d \approx 9.42$ (Why?)

$d = \frac{9.42}{3.14}$ (Why?)

$d \approx 3$

$$\begin{array}{r|l} 314\overline{)942} & 3 \\ \underline{942} & \end{array}$$

The diameter of the circle measures about 3″.

b $\frac{C}{d}=\frac{\pi}{1}$. For π use 3.14.

$\frac{9.42}{d} \approx \frac{3.14}{1}$

$9.42 \times 1 \approx d \times 3.14$

$9.42 \approx 3.14 \times d$ (Why?)

$3.14 \times d \approx 9.42$ (Why?)

$d \approx \frac{9.42}{3.14}$ (Why?)

$d \approx 3$

The diameter of the circle measures about 3″.

13. Find the circumference of a circle whose diameter measures:

a 4″	**b** 6 ft.	**c** 2 yd.	**d** 10″	**e** $1\frac{3}{4}$″
f 3.5″	**g** 6.28 yd.	**h** $7\frac{1}{3}$ ft.	**i** 376.8 yd.	**j** 24 ft.

14. Give the circumference of a circle if its radius measures:

a 4″	**b** $1\frac{3}{4}$″	**c** $\frac{3}{4}$ ft.	**d** 63 yd.
e 10 yd.	**f** 15.4 yd.	**g** 20.5 ft.	**h** $17\frac{1}{2}$″

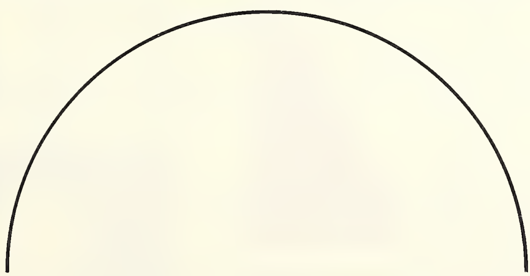
15. An airplane propeller is 7 ft. long. How far does the tip of this propeller travel when it makes 10 revolutions?

16. The diameter of each wheel of a bicycle is 28″. How far does the bicycle travel when the wheels make one complete revolution?

17. A basketball hoop is circular in shape and has a diameter of 18″. What length of metal bar is needed to make 2 basketball hoops?

18. An indoor running track is circular in shape. An athlete must run 12 laps of this track to cover one mile. What is the diameter of the track?

19.



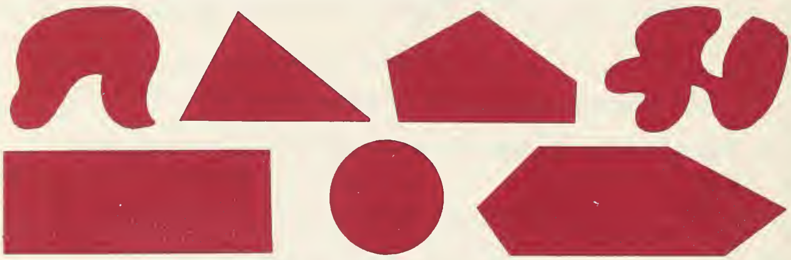
The plane figure above is called a **semicircle**. It is half a circle. If the length of the curved line making the semicircle is 4.71″, what is its radius?

TWO-DIMENSIONAL FIGURES

1. We have seen that a one-dimensional figure can be:

   a drawn without taking your pencil from the paper.

   b *straightened out* to form a line or part of a line. Now we want to consider what is meant by a 'two-dimensional figure.
   Here are some examples of two dimensional figures:



   What do you notice about them?
   Can they be *straightened out* to form lines or part of lines?
   What do you call the boundaries of these figures?

2. Important facts we should know about two-dimensional figures are that:

   a they are sets of points;

   b they can be *flattened out* to form a plane or part of a plane;

   c their boundary is a closed curve;

   d they contain all the points inside the closed curve;

   e the set of points consisting of the points on the closed curve, together with the points of the interior of the closed curve, is called a **region.**

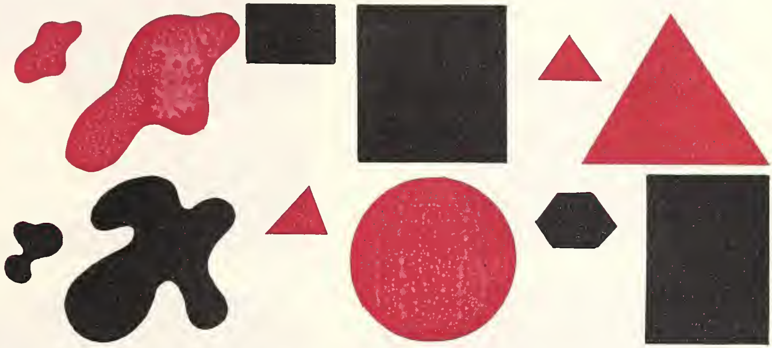
3. Study the diagrams below:

one-dimensional figure | two-dimensional figure | one-dimensional figure | two-dimensional figure

4. How thick is a plane? How thick is a two-dimensional figure?

5. We learned how to compare the sizes of one-dimensional figures. We now shall discover a way to measure and compare the sizes of two-dimensional figures.

6. Examine the pairs of two-dimensional figures below. Say which region is greater in each of the pairs.

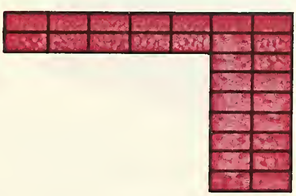


Why is it easy to find which of the regions is the larger of each pair?

7. Here is a region to be measured:

To measure it, we shall compare its size with the size of a region that we shall take as a unit size of region. Suppose we take the size of this region as a unit of measure: 

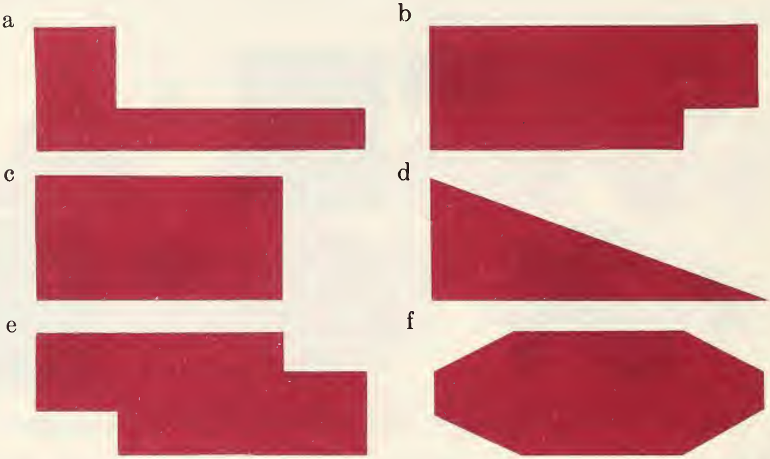
Now we can see how many of these units we need to *fill up* the region we are measuring:



By counting, we can see that we need 28 units of measure to fill up the region.

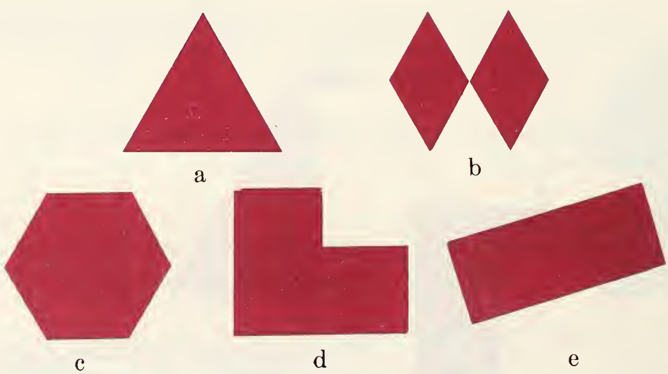
We say that the region has a measure of 28. Its size is 28 units of measure.

8. Copy this region on a piece of cardboard. Cut it out and use it as a unit of measure; then use it to measure the regions below:



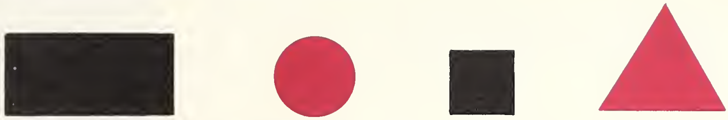
9. Copy this triangular region and use it to measure the interior of the polygons:

unit of measure 



Which regions did you find difficult to measure? Why?

10. Copy the following units of measure:



Now use them to measure the sizes of the following regions:

a

b

c

d

11. Which unit of measure did you find the easiest to use in question 10? Why?

12 You have been using different types of units to measure the sizes of various regions. You may have noticed that some units were easy to use in covering the region to be measured. Just as we did when measuring one-dimensional figures, we need to define a unit for measuring the sizes of two-dimensional regions. We could use the size of any region, but the one we usually use is a square region. The smallest unit is the region of a one-inch square.

The size of this region is called one **square inch.**

The name we gave to the size of a one-dimensional figure was length. We call the size or measure of a two-dimensional region its **area.**

13 Area is the measure in unit regions of a two-dimensional region.

14 What shape is a square inch?
Remember: a square inch is a unit of measure:
Begin with a one-inch square.

Now imagine it cut into two parts:

Put the parts together this way:

or this way:

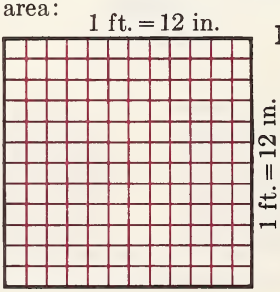
Do we still cover the same size of region?
Can we cut the region covered by a square inch and arrange it in different shapes?

15 A square inch is a unit of area. A square inch has no fixed shape. It tells us the extent of a region.

16. The diagram below helps us to see the relationship between different units of area:

A

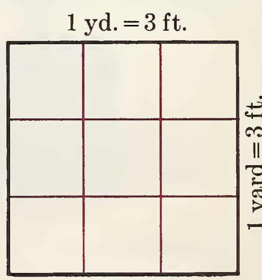
1 square inch
or 1 sq. in.



1 square foot
1 sq. ft.
144 sq. in.

B

1 sq. ft.



1 square yard
1 sq. yd. 9 sq. ft.

17. **a** Draw a rectangle 4 in. long and 3 in. wide.

b Use the square inch as a unit and see how many are needed to fill this rectangular region.

c How many squares are needed to fill a row?

d Does the number of inches in the length tell you the number of square inches needed to fill one row?

e How many rows of squares did you need to fill the region?

f Does the number of inches in the width tell you how many rows of square inches you needed to fill the rectangular region?

g Can you see a short way of finding the area of a rectangular region?

18 We often use short ways of saying things in mathematics. For 'area of a triangular region' we often say 'area of a triangle'. We have seen that to find the number of square inches in a rectangular region we can multiply the number of inches in the length by the number of inches in the width. This can be written as

$$\mathbf{A = l \times w}$$

number of units in the measure of the area of rectangular region	number of units of length in the length of the rectangle	number of units of length in the width of the rectangle.

$A = l \times w$ is a formula. It is a shorthand way of making a mathematical statement.

19 **a** What do we know about the length and the width of a square? If we let s be the number of units of length in each side of the square, then $A = s \times s = s^2$.

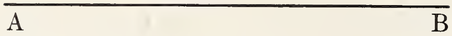
b What is the area of a square with sides of 3″?

The area is given by, $A = 3^2$;

therefore, the area $=$ 9 sq. in.

Because expressions like 3^2, s^2, 4^2, 1.5^2 etc. are given as the measure of the areas of squares with sides of 3, s, 4, 1.5 etc., we often say *3 squared* for 3^2, *s squared* for s^2, *4 squared* for 4^2, *1.5 squared* for 1.5^2, and so on. Previously, we learned to say *3 exponent two* for 3^2. We have now learned another way of naming a base to exponent two.

20 What is the area of segment $\overline{AB}$ if $\overline{AB}$ is 6 in. long?



Think of $\overline{AB}$ as a 'rectangle'. What is the width of $\overline{AB}$?
What is the width of any line?

$A = l \times w$ Since $w = 0$ then

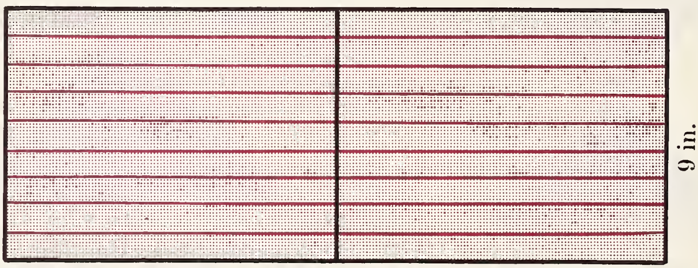
$A = 6 \times 0 = 0$

What is the area of segment $\overline{AB}$?
What is the width of any one-dimensional figure?
What is the area of any one-dimensional figure? Why?

21 The area of any one-dimensional figure is zero.

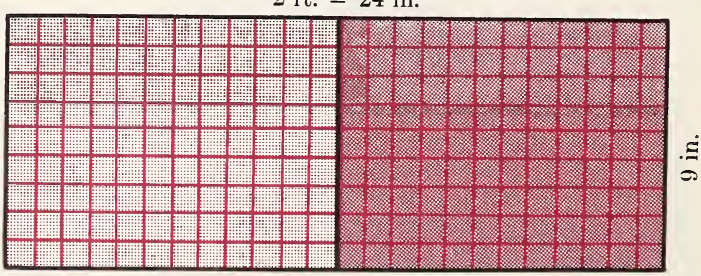
22 Find the area of a rectangular region which is 2ft. long and 9 in. wide.

2 ft.



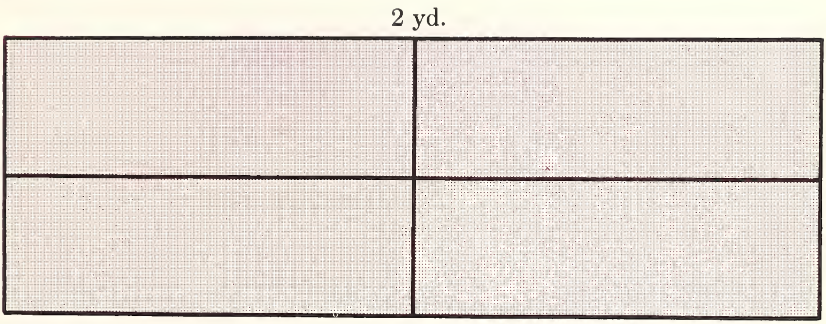
Into what size of rectangular regions have we divided the 2ft. by 9 in. rectangle? Would 18 one-foot by one-inch rectangular regions be a measure of the area of this region? Why would it not be a good measure? Is a 'one-foot by one-inch rectangular region' a standard unit of area? Into what units of length could we divide the length in order to get standard units of area?

2 ft. = 24 in.

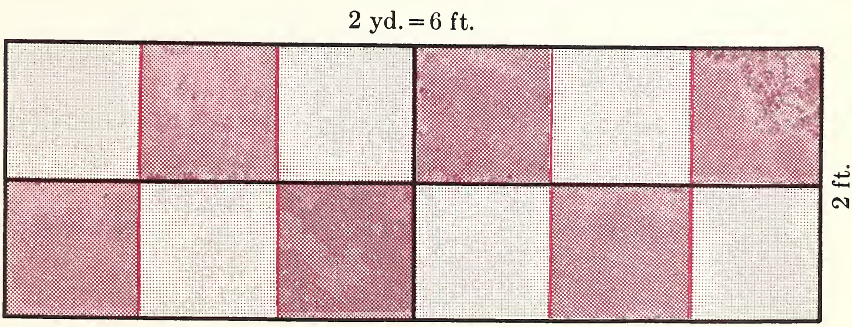


Area $= 24 \times 9$ sq. in. $= 216$ sq. in.

23 Find the area of a rectangular region 2 yd. long and 2 ft. wide.



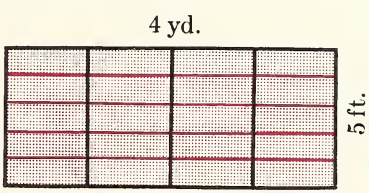
The area is 4 'one-yd.-by-one-ft.' rectangular regions. What is the area in standard units of area?

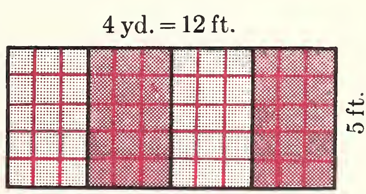


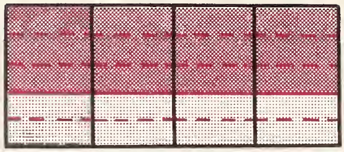
Area = 6×2 sq. ft.
= 12 sq. ft.

24. Can you make a rule about the units of length to use if you want to find the area of a rectangle in standard units of area?

25. Find the area of a rectangular region 4 yd. long and 5ft. wide. Study the diagrams below.







26. Find the areas of rectangular regions with the following lengths and widths: (Express your answers as (i) square inches (ii) square feet (iii) square yards.)

a l = 24 in.	w = 1 ft.	**b** l = 2 yd.	w = 9 in.
c l = 4 yd.	w = 1 ft. 6 in.	**d** l = 10 yd.	w = 1 ft.
e l = $1\frac{1}{2}$ yd.	w = $\frac{3}{4}$ yd.	**f** l = $4\frac{1}{2}$ ft.	w = $1\frac{2}{3}$ in.
g l = $6\frac{1}{4}$ in.	w = $1\frac{3}{5}$ in.	**h** l = $20\frac{1}{2}$ in.	w = $13\frac{1}{3}$ in.

27. An **acre** is a unit of measure used to measure large regions. A square region having a one-mile side has an area of 1 square mile. If this region is divided into 640 regions having the same measurement, each of these smaller regions has a measure of 1 acre.

640 acres = 1 sq. mi.

28. How many yards are there in a mile?

29. How many square yards are there in a square mile?

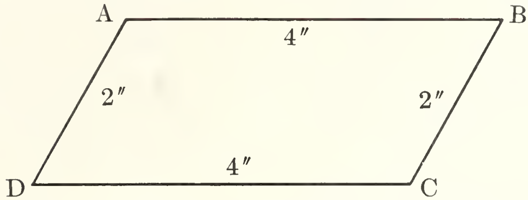
30. How many square yards are there in an acre?

31. How many square feet are there in 1 square mile? in $1\frac{1}{2}$ acres?

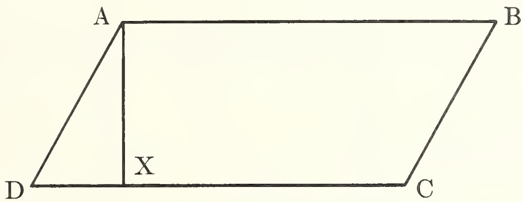
32. The area of a certain playground is two acres. About how many gallons of disinfectant are needed to treat the playground if $\frac{1}{3}$ gallon is used per square yard?

AREAS OF PARALLELOGRAMS

1. Draw parallelogram ABCD below on a piece of cardboard and cut it out.

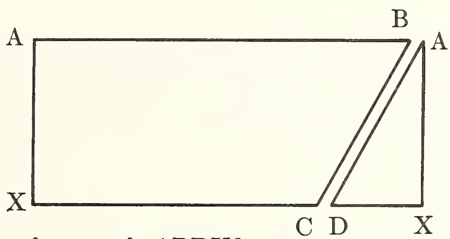


2. Now draw segment $\overline{AX}$ so that $\overline{AX}$ is perpendicular to $\overline{DC}$.



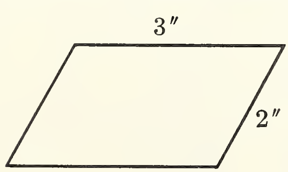
3. Cut along $\overline{AX}$. We now have quadrilateral ABCX and triangle AXD.

4. Fit side $\overline{AD}$ of triangle ADX along side $\overline{BC}$ of quadrilateral ABCX. What do you notice? What kind of figure is quadrilateral ABDX?

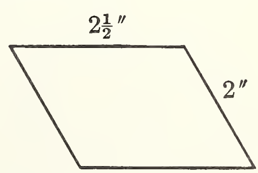


5. What is the area of rectangle ABDX?
What is the area of parallelogram ABCD? Why?

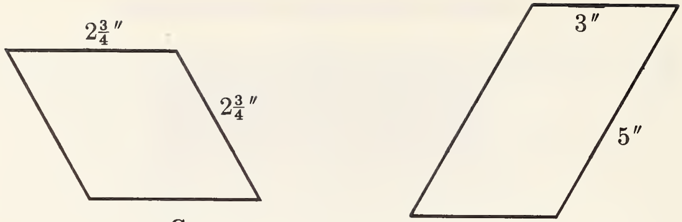
6. Copy the parallelograms below on pieces of cardboard. Cut them out and find their areas by the method we have used above:



A

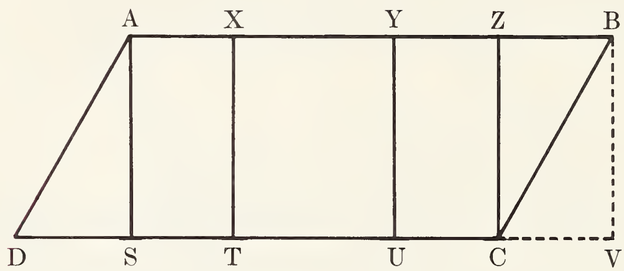


B



C D

7 Study the parallelogram below:



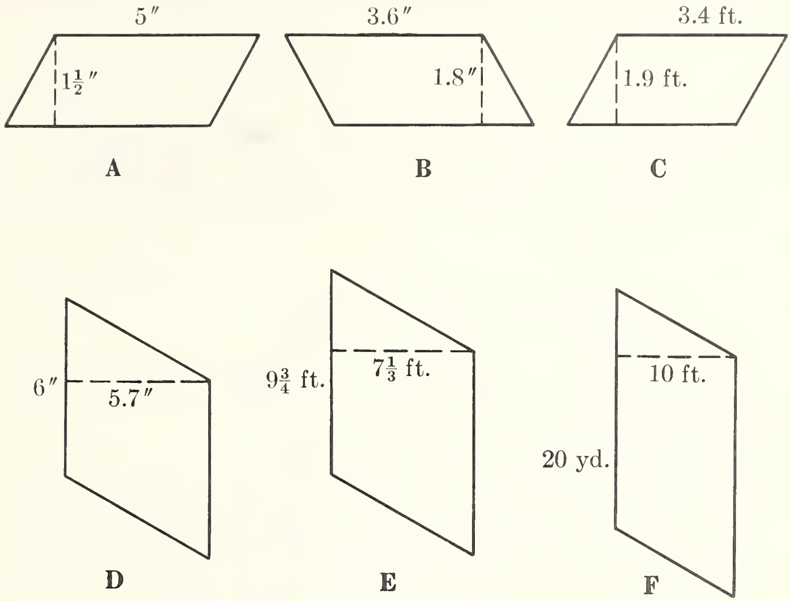
Find the lengths of $\overline{AS}$, $\overline{XT}$, $\overline{YU}$, $\overline{ZC}$, $\overline{BV}$. What do you notice? Each of the segments $\overline{AS}$, $\overline{XT}$, $\overline{YU}$, $\overline{ZC}$, $\overline{BV}$ is an **altitude** of parallelogram ABCD. $\overline{DC}$ is the **base** of parallelogram ABCD.

8 Can you now state a way of finding the area of a parallelogram? **The area of a parallelogram can be found by multiplying the number of units of length in its base and the number of units of length in its altitude.**

$$\mathbf{A} = \mathbf{b} \times \mathbf{h}$$

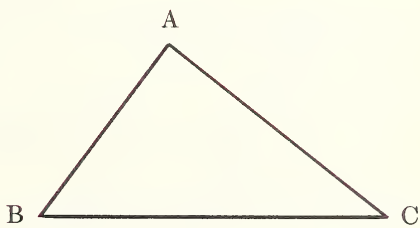
A	b	h
number of units in the measure of the area of a parallelogram	number of units of length in base	number of units of length in altitude

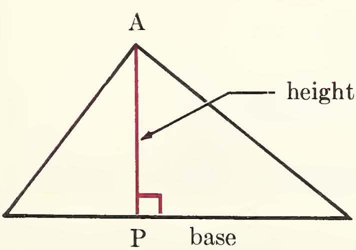
9. Find the areas of the parallelograms below:



ALTITUDES AND BASES OF TRIANGLES

1 Sometimes we need to know the height of a triangle. In triangle ABC what is the height?



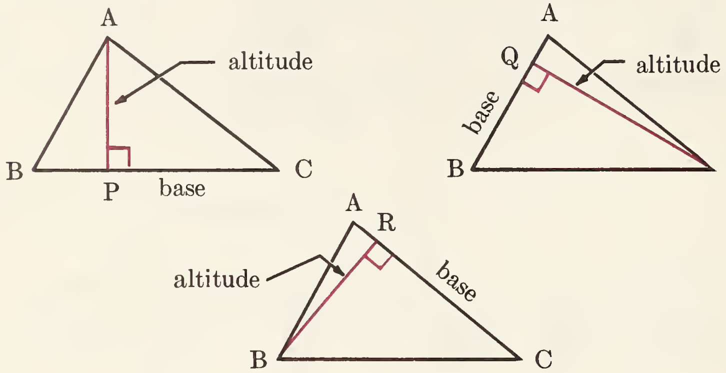


a we first agree to call one side of the triangle a **base.** Let $\overline{BC}$ be the base.

b Now we need to know the height of the triangle if we think of $\overline{BC}$ as its base. Segment $\overline{AP}$ is drawn so that $\overline{AP}$ is $\perp$ to $\overline{BC}$.

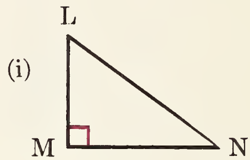
c For height we can use the word **altitude.**

2. In geometry, we make an agreement to use any one of its sides as a base. How many bases can we consider a triangle as having? how many altitudes?

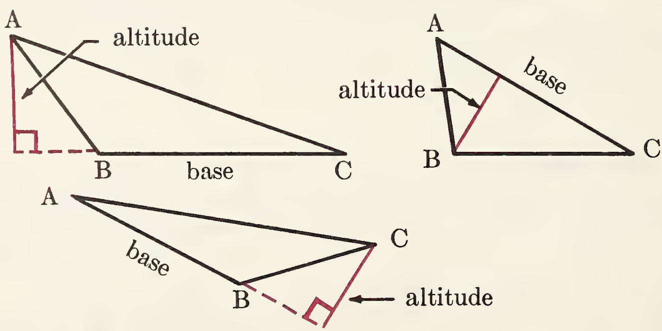


3. In the right triangle to the right, what sides can be thought of as altitudes?

If we use $\overline{MN}$ as the base, what is the altitude? If we use $\overline{LM}$ as the base, what is the altitude?

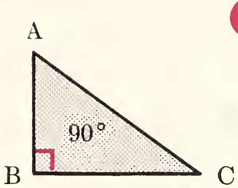


4 Sometimes the altitude of a triangle may lie outside the triangle.



When this happens, we have to think of the base as being extended to make a ray.

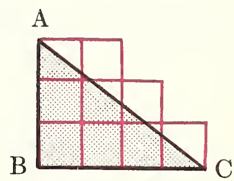
5. What kind of triangle, do you think, has an altitude that lies outside the triangle?



1 Draw a triangle ABC such that $\overline{AB}=3$ in., $\overline{BC}=4$ in., and $\angle ABC=90°$.

What is the area of this triangular region? One way to try to find the area would be to try to cover the region with squares that had areas of 1 sq. in.

This is what we may see:



How many unit regions were needed? We notice that 9 unit regions were used.

But do all of these square regions lie completely inside the triangular region we are measuring? How many unit regions overlap the triangular region? From the diagram we can see

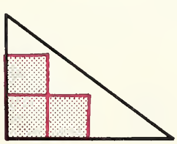
(i) the area is certainly more than 3 sq. in.

(ii) the area is certainly less than 9 sq. in.

All we know so far is that the area is between 3 sq. in. and 9 sq. in. Can you think of a more accurate method of covering the triangular region with standard units of area?

a Make several one-inch squares and use them as standard units of area.

b Using uncut 1-in. squares cover as much of the triangular region as you can.

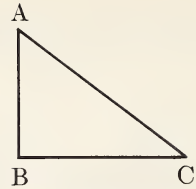


c How many whole squares have you used?

d Now cut the other squares and try to fill the remainder of the triangular region.

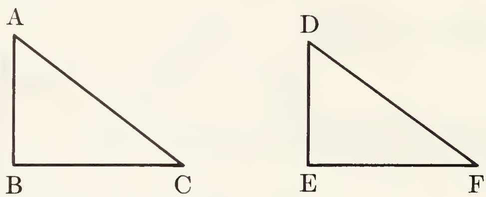
e Remember we have seen that the area is > 3 sq. in. but < 9 sq. in. Can you get a better estimate by cutting the squares? What is your estimate now?

2 Now draw triangle ABC on a piece of cardboard and cut it out. We have:



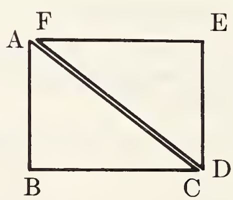
Cut out another triangle of exactly the same size. Name this DEF.

We now have:



Do these triangular regions have the same area? Can you put the triangles together to form a simple polygon whose area you can measure? Can you make a rectangle from the two triangles?

The diagram below shows how our new figure should look.



Does the length of $\overline{BC}$ = the length of $\overline{FE}$?
Does the length of $\overline{AB}$ = the length of $\overline{DE}$?
Are $\angle$ABC and $\angle$EDF right angles?
Does $\overline{AC}$ touch $\overline{FD}$ along its entire length?
Measure the angles where corners F and A meet and where corners E and C meet. What size are they?
Is this new figure a rectangle?
Let a be the number of units of length in $\overline{BC}$.
Let c be the number of units of length in AB.
What is the area of this rectangle?
The area is given by $a \times c$ or ac.
How many triangular regions with the area of triangular region ABC did we need to make the rectangular region?

What fraction of the area of the rectangular region is the area of triangular region ABC?

We see that

Area of ABC $= \frac{1}{2}$ of area of rectangular region

$= \frac{1}{2}$ of ac

$= \frac{1}{2}$ ac or $\frac{ac}{2}$

a is the length of $\overline{BC}$, which is the base of ABC.

c is the length of $\overline{AB}$, which is the altitude of ABC.

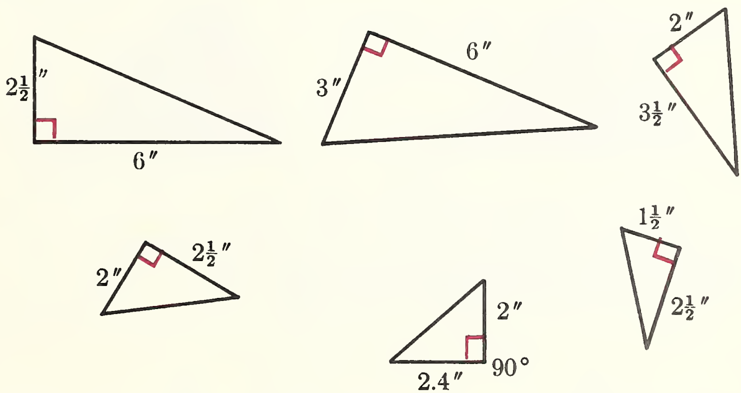
So the area of ABC $= \frac{1}{2}$ ac, where

a is the number of units of length in the base of ABC, and c is the number of units of length in the altitude of ABC. We know that $a=4$, $c=3$; so the area of ABC $= \frac{1}{2} \times 4 \times 3$ sq. in.

$= 6$ sq. in.

How does this answer compare with the estimates you made?

3. Find the areas of the triangular regions represented below. Find these areas by the method of drawing triangles and cutting them out as we did in Ex. 2.



Is each of the triangles above a right triangle? How do you know?

4. Make a rule for finding the areas of regions contained by right triangles.

5. We can use a formula to find the area of right triangles. It is

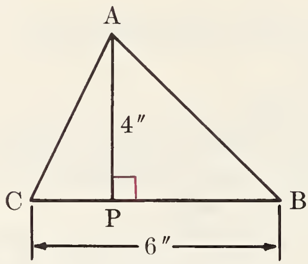
$$\mathbf{A = \tfrac{1}{2}ac}$$

A: number of units in the measure of the area of a right-triangular region

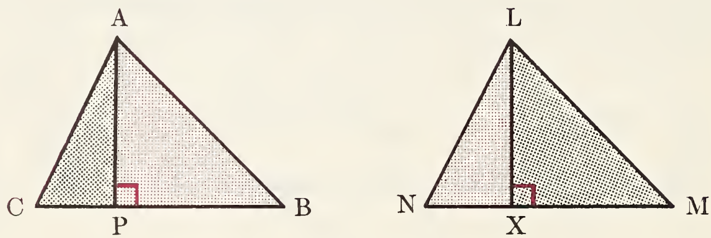
a: number of units of length in base

c: number of units of length in altitude

6 Find the area of triangle ABC below, which has a base of 6 inches and an altitude of 4 inches.



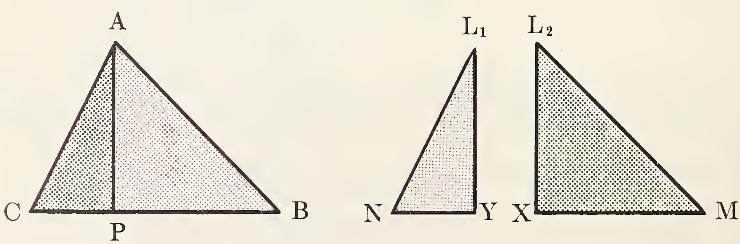
a Draw on a piece of cardboard two copies of triangle ABC. Label the copies as below:



A B C P L M N X

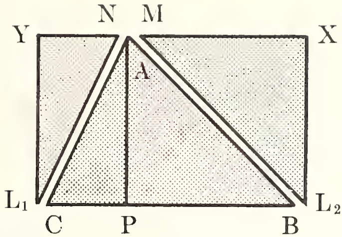
Will each of these triangles have the same area? Why?

b Take triangle LMN and cut along $\overline{LX}$.



Label the two triangles we have made from triangle LMN as L_2MX and L_1YN.

c Let triangles L_2MX and L_1YN be fitted to triangle ABC as shown below:



What is the size of the angle at the corner where Y is? Where X is? Measure the angles at the corners where L_1 and C meet and where L_2 and B meet. What do you notice?

d Does $\overline{L_1N}$ fit exactly along $\overline{CA}$?
Does $\overline{L_2M}$ fit exactly along $\overline{BA}$?

e Measure the lengths of $\overline{YL_1}$, $\overline{AP}$, and $\overline{XL_2}$. What do you notice? What do the lengths equal?

f Measure the angle where points N, A, and M come together. What kind of angle is this?
Does $\overline{YN}$ fit with $\overline{MX}$ to make line segment $\overline{YX}$?

g What kind of polygon is YXBC?

h Let the length of $\overline{CB}$ be a, let the length of $\overline{YL_1}$ be b.
What is the area of rectangle YXBC?

i We see that the area of rectangle YXBC is **ab** sq. in.

j We made this rectangle from two triangles having equal areas. What fraction of the area of the rectangle is the area of triangle ABC?

k Area of triangle ABC $= \frac{1}{2}$ of area of rectangle YXBC
$= \frac{1}{2}$ of ab
$= \frac{1}{2}$ ab or $\dfrac{ab}{2}$

l a is equal to the length of $\overline{BC}$; b is equal to the length of $\overline{YL}$, which is equal in length to $\overline{AP}$. Why?

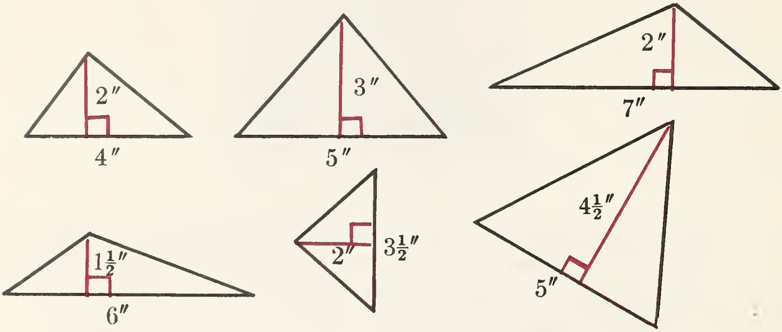
m $\overline{BC}$ is the base of triangle ABC. $\overline{AP}$ is the altitude of triangle ABC.
Number of units in the measure of the area of
triangle **ABC** $= \frac{1}{2}$ **ab**

number of units of length in the base of triangle ABC

number of units of length in the altitude of triangle ABC

But a = 6 and b = 4.
So area of triangle ABC $= \frac{1}{2} \times 6 \times 4$ sq. in. = 12 sq. in.

7. The triangles below are drawn to scale. Make full-sized copies of them in cardboard and use the method we have just used in Ex. 6 to find their areas.



8. Can you make a rule about finding the area of a triangle when you know the length of its base and its altitude?

9 Number of units in the measure of

the **area of a triangle** = $\frac{1}{2}$**ab**

number of units of length in the base — **a**; **b** — number of units of length in the altitude.

UNITS OF SQUARE MEASURE

1.

144 sq. in.	=	1 sq. ft.
9 sq. ft.	=	1 sq. yd.
$30\frac{1}{4}$ sq. yd.	=	1 sq. rd.
160 sq. rd.	=	1 acre (ac.)
640 acres	=	1 sq. mile (sq. mi.)

2. Copy and complete:

a 432 sq. in = ▩ sq. ft.

b ▩ sq. in = 5 sq. ft.

c 108 sq. ft. = ▩ sq. yd.

d 7 sq. yd. = ▩ sq. ft.

e 121 sq. yd. = ▩ sq. rd.

f ▩ sq. yd. = 10 sq. rd.

g 6400 sq. rd. = ▩ ac.

h 6 ac. = ▩ sq. rd.

i 1280 ac. = ▩ sq. mi.

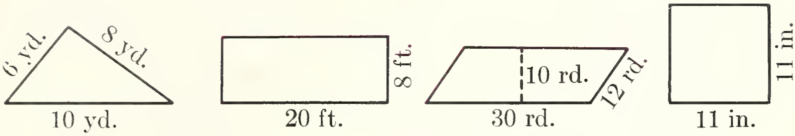
j ▩ ac. = 8 sq. mi.

k 1 ac. = ▩ sq. yd.

l ▩ sq. yd. = 1 sq. mi.

PRACTICE IN FINDING AREAS

1.



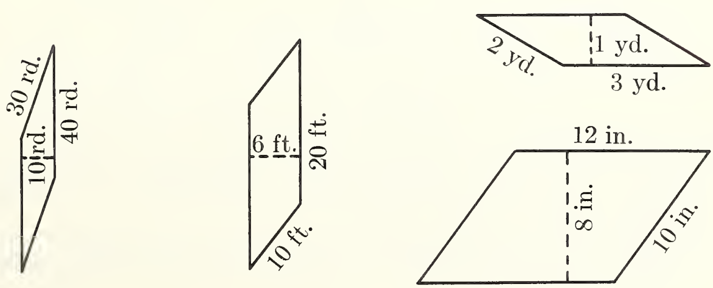
a Name each of the figures drawn above.

b Find the area of each of the figures above.

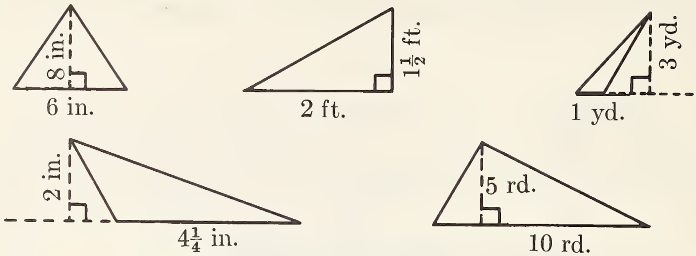
2. Find the areas of the rectangles having the dimensions given below. Give the areas in the units stated.

Length	Width	Unit of Area
6 in.	3 in.	square inches
18 in.	1 ft.	square feet
3 yd.	$1\frac{1}{2}$ ft.	square yards
$2\frac{1}{2}$ yd.	9 in.	square feet
1 ft. 6 in.	$\frac{1}{4}$ yd.	square inches
1 rd.	$\frac{1}{2}$ rd.	square yards
220 yd.	22 yd.	acres
30 rd.	20 rd.	acres
1 mi. 880 yd.	1320 yd.	square miles
$2\frac{1}{4}$ in.	$1\frac{2}{3}$ in.	square inches
$1\frac{5}{6}$ ft.	$1\frac{1}{8}$ ft.	square feet
90 rd.	80 rd.	acres

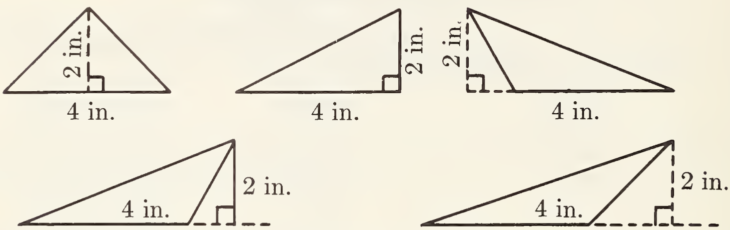
3. Find the areas of the parallelograms drawn below:



4. Find the areas of the triangles drawn below:

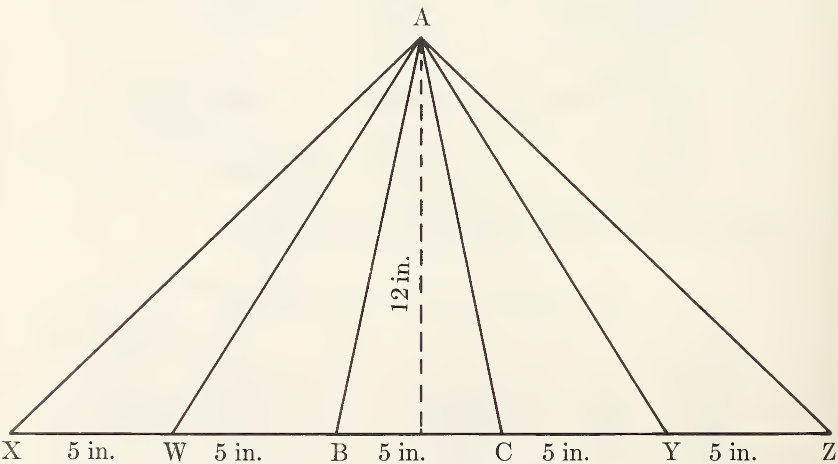


5. Find the area of each triangle drawn below:



What are you led to believe about the areas of triangles which have the same base and the same altitude?

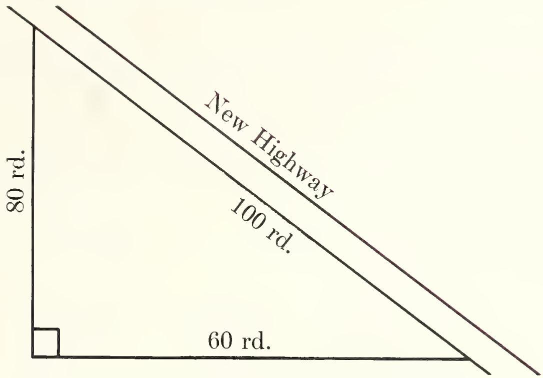
6.



a What is the area of △ ABC?

b What do you think will be the areas of △'s, ABW, AWX, ACY, AYZ? Why?

1. Below is a sketch of the corner of a farm:



This part of the farm was sold for \$250 an acre for building a factory. What was the selling price of this corner of the farm?

Let the number of dollars it was sold for be n.

Then n will equal the number of acres of land times the number of dollars paid for 1 acre.

[Think: "I can find the area of the farm because it is triangular and I know the length of the base and the altitude. This area will be in square rods. To change square rods to acres I divide by 160. Now I shall have the area in acres. I find the product of the number of acres and the number of dollars paid for 1 acre to find n.]

Then $n = \dfrac{\frac{1}{2} \times 80 \times 60}{160} \times \$250$

$= \dfrac{\overset{1}{\cancel{40}} \times \overset{15}{\cancel{60}} \times \$250}{\underset{\cancel{4}\,1}{\cancel{160}}}$

$= \$3{,}750$

$$\begin{array}{r} 250 \\ \underline{15} \\ 1250 \\ \underline{2500} \\ 3750 \end{array}$$

The corner of the farm was sold for \$3,750.

2. A floor 18 ft. long and 15 ft. wide was covered with wall-to-wall carpeting at \$16.25 a square yard. What was the cost of carpeting?

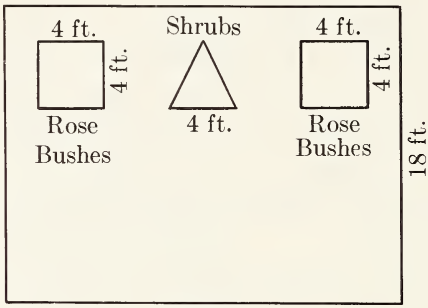
3. What number of gallons of paint will be needed to paint a board fence which is 125 ft. long and 6 ft. high if 1 gallon of paint covers 200 sq. ft. of fencing?

4. The base of the Great Pyramid in Egypt is a square. If the side of the square is 745 feet, what area does the pyramid cover? Give the answer in acres and work to two places of decimal.

5. What is the cost of paving a driveway that is 30 ft. long and 10 ft. 6 in. wide if it costs $4.95 to pave 1 square yard of driveway?

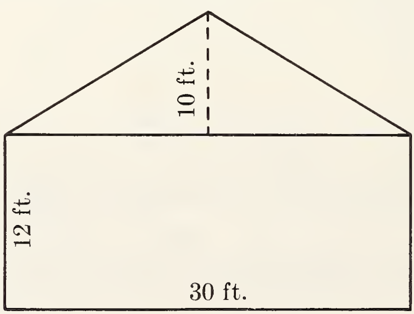
6. A section of lawn is in the shape of a parallelogram with a base of 36 ft. and an altitude of 30 ft. One pound of grass seed costs $0.89 and covers an area of 120 sq. ft. What will be the cost of seeding this lawn?

7. Below is a diagram of a garden. Find the area of the garden that is lawn.



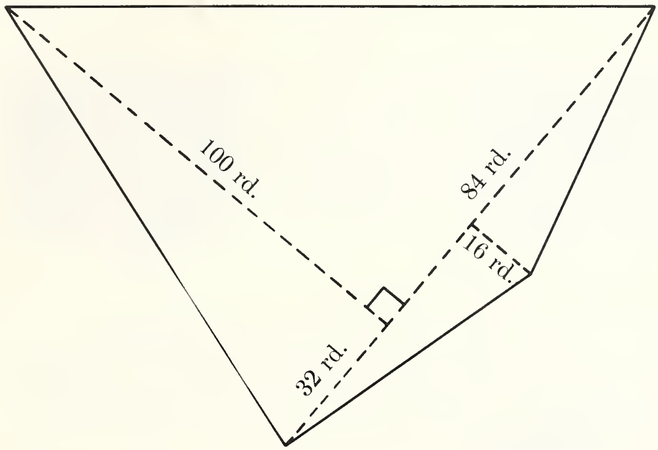
8. A triangular piece of land was sold for $4,500. If the triangle had a base of 160 rods and an altitude of 45 rods, what was the selling price per acre of land?

9. The end of a building is shown on the diagram below. Find the cost of covering both ends of the building with clapboard if clapboard to cover one square foot costs 22 cents.

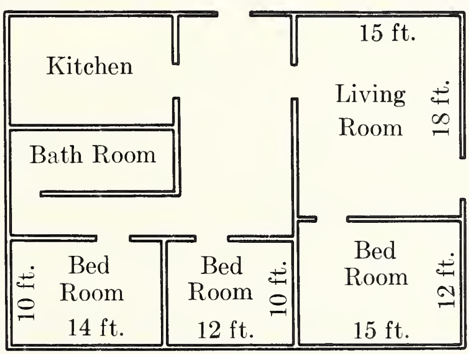


10. The roof of a house consists of two rectangular surfaces 40 ft. long and 16 ft. wide. What would be the cost of re-shingling the roof if it costs $11.95 to cover 100 sq. ft. of the roof?

11. A field has the shape and dimensions shown below. Find the area of the field in acres. Give your answer correct to two places of decimal.



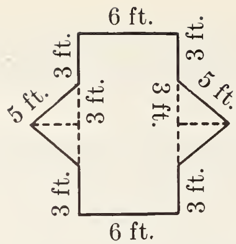
12. The ground floor plan of a summer cottage is shown below:



Find the cost of flooring the living room and the three bedrooms with plywood at 9¢ a square foot?

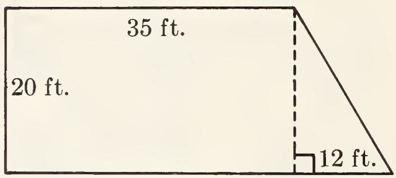
13. A bundle of shingles costs $2.79 and three bundles cover 100 sq. ft. of roof. What is the cost of covering the two sections of a roof if each section measures 36 ft. by 10 ft.?

14.



Find the area, in square yards, of the figure above.

15. A lawn has the shape and dimensions shown below. Find the cost of laying new sod if 1 sq. yd. of sod costs \$2.25.



16. Which has the greater area and by how much: a parallelogram with a $5\frac{1}{4}''$ base and an altitude of $3\frac{1}{3}''$ or a triangle with a base of $9\frac{1}{3}''$ and an altitude of $5\frac{1}{4}''$?

THREE-DIMENSIONAL FIGURES

1 So far we have considered one-dimensional figures and two-dimensional figures. We have seen that one-dimensional figures can be *straightened out* to form segments, rays, or lines. Two-dimensional figures can be *flattened* to form planes or parts of planes. Now consider the set of all points in the room where you are. Can this set of points be *straightened* into a line or *flattened* into a plane? Why not? A set of points that cannot be bent into a line or a plane (or parts of lines or planes) is a three-dimensional figure. We often call such a set of points a **solid figure** or a **solid**.

Although all geometric figures are ideas, there are models of geometric figures in the world around us. Think of your desk. The desk is a model of a three-dimensional figure. The set of all points that your desk contains is a three-dimensional figure.

What kind of geometric figure is suggested to you by the following:

(i) an orange, (ii) a brick, (iii) the water in a swimming pool, (iv) a piece of chalk, (v) a baseball, (vi) a chair?

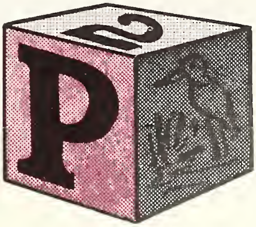
Are they all models of three-dimensional figures? Why?

2 Now think of the set of all the points that there are. This will include the set of all the points inside your room, outside your room, in fact, all points everywhere. The set of *all* points is called **space.**

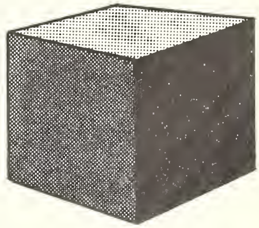
3 Space is the set of all points.

4. Think of the set of points that is suggested to you by a child's building brick.

Building brick

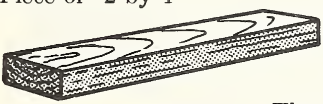


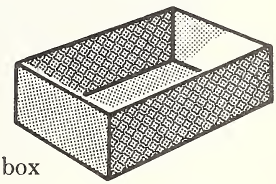
Drawing of geometric figure suggested by a building brick



5 Think of the sets of points that are suggested to you by (a) a piece of '2 by 4',(b) the inside of a box. We might represent these sets of points in this way:

Piece of '2 by 4'

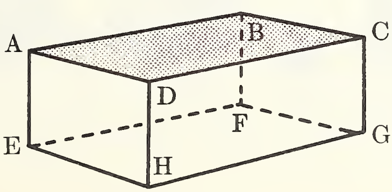




The inside of a box

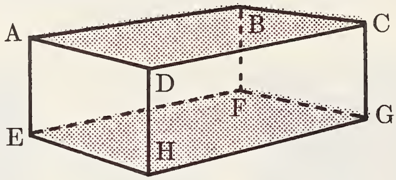
In what ways are the two geometric figures alike? Figures like these and like the geometric figure suggested by a child's building brick are called **rectangular prisms.**

6 **a** How many corners does a rectangular prism have?
We call the point at the very corner of a rectangular prism a **vertex.**
The plural of vertex is **vertices.**
How many vertices does a rectangular prism have?
Consider the rectangular prism drawn below. Name the points that make up the vertices A, B, C, D, E, F, G, H.



b How many edges does the figure have? The edges of the figure are segments. Name the edges.

c What kind of region is the one consisting of the rectangle ABCD and the points inside this rectangle? This region is called a **face** of the rectangle. This face is part of a plane. How many faces does the rectangular prism have? Name them.



7. Two of the faces are usually called the bases of the figure. In the drawing, two faces have been coloured. These faces can be considered as bases.

8. Copy and complete the following:

a A rectangular prism is a ______ -dimensional figure.

b It has ______ vertices, ______ edges and ______ faces.

c The vertices are ______ .

d The edges are ______ .

e The faces are ______ .

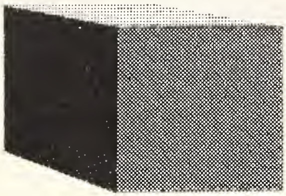
MEASURING VOLUMES

1. We have learned how to find the measure of one-dimensional figures and of two-dimensional figures. Now we need to discover how to find the measure of three-dimensional figures. Below are diagrams of pairs of three-dimensional figures. State which one of each pair is the larger.

(i) (ii) (i) (ii) (i) (ii)

a **b** **c**

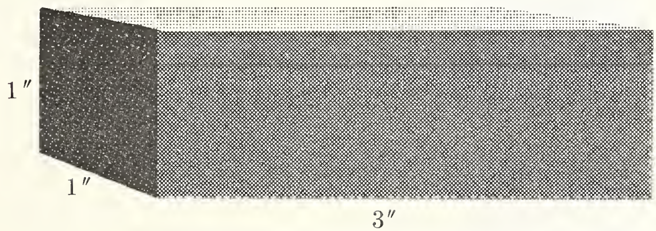
How can you tell which is larger?
How many times as large as **a**(i) is **a**(ii)?
Why is it difficult to tell?



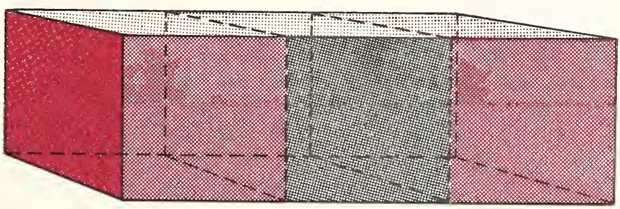
2 When we measure the size of three-dimensional figures, we need a measure. Look at the drawing. What kind of figure is this? What do you notice about the length of all its edges? What figure is each face?

We call a rectangular prism which has all its edges the same length, a **cube.** The cube we have drawn has 1″ edges. It is called a 1-inch cube. The amount of space that a one-inch cube contains is used as the unit of measure for three-dimensional figures.

3 How many times as big as a 1-inch cube is the figure drawn below?



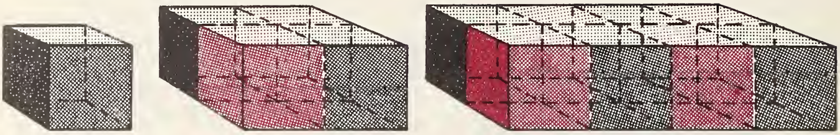
Think of the figure as divided into 1-inch cubes.



How many times as much space does this figure contain as a one-inch cube? Its measure is 3. We say it contains as much space as 3 one-inch cubes.

4 The amount of space a one-inch cube contains is called 1 cubic inch. When we find the amount of space a three-dimensional figure contains, we are measuring its **volume.** What is the volume of the figure in Ex. 3?

5. Figure A represent a one-inch cube. Compare the sizes of figures B and C with that of figure A.

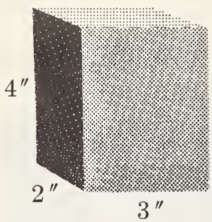


A B C

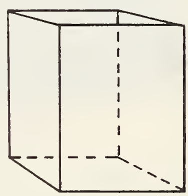
Complete:

a Figure B is ______ times as big as a 1-inch cube. The volume is ______ .

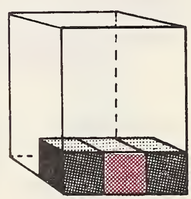
b Figure C is ______ times as big as a 1-inch cube. Its volume is ______ .



6. Find the amount of space contained in a rectangular prism 3 in. long, 2 in. wide, and 4 in. high.



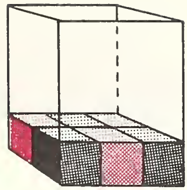
a Imagine that we have a box in the shape of a rectangular prism measuring 3″ by 2″ by 4″ on its inside. Now suppose that we have a supply of 1-inch cubes. How could we find the volume of the inside of this box? Would the volume of the inside of this box be the same as the volume of the rectangular prism above?



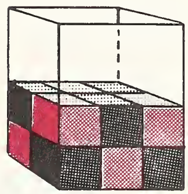
b Will the number of 1-inch cubes needed to fill the box be the measure of its volume? How many cubes will be needed to fill one row of the bottom layer?

Will the number of inches in the length of the box tell us the number of cubes we have in the first row?

c How many rows of cubes will be needed to fill completely the bottom layer? How many cubes will there be in the bottom layer?

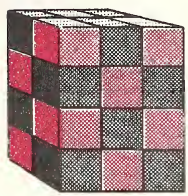


Will the number of inches in the width of the box let us know the number of rows of cubes we have in the first layer? Will the number of inches in the length multiplied with the number of inches in the width give us the number of cubes in the first layer?



d How many cubes will there be in the second layer?

Are there as many cubes in the second layer as there are in the first layer? How many cubes will be needed for the first two layers?



e How many layers will be needed to fill the box? If each cube is 1 inch high, will the number of inches in the height give us the number of layers?

How many layers are there?

How many cubes are there in each layer? How many cubes are needed to fill the box? What is the volume of the box? What is the volume of the rectangular prism?

f Check these answers:

How many inches are there in the length? 3.
How many cubes are there in the first row? 3.
How many inches are there in the width? 2.
How many rows of cubes are there in the first layer? 2.
How many cubes are there in the first layer? 3×2.
How many inches are there in the height? 4.
How many layers of cubes will be needed to fill the box? 4.

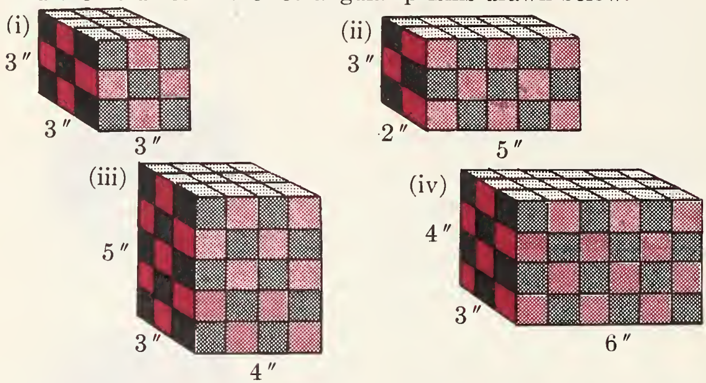
If there are (3×2) cubes in each layer, how many cubes will there be in the box? $3 \times 2 \times 4$

Number of cubes in the box

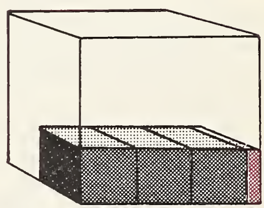
$=3$	$\times$	2	$\times$	4
number of inches in the **length**		number of inches in the **width**		number of inches in the **height**

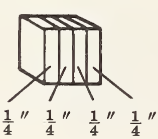
$= 24$
number of cubic inches in the **volume**

7. Find the volumes of the rectangular prisms drawn below.



8. Find the volume of a rectangular prism having a length of $3\frac{1}{4}$ inches, a width of 2 inches, and a height of 3 inches.
The figure is $3\frac{1}{4}$ inches long. How many one-inch cubes will be needed to fill the first row? Does the diagram below help you?

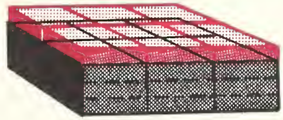




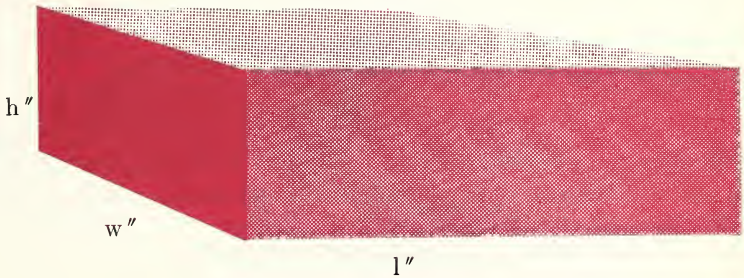
The width is 2 inches. How many rows will there be in the first layer? The height is 3 inches. How many layers will there be? Show that the number of cubic inches is $3\frac{1}{4} \times 2 \times 3$. What is the volume?

9. A rectangular prism is 3″ long, 2″ wide, and $\frac{3}{4}$″ high. What is its volume?

What part of 1 cubic inch do we have in each row? Show that we have $3 \times 2 \times \frac{3}{4}$ cubic inches. Now find the volume.



10.



Show that the volume of the rectangular prism above is given by

$$\mathbf{V} = \mathbf{l} \times \mathbf{w} \times \mathbf{h}$$

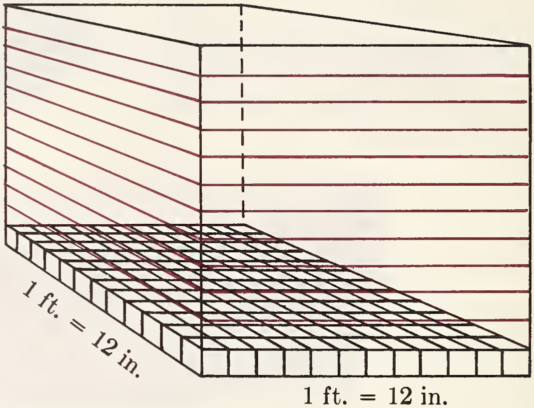
V	l	w	h
number of cubic inches in volume	number of inches in length	number of inches in width	number of inches in height.

11. Below are given the dimensions of rectangular prisms. Find their volumes:

a	l. = 4″,	w. = $2\frac{1}{2}$″,	ht. = 2″
b	l. = $3\frac{1}{2}$″,	w. = 3″,	ht. = 3″
c	l. = $6\frac{1}{4}$″,	w. = 3″,	ht. = $1\frac{1}{5}$″
d	l. = 3.2″,	w. = 2″,	ht. = 5″
e	l. = 4.5″,	w. = 2″,	ht. = 2.25″
f	l. = $5\frac{1}{4}$″,	w. = $2\frac{2}{3}$″,	ht. = 3″
g	l. = 10″,	w. = $4\frac{1}{2}$″,	ht. = $4\frac{1}{2}$″
h	l. = 12″,	w. = $3\frac{1}{3}$″,	ht. = $5\frac{1}{2}$″
i	l. = 6.5″,	w. = 4.6″,	ht. = 3.8″
j	l. = $4\frac{1}{4}$″,	w. = $2\frac{2}{3}$″,	ht. = $1\frac{4}{5}$″

12. How many cubic inches are there in a 1 foot cube? We use the volume of a 1 foot cube as a unit volume. We call it **1 cubic foot.** We can write 1 cu. ft.

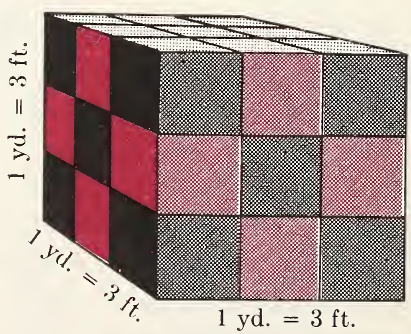
1 ft. = 12 in.



How many 1″ cubes are there in 1 row?
How many rows are there in each layer?
How many 1″ cubes are there in 1 layer?
How many layers are there?
How many 1″ cubes will fill the space of a 1-ft. cube?
What is the volume of a 1-ft. cube?
The volume of a 1-ft. cube is 1 cubic foot.
It is the same volume as 1728 cu. in.

13. The volume of a 1-yard cube is called a **cubic yard.**
How many cubic feet are there in 1 cubic yard?

1 yd. = 3 ft.



How many cubic feet are there in a row?
How many rows are there in a layer?
How many cubic feet are there in a layer?
How many layers are there?
How many cubic feet are there in one cubic yard?

14. We now have a table of measures for volumes:
1728 cu. in. = 1 cu. ft.
27 cu. ft. = 1 cu. yd.

15. A block of wood has a volume of $2\frac{1}{4}$ cu. ft.
What is its volume in cubic inches?
Since 1 cu. ft. = 1728 cu. in., we know that $2\frac{1}{4}$ cu. ft. = $2\frac{1}{4} \times 1728$ cu. in. Now complete the problem.

16. A room has a volume of 2835 cu. ft.
What is its volume in cubic yards?
We know that 27 cu. ft. = 1 cu. yd. Therefore, every time we can take 27 cu. ft. out of 2835 cu. ft. we have the equivalent of 1 cu. yd. The number of cubic yards in 2835 cu. ft. = $\frac{2835}{27}$
Complete the problem.

USING VOLUMES

1. A classroom is 35 ft. long, 20 ft. wide, and 14 ft. high. How many cubic feet of air space does the room contain?

2. A contractor digs a ditch having perpendicular sides. It is 3 ft. 6″ wide at the top and at the bottom. If the ditch is 6 ft. deep and 72 ft. long, how many cubic feet of earth are removed? How many cubic yards of earth are removed?

3. Water weighs approximately $62\frac{1}{2}$ lb. per cubic foot. What weight of water can a tank hold, if the tank is in the shape of a rectangular prism 4 ft. long, 3 ft. wide, and 3 ft. high?

4. A steel bar is 24 ft. long. It has a square section of 1 inch. What is its weight if 1 cu. ft. of steel weighs about 480 lb.?

5. The inside measurements of a freight car are: length, 39 ft. 6 in.; width, 8 ft. 10 in.; height, 12 ft. What is the capacity of the freight car in cubic feet?

6 A rectangular prism has a volume of 60 cu. in. Its length is 5 in.; its width, 3 in. What is its height?

Think: **V** = **l** × **w** × **h**

Think:	**V**	=	**l**	×	**w**	×	**h**
	number of cubic inches in volume		number of inches in length		number of inches in width		number of inches in height
So	60	=	5	×	3	×	h
	60	=	15	×	h		
	15×h	=	60	(Why?)			
	h	=	$\frac{60}{15} = 4$				

The height of the prism is 4 in.

7. Find the missing dimensions of the rectangular prisms below:

	A	B	C
Volume	120 cu. in.	6 cu. yd.	30 cu. ft.
Length	5″	3 yd.	2′ 6″
Width	?	2 yd.	?
Height	6″	?	4′

	D	E	F
Volume	580 cu. in.	1 cu. ft.	6 cu. yd.
Length	?	12″	6′
Width	8″	12″	?
Height	$7\frac{1}{4}$″	?	9′

8. A cubic foot is about the same volume as $6\frac{1}{2}$ gallons. About how many gallons does a tank hold which is 15 ft. long, 10 ft. wide, and 6 ft. high?

9. A truck can carry 5 cu. yd. of earth. How many truckloads would be needed to remove the earth cut from a ditch 375 ft. long, 3 ft. wide, and 4 ft. deep?

10. The edge of a cube measures 40 in. What is its volume in cubic feet?

11. Topsoil was being sold for $2.25 a cubic yard. What was the cost of topsoil needed to cover a garden 75 ft. long and 45 ft. wide to a depth of 4 inches?

12. Concrete costs $5.00 a cubic yard. What will it cost to pave a driveway with concrete, if the driveway is 30 ft. long and 12 ft. wide, and is to be paved to a depth of 6 in?

9

Graphs

READING AND PREPARING BAR GRAPHS

1. Here are the records of a whole class on a final arithmetic test. The maximum possible score was 100.

Jean	62	Fern	19
Bonnie	93	Mary	24
Emily	88	Mae	75
John	42	Bill	32
Ila	83	Betty	63
Rene	81	Leo	66
Dave	70	Myrna	61
Bev	76	Doug	65
Bert	52	Patty	50
Diane	39	Cathy	41
Anne	30	Fred	64
Brent	48	Jim	60
Clay	71	Lynda	43
Dora	69	Sam	96
Anna	85	Dick	51

Find the following:

a the lowest score

b the third lowest score

c the highest score

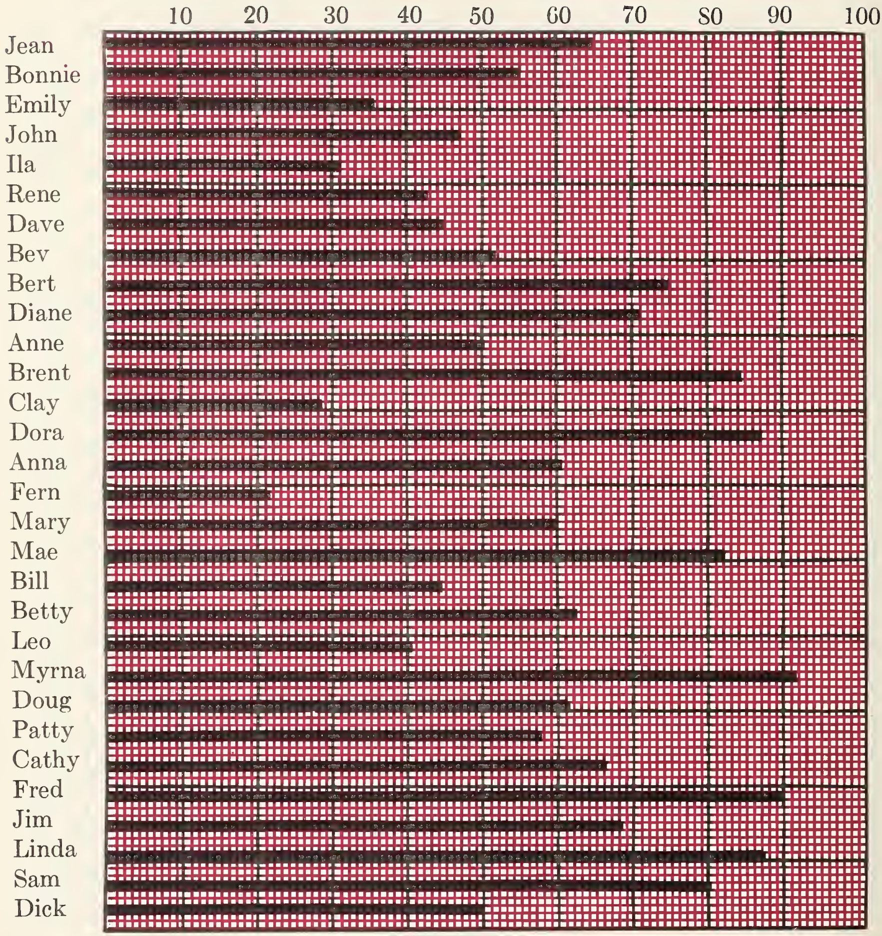
d the second highest score

e the second lowest score

f the third highest score

2 Here are their English marks represented on a graph.

The Final English Marks of Division 26, June 1963.



Answer the questions below.

a What is the highest score?

b What is the third lowest score?

c What is the lowest score?

d What is the second highest score?

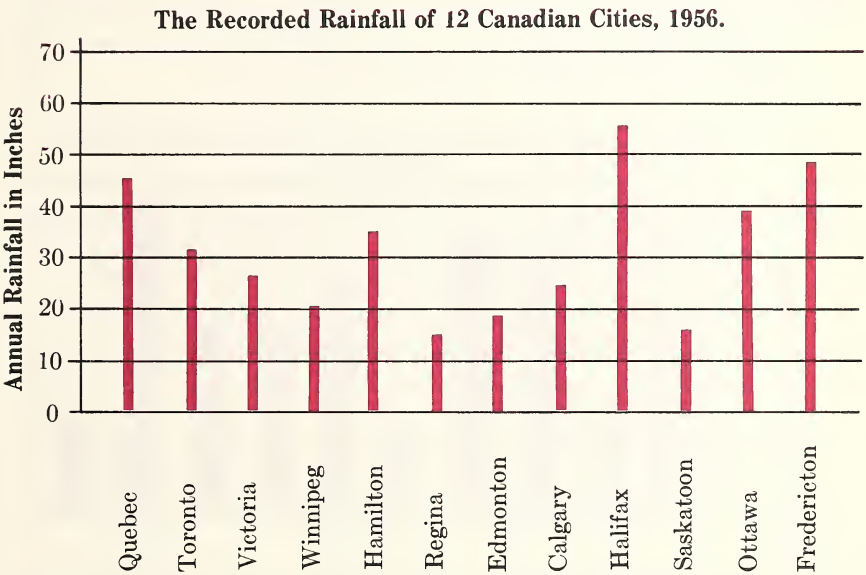
e What is the second lowest score?

f What is the third highest score?

Was it easier for you to find the answers this time? When there are many scores, most people find it easier to compare scores if they are graphed.

The whole purpose in using bar graphs is to compare scores. If there are only 5 or 6 scores, there is no point in graphing them. We can find out all we want to know just by looking at the scores. If there are hundreds of scores, again there is no point graphing them. In such cases, we need ways of finding a few scores to represent all of them.

3 Study this bar graph:



Note the following:

a The graph has a title. Every graph must have a title stating as clearly as possible the nature of the information in the graph.

b It has a **date**. The information in most graphs is useless without a date.

c The **bars** are clearly *named*. We must be able to find, without difficulty what each bar refers to.

d The **scale** is given. We must be able to find the number that each bar represents. (The scale starts at 0. Except in special cases, bar graphs should start at 0.)

Almost all bar graphs should have these four characteristics:

a clear title

a date

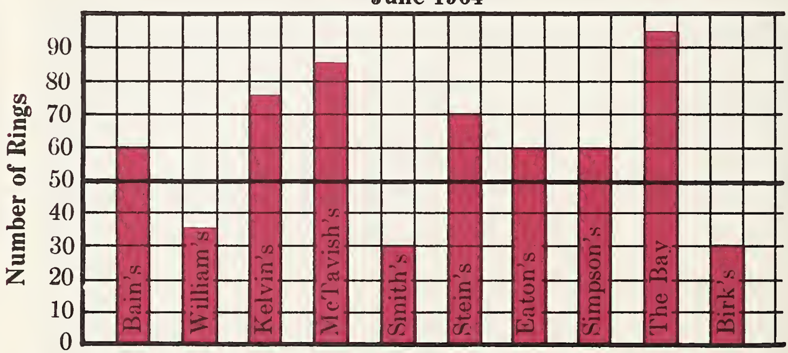
names on the bars

a scale that begins at 0

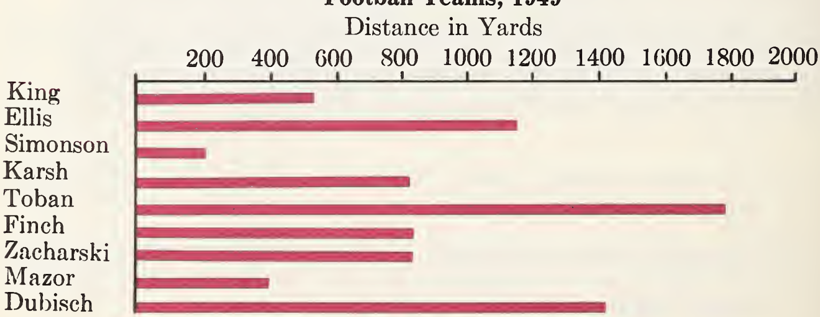
When you draw bar graphs, you should make sure that your graphs have these characteristics: they should be simple, easily read, and well balanced.

4 The way we represent the scale on a bar graph depends upon the kind of paper on which we are to draw the graph. Your teacher will show you how to place scales on the paper you will use. Study these bar graphs, noting the scales in particular.

a **Sales of Rings in the Jewellery Stores of Uniontown, Ontario—June 1964**



b **Yards Gained Passing By Quarterbacks of Canadian Professional Football Teams, 1949**



5. Draw bar graphs to represent the information given below. Remember, each graph should have a title, a date, a scale, and clearly labelled bars. In addition, each graph should be simple, neat, and easy to read.

a The money collected for the Junior Red Cross in Vanier Elementary School, September, 1964.

Division	Amount	Division	Amount
1.	\$3.20	8.	\$5.01
2.	\$4.19	9.	\$1.15
3.	\$7.35	10.	\$8.70
4.	\$5.13	11.	\$0.60
5.	\$2.65	12.	\$4.72
6.	\$7.50	13.	\$6.05
7.	\$2.11		

(i) What was the *largest* amount collected?

(ii) What was the *smallest* amount collected?

(iii) From your graph, what seems to be about the average amount collected?

(iv) By how much does the largest amount exceed the smallest?

(v) What is the *ratio* of the largest amount to the smallest?

b The populations of the Canadian Provinces in 1956 were approximately these:

British Columbia	1,400,000	Quebec	4,630,000
Alberta	1,120,000	New Brunswick	560,000
Saskatchewan	880,000	Nova Scotia	700,000
Manitoba	850,000	Prince Edward Island	100,000
Ontario	5,400,000	Newfoundland	420,000

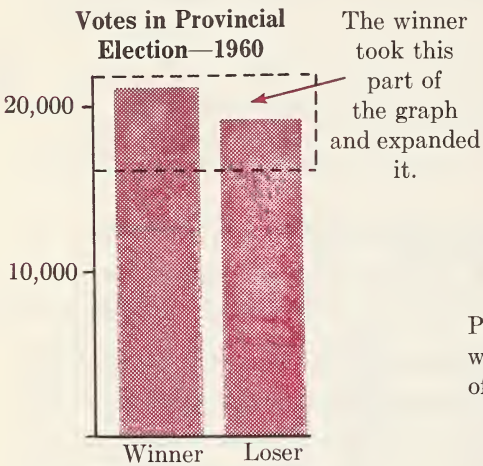
(i) Which 2 provinces had the largest concentration of Canada's population?

(ii) Approximately what fraction of Canada's population lived west of Ontario in 1956?

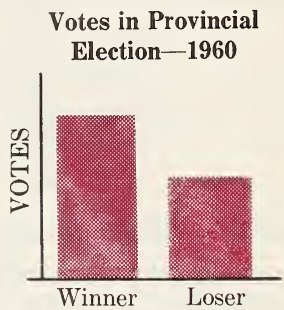
(iii) By how much does the population of Alberta exceed that of New Brunswick?

6 Bar graphs are very useful for presenting information clearly. They can also be used to mislead the reader. Study carefully the following *misuses* of graphs:

a Study the graphs below. Can you see how the winner of an election misused a graph to make people think he had a clear victory?



Graph showing the total votes cast for each candidate.

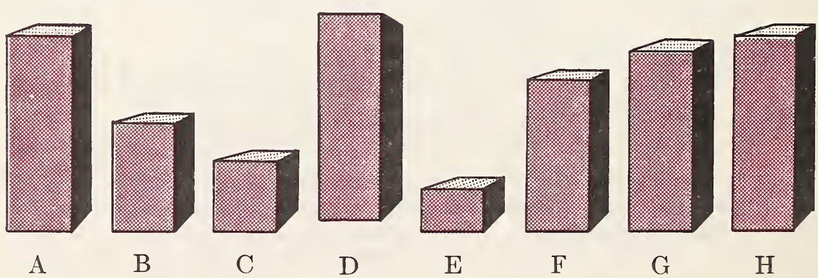


Part of the graph taken by the winner to exaggerate the size of his victory.

Notice–No Scale

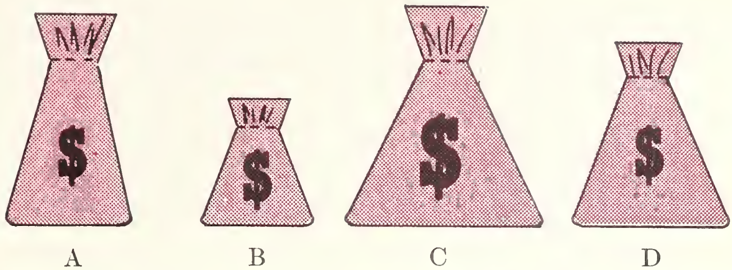
Always look for the scale on a bar graph. If it is not there, you may be misled.

b It is possible to give an incorrect impression on a bar graph by tampering with the bars. Study this example:



What has been done to make bar D seem much taller than the others?

c Study this example:

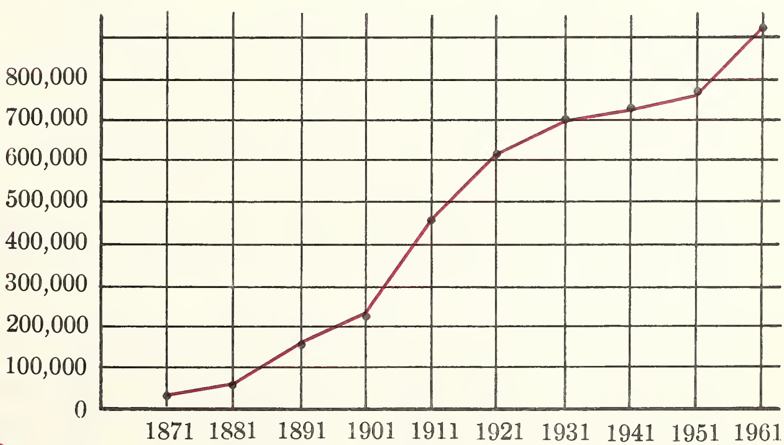


Bar A is almost as tall as bar C. What has been done to make bar C look larger? Sometimes special effects are used to make a graph clearer, but we should always be suspicious of special effects.

READING AND PREPARING LINE GRAPHS

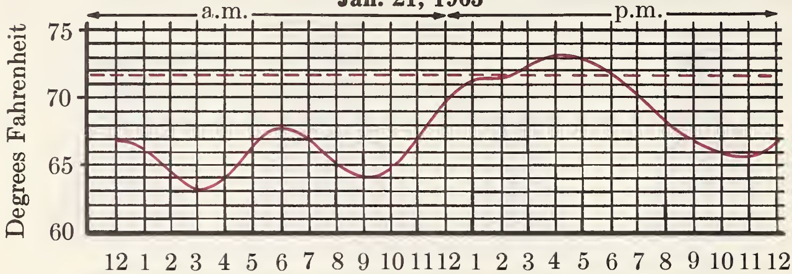
1 Have you noticed that bar graphs are used to compare *different things* at the *same time*? Often we wish to see how one thing changes from time to time. For this, we use a different kind of graph called a line graph. Study this graph:

The Population of Manitoba 1871 - 1961



2 Note that this graph also has a title, a date, clearly labelled points along the base, and a scale. It is sometimes permissible for the scale of a line graph not to begin at zero, but if this is done the reason should be made quite clear.

The Temperature of the Auditorium of MacDonald High School
Jan. 21, 1965



There is no real need in this case, to give scale temperatures below 60. No one is likely to be misled. Note however, that the scale is clearly labelled.

3. From the information given below, construct your own line graphs.

a In the 24 hours of April 3, 1952, these were the tide levels at Coolachuck, B.C.:

12 M.	3′4″	6 a.m.	4′3″	12 n.	3′9″	6 p.m.	7′9″
1 a.m.	5′10″	7 a.m.	2′6″	1 p.m.	6′1″	7 p.m.	5′7″
2 a.m.	7′9″	8 a.m.	1′0″	2 p.m.	7′6″	8 p.m.	3′4″
3 a.m.	9′8″	9 a.m.	6″	3 p.m.	9′7″	9 p.m.	1′4″
4 a.m.	8′9″	10 a.m.	4″	4 p.m.	10′2″	10 p.m.	8″
5 a.m.	6′2″	11 a.m.	1′7″	5 p.m.	9′8″	11 p.m.	2″
						12 M.	2′1.″

(Note that most of these tides are positive. For convenience, in Canada, the '0' point for tides is selected so that negative tides would usually only occur about 13 times per year.* In other words, it would be unusual to have such extreme tides as are recorded above. In the United States, '0' for tides is chosen somewhat higher; so negative tides are quite common.**
The behaviour of the tides at any one point is complicated, depending upon three things: the position of the Sun, the position of the Moon, and the nature of the seacoast.)

b In the years 1941-1960, the city of Gopher Prairie reached the following percentages of their Community Chest objectives:

1941	72%	1946	93%	1951	72%	1956	106%
1942	65%	1947	87%	1952	72%	1957	91%
1943	62%	1948	72%	1953	83%	1958	81%
1944	58%	1949	63%	1954	99%	1959	72%
1945	88%	1950	72%	1955	104%	1960	61%

* '0' for tides in Canada is the mean of low spring tides.
** '0' for tides in the U.S.A. is the mean of low tides.

c The sales of Polypaste toothpaste changed markedly after June 1961 when the makers of Polypaste first advertised that they had added silicon dioxide to their product. Graph this information and decide whether or not the addition of silicon dioxide was a good idea.

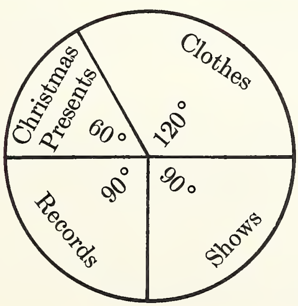
Date	Tubes Sold	Date	Tubes Sold
Jan. 1961	865,420	Jan. 1962	346,088
Feb. 1961	871,106	Feb. 1962	530,816
Mar. 1961	852,763	Mar. 1962	507,702
Apr. 1961	829,129	Apr. 1962	592,219
May 1961	817,400	May 1962	645,560
June 1961	859,400	June 1962	700,019
July 1961	988,662	July 1962	723,327
Aug. 1961	1,502,229	Aug. 1962	711,088
Sept. 1961	1,671,004	Sept. 1962	745,000
Oct. 1961	823,062	Oct. 1962	799,916
Nov. 1961	312,488	Nov. 1962	803,816
Dec. 1961	298,019	Dec. 1962	843,090

d The student population of Laurier Elementary School is given below for the years 1945-1964. Graph this information on a line graph. In which years do you think new schools were opened?

1945	426	1950	500	1955	739	1960	906
1946	439	1951	523	1956	922	1961	948
1947	468	1952	539	1957	621	1962	1028
1948	592	1953	573	1958	698	1963	716
1949	675	1954	624	1959	743	1964	738

READING AND PREPARING CIRCLE GRAPHS

1 How Clara spent her allowance one year:



Clara drew this graph to show how she spent her allowance for a year.

The portion shown for clothes occupies 120°. This shows that she spent $\frac{120}{360}$ or $\frac{1}{3}$ of her allowance on clothes, for the full circle contains 360°. What fraction of her allowance did she spend on records? On Christmas presents?

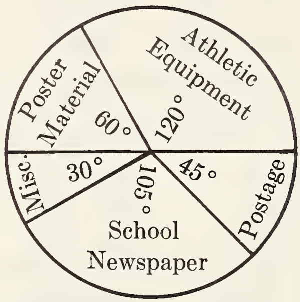
2 Suppose we were told that the student council spent \$120 last year in the following way:

Poster material	\$20.
Athletic equipment	\$40.
Postage	\$15.
School newspaper	\$35.
Misc.	\$10.

We could graph these expenditures this way:

The Expenditures of the Student Council, Sept.—June

$$\frac{20}{120} \times 360° = 60°$$
$$\frac{40}{120} \times 360° = 120°$$
$$\frac{15}{120} \times 360° = 45°$$
$$\frac{35}{120} \times 360° = 105°$$
$$\frac{10}{120} \times 360° = 30°$$



2. Construct your own circle graphs to illustrate the following:
   a In a certain class of 36 students, 9 walk to school, 10 ride bicycles, 11 come by bus, 5 are brought by their mothers by car, and 1 is driven to school by a chauffeur.
   b In a certain prison, there are 60 prisoners. 27 are imprisoned for breaking and entering, 11 for car theft, 13 for disturbing the peace, 6 for embezzlement, 2 for impersonating police officers, and 1 for murder.
   c In a television viewing survey in Fort Douglas, B.C., 38% of the viewers were watching 'Cowboy Bob', 26% were watching 'The Orderlies', 20% were watching 'Wrestling', 13% were watching a movie on World War I, and 3% were watching the 'United Nations Report'.
   d Use the information given in section 5b, page 351, to show the distribution of Canada's population in each province in 1956.

10 Review Exercises

Exercise 1

1. Which of the following numerals represent the number 20?

   a $(3\times 10)-10$

   b $4\times(10\div 2)$

   c $30-(2\times 5)$

   d $2\times(10+0)$

   e $\frac{1}{5}$ of 100

   f $3+(10-3)+(7-3)+(7-1)$

   g $\frac{20}{20}-19$

   h $2+0$

   i $\frac{1}{2}$ of ($\frac{1}{2}$ of 80)

   j $20-(100-99)+1$

2. Write five different numerals to represent each of the following numbers:

   a six

   b one hundred

   c thirty-five

   d one

   e zero

   f twenty-five

3. Work the following:

   a $(4\times 6)\times(6-4)$

   b $(\frac{1}{2}\text{ of }8)\times(10-10)\times(3+4)$

   c $(3\times 19)+(7\times 19)$

   d $(19-4)\div(25\div 5)$

   e $\dfrac{264+(72\div 12)}{10}$

   f $47-2\times 5\times 4+3$

   g $27+3-(15-3)$

   h $\dfrac{(44+16)-(15\times 4)}{26\times 14}$

   i $15\times(14-3)\times 2$

   j $53\times 6+53\times 4$

4. Write expanded numerals for the following:

   a 500

   b 9010

   c 6935

   d 7021

   e 10,000

   f 9,649

   g 100,000

   h 729,004

   i 589,000

   j 1,000,000

5. Write decimal numerals for each of the following:

   a $(9\times 10\times 10)+(6\times 10)+(7\times 1)$

   b $(5\times 10\times 10\times 10)+(1\times 10)$

   c $(7\times 10\times 10\times 10\times 10\times 10)$

   d $(2\times 10\times 10)+(3\times 10\times 10\times 10)+(1\times 10)$

   e $(0\times 10)+(4\times 10\times 10\times 10)+(1\times 10)+(2\times 10\times 10\times 10)$

Exercise 2

1. Write the following as expanded numerals using exponential notation:

   a 300
   b 1000
   c 10,000
   d 315
   e 10,086
   f 909,099
   g 1,100,000
   h 10,708,697

2. Write the following using exponential notation:

   a $3\times3\times3\times3$
   b 10×10
   c $9\times9\times9\times9\times9$
   d $4\times4\times4\times4$
   e $50\times50\times50\times50\times50$
   f $2\times2\times2\times2\times2\times2$

3. Write the following using decimal numeration:

   a 3^6
   b 2^4
   c 8^2
   d 11^3
   e 12^2
   f 9^3
   g 10^4
   h 100^2
   i 5^3
   j 6^4

4. What numeral does ^{n} represent in each of the examples below?

   a $2^n=4$
   b $4^n=4\times4\times4$
   c $5^n=5\times5\times5$
   d $3^n=81$

5. Which is larger, and by how much, 2^3 or 3^2?

Exercise 3

1. Copy and complete the following:

   a $27-x=15$ means $27=15+x$ so $x=\boxed{\quad}$
   b $y-19=20$ means $\boxed{\qquad\qquad}$ so $y=\boxed{\quad}$
   c $100-58=z$ means $\boxed{\qquad\qquad}$ so $z=\boxed{\quad}$
   d $n-86=115$ means $\boxed{\qquad\qquad}$ so $n=\boxed{\quad}$
   e $615-t=287$ means $\boxed{\qquad\qquad}$ so $t=\boxed{\quad}$

2. Copy and complete the following:

   a $35\div7=x$ means $35=x\times7$ so $x=\boxed{\quad}$
   b $54\div6=y$ means $\boxed{\qquad\qquad}$ so $y=\boxed{\quad}$
   c $p\div5=7$ means $\boxed{\qquad\qquad}$ so $p=\boxed{\quad}$

d $144 \div s = 12$ means $\boxed{}$ so $s = \square$

e $121 \div v = 11$ means $\boxed{}$ so $v = \square$

3. Which of the following sets of numbers are closed under the operation of (a) addition (b) multiplication?
   (i) $\{1, 1, 1, 1, 1, \ldots\}$
   (ii) $\{1, 0, 1, 0, 1, 0, 1, \ldots\}$
   (iii) $\{7, 14, 21, 28, 35, \ldots\}$
   (iv) $\{2, 4, 8, 16, 32, \ldots\}$
   (v) $\{3, 3, 3, 3, 3, \ldots\}$

4. Use short division to work the following:

a $9\overline{)20646}$	**f** $8\overline{)3189624}$
b $7\overline{)35091}$	**g** $9\overline{)7358166}$
c $8\overline{)71264}$	**h** $4\overline{)1397528}$
d $5\overline{)21330}$	**i** $2\overline{)5004602}$
e $7\overline{)68957}$	**j** $6\overline{)1103214}$

Exercise 4

1. Use curly brackets and write the elements of the following sets:
   a The natural numbers less than 10.
   b The whole numbers less than 10.
   c The natural numbers less than 1.
   d The whole numbers less than 1.
   e The whole numbers less than 15 but greater than 9.
   f The factors of 48.
   g The natural numbers which are factors of:
   (i) 25 (ii) 91 (iii) 36 (iv) 62 (v) 63 (vi) 41 (vii) 72.
   h The multiples of 8 which are greater than 15 but less than 60.
   i The whole numbers greater than 5 and less than 6.
   j The factors of 256 which are greater than 10 and less than 100.
   k The whole numbers less than 100 which are divisible by 11.
   l The factors of 36 which are even numbers.

m The odd numbers which are divisible by 2.

n The 3-digit numerals you can write using the digits 1, 7, 5 once each in each numeral.

o The numbers less than 100 which are multiples of both 3 and 4.

p The factors of 84 that are also factors of 63.

q The factors of 27 that are also factors of 42.

r The factors of 36 that are also factors of 35.

s The multiples of 5 less than 100 which are also multiples of 6.

2. Study the following sequences. What are likely to be the next three numbers in each sequence?

a $\{13, 26, 39, 52, \ldots\}$

b $\{1, 4, 5, 8, 9, 12, 13, \ldots\}$

c $\{2, 4, 7, 11, 16, 22, \ldots\}$

d $\{0, 2, 6, 12, 20, 30, 42, \ldots\}$

e $\{0, 1, 1, 2, 3, 5, 8, 13, 21, \ldots\}$

f $\{0, 1, 4, 9, 16, 25, \ldots\}$

g $\{1, 2, 3, 6, 7, 14, 15, 30, 31, \ldots\}$

h $\{15, 13, 20, 18, 25, 23, 30, \ldots\}$

i $\{2, 9, 18, 25, 50, 57, 114, \ldots\}$

j $\{2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots\}$

3. Draw the graph of each set below:

a $\{0, 1, 3, 5\}$

b $\{4, 8, 12\}$

c $\{5, 9, 13\}$

d $\{2, 2\times2, 2\times2\times2\}$

e $\{0, 1+1, 2+2, 3+3\}$

f $\left\{\frac{1+1}{2}, \frac{2+2}{2}, \frac{3+3}{2}\right\}$

g $\{\ \ \}$

h $\{0\}$

4. Which of the sets below are infinite sets?

a The whole numbers less than one billion.

b The whole numbers greater than one billion.

c The multiples of 2.

d The factors of 2.

e The common fractions equal to or less than $\frac{1}{2}$.

Exercise 5

1. For each question below, find the whole number or numbers, which make the open number sentences true.

a $\square + \square = 18$

b $\square + \square = 6 + \square$

c $\square \times \square = 81$

d $\square \times 7 = \square \times \square$

e $(\square \times \square) \div 4 = 16$

f $(7 \times \square) + 8 = 29$

g $\square + 2 < \square + \square$

h $5 \times \square < 10 + \square$

i $25 \div \square = \square$

j $9 \times \square = (3 \times \square) + (6 \times \square)$

2. List the whole numbers which make each of the open number sentences true.

a $\square \times \square = 144$

b $(\square \times \square) + (3 \times \square) = 10$

c $(\square \times \square) + 9 = 90$

d $(\square \times \square) - \square = 30$

e $\square + 9 + 11 + \square = 40$

f $(\square \times \square) - (2 \times \square) = 3$

g $\square \times \square \times \square = 125$

h $(\square \times \square) + \square = 42$

i $(\square \times \square) - (2 \times \square) = 48$

j $\square + 14 + \square = 4 \times 4$

Exercise 6

Write open number sentences for the problems below and find the numbers which make each open sentence true.

1. Three times a number plus 7 equals 43. What is the number?
2. If we take 8 times a number and subtract 9 we get 47. What is the number?
3. If we take twice a number from 25 we get 5. What is the number?
4. A number multiplied by itself is 6 less than 70. What is the number?
5. If three times a number is less than 29, what may the number be?

Exercise 7

Write the number pairs which make the following open sentences true. *Replacement set: the set of whole numbers for each place holder.*

1. $\square + \triangle = 10$
2. $\square + 6 + \square = 15$
3. $5 + \square = 16 - \triangle$
4. $4 \times \square = 3 \times \triangle$

5. $(3\times\square)+\triangle=24$

6. $(2\times\square)+(14\times\triangle)=30$

7. $\square+\triangle=1$

8. $26-\square=8+\triangle$

9. $(3\times\square)+(2\times\triangle)=15$

10. $\square=11-(2\times\triangle)$

Exercise 8

Write solution sets for the open sentences below.
Replacement set: the set of whole numbers for each place holder.

1. $\square=3\times\triangle$
2. $\square=10+(2\times\triangle)$
3. $\square+\triangle=\triangle+\square$
4. $\square+\triangle<5$
5. $(2\times\square)-5<\triangle$
6. $(3\times\square)+(2\times\triangle)<20$
7. $\square=36\div\triangle$
8. $(2\times\square)-(3\times\triangle)=1$
9. $\square\times\triangle=24$
10. $\square+9=\triangle+6$
11. $\square\div5=\triangle\div3$
12. $3\times\square=\frac{1}{2}$ of $\triangle$
13. $(2\times\square)+\triangle>10$
14. $2\times\square=5\times\triangle$

Exercise 9

1. Write the sets of whole numbers represented by x in each example below:

   a $x<5$ but $x>4$
   b $x>4$ but $x<14$
   c $16<x+15$
   d $x+7<x+9$
   e $x+4>x^2$

2. Find the solution sets for the open sentences below.
   Replacement set: the set of whole numbers

   a $x+5=17$
   b $m+m+m=3\times m$
   c $y-9=18$
   d $64\div x=x$
   e $x+x=22$
   f $x>y+2$
   g $(3\times z)+9=\frac{1}{2}$ of 78
   h $x+y=2\times y$
   i $x^2+6<12$
   j $x^2=16-7$

3. Write solution sets for the following.
Replacement set: the set of whole numbers

a $3+x=2x+2$
b $x^2+1=17$
c $2x+4=3x-5$
d $x+x+x=x+x$
e $5-x<8$
f $x+15<22$
g $2x+18>30$
h $2x+5>x+12$
i $3x-1<2x+6$
j $2x^2=50$
k $x^2 \times x^2=1$
l $2x^2+3x=5$
m $x^2+2x=3$
n $x+15=x^2+15$
o $x+6<2x$
p $2+x>x$
q $2+x<x$
r $x^2+4=4x+1$
s $5x>25+x$
t $x^2=3x^2-2x^2$

4. Find solution sets for the following open sentences.
Replacement set: the set of whole numbers

a $5x+2x=28$
b $20 \div 2a=10$
c $2x^2=18$
d $m+9+m=3m$
e $4y=16-2y$
f $3p=60-p$
g $2a-6=a+3$
h $2n+6=25+7$
i $5a \div 3=10$
j $3n<10$

5. Write the sets of number pairs which make the following open sentences true. *Replacement set: the set of whole numbers for each place holder*

a $a+b<10$
b $(2 \times x)-y=0$
c $x+(2 \times y)=12$
d $a \div b=1$
e $15 \div p=q$
f $w>\frac{1}{2}$ of n
g $m+5>m+n$
h $(a \times a)+b=20$
i $(4 \times x)=y$
j $r-s=4$

6. Write solution sets for the following.
Each place holder represents a whole number

a $x+y=10$
b $2x-y=3$
c $x<y+1$
d $2x-3=2y+1$
e $x+y=y$
f $2x+1=y+3$

g $3y+1<x$
h $x>y$
i $5+x=y+4$
j $x^2=y^2$
k $x+y>10$
l $x^2-y^2=5$
m $x+y=y+x$
n $xy=yx$
o $(x+y)+5=x+(y+5)$
p $xy=24$
q $2xy<4$
r $x-y=6$
s $3x+2<2y+3$
t $5x=3y$

Exercise 10

1. Copy the open sentences below. Replace each ▩ with a numeral that makes the open sentence true, then state the property you used to do it.
   a $9+36=▩+9$
   b $(300+15)+18=300+(▩+18)$
   c $10\times 26=(6\times 26)+(▩\times 26)$
   d $5\times 87=▩\times 5$
   e $37\times 115=(30\times 115)+(▩\times 115)$
2. Draw your own number line with the whole numbers 0, 1, 2, . . . 20 on it. Use it to evaluate the following:
   a $7+9-4$
   b $6-4+7$
   c 4×5
   d $3+3+3+3-2$
   e $8-3+5$
   f $9+4-3+5-15$
   g $1+1+1-3+1-1$
   h $2-1+6-1+5$
   i $10-2+6$
   j $14-7+5-2$

Exercise 11

1. a What are the prime factors of 161?
   b Write the following as the products of prime factors:
   49 168 1000 156 128
   c Name the primes which are greater than 50 but less than 70.
   d Write *all* the factors of
   48 144 100 78 198
   e What is the largest prime number that will divide both 48 and 36?

2. Find the missing factors:

a $308 = 2 \times 2 \times \square \times 11$

b $390 = 2 \times 3 \times \square \times 13$

c $986 = 2 \times \square \times 29$

d $1029 = 3 \times 7 \times \square \times 7$

e $858 = 2 \times 3 \times 11 \times \square$

f $1428 = 2 \times 2 \times 3 \times 7 \times \square$

3. You are told that $4620 = 2 \times 2 \times 3 \times 5 \times 7 \times 11$. Which of the following are factors of 4620? Explain your answers.

a 12　b 55　c 9　d 231　e 40

f 44　g 88　h 132　i 770　j 660

4. Use factoring to find answers to the following division questions:

a $1820 \div 65$

b $1254 \div 33$

c $1540 \div 28$

d $9240 \div 42$

e $17290 \div 130$

f $3675 \div 49$

g $30030 \div 231$

h $2280 \div 76$

5. Factor the following numbers and then find their square roots:

a 900　b 2500　c 324　d 576

e 361　f 4356　g 3136　h 2304

i 1936　j 3600　k 625　l 225

6. The factors of 6 are $\{1, 2, 3, 6\}$. If we add these factors, excluding 6, we have $1+2+3=6$. 6 was called a "perfect number" by the Greeks. Which of the following numbers are "perfect numbers"?

a 16　b 24　c 15　d 4　e 12　f 28

Exercise 12

1. Write the following pairs of numbers as products of primes. Use these products to find the greatest common divisor of each pair of numbers.

a 54 and 72　b 18 and 24　c 132 and 144

d 147 and 105　e 108 and 180　f 294 and 168

g 182 and 110　h 165 and 240　i 330 and 308

2. The set of factors of 136 is $\{1, 2, 4, 8, 17, 34, 68, 136\}$
The set of factors of 102 is $\{1, 2, 3, 17, 34, 51, 102\}$
The set of common factors is $\{1, 2, 17, \text{and } 34\}$
The greatest common factor is $\{34\}$
Use this method to find the greatest common factors of the pairs of

numbers below:

a 84 and 108 b 156 and 195 c 42 and 56
d 125 and 175 e 200 and 500 f 106 and 212
g 63 and 63 h 144 and 126 i 135 and 165

3. By writing the sets of all factors of the following pairs of numbers, find the *highest common factor* for each pair.

a 24; 36 b 39; 65 c 51; 34 d 90; 60
e 48; 96 f 121; 66 g 27; 24 h 38; 57
i 140; 84 j 126; 189 k 166; 123 l 125; 150
m 72; 108 n 144; 240 o 64; 96 p 81; 135
q 294; 196 r 108; 156 s 36; 49 t 288; 432

4. Find the least common multiples of

a 18 and 12 b 15 and 25 c 24 and 32
d 21 and 28 e 4, 10 and 12 f 15, 20 and 25
g 12, 16 and 18 h 8, 10 and 18 i 10, 15 and 21

5. By writing the sets of all multiples of the following pairs of numbers, find the *least common multiple* for each pair:

a 3; 6 b 12; 8 c 15; 12 d 8; 12
e 15; 10 f 24; 27 g 9; 8 h 11; 3
i 4; 5 j 18; 15 k 15; 9 l 10; 25
m 8; 16 n 7; 35 o 14; 21 p 63; 42
q 48; 60 r 5; 13 s 4; 9 t 24; 72

Exercise 13

1. Copy and complete the following:

a $\frac{1}{5}=\frac{?}{10}=\frac{3}{?}=\frac{?}{25}$ b $\frac{5}{9}=\frac{15}{?}=\frac{?}{54}=\frac{60}{?}$
c $\frac{4}{4}=\frac{16}{?}=\frac{?}{64}=\frac{?}{256}$ d $\frac{7}{12}=\frac{?}{24}=\frac{21}{?}=\frac{?}{48}$
e $\frac{9}{11}=\frac{18}{?}=\frac{27}{?}=\frac{36}{?}$ f $\frac{4}{1}=\frac{?}{2}=\frac{?}{3}=\frac{?}{4}$
g $\frac{5}{8}=\frac{?}{16}=\frac{15}{?}=\frac{20}{?}$ h $\frac{5}{6}=\frac{10}{?}=\frac{?}{18}=\frac{20}{?}$
i $\frac{3}{7}=\frac{?}{14}=\frac{9}{?}=\frac{?}{28}$ j $\frac{11}{33}=\frac{22}{?}=\frac{?}{39}=\frac{44}{?}$
k $\frac{5}{9}=\frac{10}{?}=\frac{?}{27}=\frac{20}{?}$ l $\frac{11}{21}=\frac{?}{42}=\frac{33}{?}=\frac{?}{84}$
m $\frac{3}{13}=\frac{6}{?}=\frac{?}{39}=\frac{?}{52}$ n $\frac{7}{15}=\frac{?}{30}=\frac{21}{?}=\frac{28}{?}$

2. Solve:

a $\frac{14}{23}=\frac{x}{46}$ b $\frac{s}{5}=\frac{18}{45}$ c $\frac{1}{1}=\frac{p}{20}$

d $\frac{2}{3}=\frac{38}{z}$ e $\frac{5}{x}=\frac{55}{121}$ f $\frac{3}{17}=\frac{x}{102}$

g $\frac{4}{23}=\frac{44}{z}$ h $\frac{m}{9}=\frac{6}{54}$ i $\frac{14}{q}=\frac{98}{105}$

j $\frac{13}{w}=\frac{65}{100}$ k $\frac{15}{19}=\frac{150}{z}$ l $\frac{11}{15}=\frac{w}{180}$

m $\frac{19}{51}=\frac{152}{x}$ n $\frac{17}{x}=\frac{68}{100}$ o $\frac{m}{95}=\frac{3}{285}$

3. Write equivalent fractions for the following fractions with the denominators stated:

a $\frac{3}{8}$ in sixteenths b $\frac{7+2}{5}$ in twenty-fifths

c $\frac{2}{11}$ in fifty-fifths d $\frac{16-4}{1}$ in halves

e $\frac{11+2}{3}$ in fifteenths f $\frac{22-22}{25}$ in fourths

g $\frac{4+9+3}{17}$ in thirty-fourths h $\frac{12-6}{3}$ in sixths

i $\frac{6-5}{6}$ in eighteenths j $\frac{16-8+4}{2}$ in eighths

4. Write the following fractions in their lowest terms:

a $\frac{4}{8}$ b $\frac{33}{11}$ c $\frac{63}{72}$ d $\frac{110}{120}$

e $\frac{84}{216}$ f $\frac{420}{630}$ g $\frac{68}{80}$ h $\frac{115}{125}$

i $\frac{500}{1000}$ j $\frac{15}{18}$ k $\frac{144}{180}$ l $\frac{27}{76}$

m $\frac{35}{42}$ n $\frac{121}{187}$ o $\frac{108}{112}$ p $\frac{161}{207}$

q $\frac{60}{75}$ r $\frac{104}{156}$ s $\frac{225}{250}$ t $\frac{100}{1000}$

5. Arrange the fractions below in order of size putting the smallest first:

a $\frac{2}{15}, \frac{7}{10}, \frac{4}{15}$ b $\frac{9}{10}, \frac{4}{5}, \frac{13}{15}$ c $\frac{2}{3}, \frac{4}{5}, \frac{1}{4}$

d $\frac{7}{8}, \frac{11}{12}, \frac{2}{3}$ e $\frac{8}{9}, \frac{9}{11}, \frac{7}{9}$ f $\frac{5}{6}, \frac{7}{8}, \frac{2}{5}$

g $\frac{3}{8}, \frac{3}{16}, \frac{3}{80}$ h $\frac{11}{18}, \frac{5}{12}, \frac{5}{8}$ i $\frac{3}{4}, \frac{5}{6}, \frac{7}{8}$

j $\frac{2}{3}, \frac{5}{8}, \frac{4}{9}$ k $\frac{11}{24}, \frac{9}{15}, \frac{7}{10}$ l $\frac{1}{5}, \frac{2}{7}, \frac{1}{2}$

Exercise 14

1. Add; write the answers in their simplest form.

a $\frac{4}{5}+\frac{3}{5}+\frac{7}{5}$ b $\frac{9}{8}+\frac{6}{8}+\frac{7}{8}$ c $\frac{11}{12}+\frac{4}{12}+\frac{13}{12}$

d $\frac{7}{2}+\frac{6}{2}+\frac{11}{2}$ e $\frac{4}{15}+\frac{27}{15}+\frac{14}{15}$ f $\frac{15}{9}+\frac{11}{9}+\frac{7}{9}$

g $\frac{11}{3}+\frac{7}{3}+\frac{19}{3}$ h $\frac{8}{7}+\frac{4}{7}+\frac{12}{7}$ i $\frac{2}{11}+\frac{11}{11}+\frac{9}{11}$

j $\frac{5}{9}+\frac{11}{9}+\frac{11}{9}$ k $\frac{12}{15}+\frac{18}{15}+\frac{15}{15}$ l $\frac{19}{30}+\frac{49}{30}+\frac{23}{30}$

2. Add:

a $\frac{3}{8}+\frac{4}{5}$ b $\frac{2}{3}+\frac{7}{9}$ c $\frac{5}{6}+\frac{11}{18}$

d $\frac{4}{9}+\frac{7}{8}$ e $\frac{9}{10}+\frac{13}{15}$ f $\frac{7}{2}+\frac{9}{5}$

g $\frac{2}{11}+\frac{13}{12}$ h $\frac{17}{8}+\frac{14}{3}$ i $\frac{21}{6}+\frac{17}{8}$

j $\frac{11}{18}+\frac{11}{10}$ k $\frac{4}{15}+\frac{7}{8}$ l $\frac{7}{4}+\frac{2}{11}$

m $\frac{7}{12}+\frac{7}{5}$ n $\frac{19}{21}+\frac{23}{21}$ o $\frac{6}{4}+\frac{7}{2}$

p $\frac{14}{9}+\frac{8}{7}$ q $\frac{22}{5}+\frac{4}{3}$ r $\frac{5}{11}+\frac{11}{5}$

3. Work the following:

a $\frac{2}{5}+\frac{3}{8}+\frac{3}{5}$ b $\frac{2}{3}+\frac{5}{6}+\frac{3}{4}$ c $\frac{2}{7}+\frac{9}{10}+\frac{2}{5}$

d $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$ e $\frac{5}{6}+\frac{3}{8}+\frac{4}{5}$ f $\frac{3}{8}+\frac{7}{10}+\frac{5}{6}$

g $\frac{11}{19}+\frac{7}{8}+\frac{8}{19}$ h $\frac{4}{9}+\frac{2}{7}+\frac{2}{3}$ i $\frac{3}{8}+\frac{5}{9}+\frac{1}{3}$

4. Work the following; put each answer in its simplest form.

a $2\frac{2}{3}+3\frac{1}{10}+1\frac{1}{2}$ b $6\frac{7}{8}+1\frac{4}{5}+2\frac{2}{3}$

c $1\frac{11}{12}+2\frac{3}{10}+1\frac{3}{4}$ d $5\frac{17}{18}+2\frac{7}{12}+5\frac{3}{4}$

e $1\frac{2}{15}+2\frac{1}{2}+1\frac{7}{10}$ f $4\frac{7}{8}+2\frac{4}{9}+1\frac{1}{8}$

g $3\frac{4}{21}+6\frac{5}{8}+1\frac{17}{21}$ h $1\frac{1}{6}+2\frac{3}{5}+1\frac{3}{4}$

i $2\frac{9}{10}+3\frac{1}{2}+1\frac{5}{9}$ j $4\frac{7}{8}+6\frac{1}{9}+2\frac{2}{3}$

5. Add and express the answers in their simplest forms:

a	b	c	d	e	f	g
$\frac{2}{3}$	$\frac{7}{8}$	$\frac{1}{6}$	$\frac{3}{4}$	$\frac{7}{9}$	$2\frac{4}{5}$	$2\frac{1}{8}$
$\frac{5}{12}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{5}{8}$	$\frac{5}{6}$	$1\frac{1}{2}$	$3\frac{5}{6}$
$\frac{5}{6}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{5}{8}$	$2\frac{3}{4}$	$4\frac{2}{3}$

h $2\frac{1}{2}+3\frac{1}{5}+1\frac{2}{3}$ i $6\frac{9}{10}+2\frac{4}{5}+2\frac{1}{8}$ j $9\frac{2}{5}+3\frac{4}{7}+4\frac{8}{9}$

k $9\frac{2}{5}+3\frac{1}{13}+4\frac{2}{3}$ l $2\frac{8}{15}+1\frac{17}{18}+5\frac{7}{12}$ m $1\frac{1}{2}+2\frac{3}{13}+1\frac{3}{5}$

n $2\frac{4}{5}+1\frac{11}{12}+2\frac{1}{2}$ o $3\frac{3}{4}+1\frac{7}{9}+4\frac{1}{8}$ p $3\frac{5}{7}+2\frac{8}{9}+2\frac{1}{2}$

6. A farmer shipped four boxes of peaches. They weighed $44\frac{1}{2}$ lb., $43\frac{3}{8}$ lb., $45\frac{1}{4}$ lb. and $42\frac{5}{16}$ lb. What was the total weight?

Exercise 15

1. Subtract and express the answers in their simplest forms:

a	b	c	d	e	f
$4\frac{1}{2}$	$12\frac{9}{16}$	8	$15\frac{1}{6}$	$4\frac{2}{3}$	$2\frac{9}{10}$
$1\frac{1}{2}$	$7\frac{11}{16}$	$2\frac{4}{5}$	$7\frac{5}{8}$	$4\frac{1}{4}$	$1\frac{11}{12}$

g $2\frac{3}{8}-1\frac{7}{9}$ h $5\frac{2}{3}-1\frac{7}{13}$ i $2\frac{1}{9}-\frac{7}{8}$ j $5\frac{2}{7}-3\frac{5}{9}$

k $2\frac{5}{8}-1\frac{17}{18}$ l $10-9\frac{9}{10}$ m $3\frac{2}{5}-1\frac{8}{9}$ n $12\frac{11}{15}-3\frac{5}{6}$

o $5\frac{3}{11}-\frac{3}{4}$ p $2\frac{9}{10}-1\frac{11}{15}$ q $4\frac{11}{16}-3\frac{13}{18}$ r $12\frac{11}{15}-4\frac{15}{16}$

2. Solve the following in the way the first example is worked:

a $\frac{7}{8}-\frac{3}{8}=x$ tells us that $\frac{7}{8}=\frac{3}{8}+x$
$\frac{3}{8}+\frac{4}{8}=\frac{7}{8}$ so $x=\frac{4}{8}=\frac{1}{2}$
$x=\frac{1}{2}$

b $\frac{6}{7}-\frac{4}{7}=n$ c $\frac{8}{9}-\frac{1}{9}=x$ d $\frac{11}{13}-\frac{4}{13}=y$

e $\frac{17}{15}-\frac{4}{15}=z$ f $\frac{8}{4}-\frac{7}{4}=r$ g $\frac{13}{8}-\frac{5}{8}=p$

h $\frac{21}{10}-\frac{7}{10}=x$ i $\frac{21}{11}-\frac{10}{11}=y$ j $\frac{11}{5}-\frac{4}{5}=n$

3. Work and prove the following in the way the first one is done:

a $\frac{4}{5}-\frac{2}{3}=x$
$\frac{4}{5}-\frac{2}{3}=\frac{12}{15}-\frac{10}{15}=\frac{2}{15}$
$x=\frac{2}{15}$
Proof: $\frac{4}{5}-\frac{2}{3}=x$ tells us that $\frac{4}{5}=x+\frac{2}{3}$
If $x=\frac{2}{15}$ then $x+\frac{2}{3}=\frac{2}{15}+\frac{2}{3}=\frac{2}{15}+\frac{10}{15}=\frac{12}{15}=\frac{4}{5}$.
$x+\frac{2}{3}=\frac{4}{5}$. The working is proved.

b $\frac{3}{4}-\frac{1}{6}=x$ c $\frac{7}{8}-\frac{2}{9}=x$ d $\frac{9}{10}-\frac{4}{5}=x$

e $\frac{23}{24}-\frac{4}{7}=x$ f $\frac{17}{8}-\frac{5}{3}=x$ g $\frac{21}{15}-\frac{11}{10}=x$

h $\frac{33}{8}-\frac{4}{2}=x$ i $\frac{16}{13}-\frac{0}{4}=x$ j $\frac{21}{8}-\frac{7}{3}=x$

k $\frac{14}{3}-\frac{5}{2}=x$ l $\frac{17}{7}-\frac{7}{5}=x$ m $\frac{11}{7}-\frac{5}{8}=x$

4. Solve for n.

a $2\frac{1}{2}-1\frac{3}{5}=n$ b $4\frac{8}{9}-3\frac{9}{10}=n$ c $6-1\frac{4}{7}=n$

d $6\frac{2}{3}-4\frac{7}{10}=n$ e $5\frac{1}{4}-5\frac{1}{8}=n$ f $4\frac{11}{15}-2\frac{7}{8}=n$

g $6\frac{2}{3}-5\frac{3}{4}=n$ h $1\frac{1}{9}-\frac{4}{5}=n$ i $2\frac{3}{4}-1\frac{2}{9}=n$

5. Solve for x.

a $\frac{3}{8}-x=\frac{1}{5}$ b $1\frac{7}{8}-\frac{9}{10}=x$ c $x-1\frac{1}{4}=2\frac{7}{9}$

d $\frac{17}{4}-x=\frac{7}{5}$ e $\frac{23}{12}-x=\frac{17}{10}$ f $\frac{41}{13}-\frac{12}{7}=x$

g $x-1\frac{3}{8}=\frac{9}{2}$ h $\frac{5}{19}-x=\frac{0}{5}$ i $\frac{37}{15}-\frac{17}{9}=x$

6. Work the following:

a $(\frac{7}{8}-\frac{1}{2})+(\frac{13}{5}-\frac{9}{10})$ b $\frac{41}{10}+(\frac{27}{2}-\frac{7}{4})+\frac{1}{12}$

c $(\frac{17}{18}-\frac{1}{2})-(\frac{1}{2}-\frac{1}{3})$ d $(\frac{27}{2}-\frac{1}{5})-(\frac{7}{8}+\frac{1}{6})$
e $(\frac{12}{7}-\frac{3}{14})-(\frac{19}{28}-\frac{1}{2})$ f $(\frac{21}{15}-\frac{1}{5})-(\frac{3}{10}+\frac{11}{20})$

Exercise 16

1. Find the following products:
   a $5\times\frac{3}{10}$ b $\frac{2}{3}\times21$ c $11\times3\frac{5}{22}$
   d $2\frac{3}{5}\times4$ e $2\frac{2}{3}\times1\frac{7}{8}$ f $\frac{11}{15}\times\frac{25}{33}$
   g $\frac{2}{7}\times1\frac{1}{6}$ h $2\frac{2}{3}\times3\frac{3}{8}$ i $\frac{2}{5}\times\frac{3}{4}\times\frac{15}{16}$
   j $1\frac{1}{2}\times2\frac{1}{3}\times\frac{11}{14}$ k $\frac{9}{10}\times2\frac{3}{4}\times1\frac{7}{18}$ l $2\frac{1}{5}\times\frac{2}{3}\times1\frac{1}{11}$
   m $\frac{5}{16}\times1\frac{1}{2}\times6\frac{2}{5}$ n $1\frac{7}{8}\times2\frac{2}{3}\times1\frac{1}{5}$ o $\frac{15}{16}\times1\frac{1}{15}\times\frac{8}{8}$
   p $2\frac{2}{3}\times1\frac{1}{8}\times1\frac{1}{2}$ q $5\frac{1}{7}\times4\frac{2}{3}\times1\frac{1}{8}$ r $2\frac{1}{2}\times1\frac{1}{5}\times2\frac{3}{4}$
   s $1\frac{1}{4}\times2\frac{1}{3}\times\frac{9}{20}$ t $2\frac{2}{3}\times4\frac{1}{6}\times3\frac{1}{15}$ u $4\frac{3}{4}\times2\frac{3}{4}\times4\frac{1}{5}$

2. Multiply and express the answers in their simplest forms:
   a $\frac{2}{3}\times\frac{4}{5}$ b $\frac{5}{6}\times\frac{9}{10}$ c $8\times\frac{3}{16}$
   d $\frac{4}{15}\times20$ e $5\frac{1}{3}\times4\frac{3}{4}$ f $2\frac{1}{7}\times6\frac{1}{9}\times4\frac{1}{5}$
   g $5\frac{1}{3}\times12\frac{3}{5}\times3\frac{3}{4}$ h $2\frac{1}{2}\times2\frac{7}{8}\times3\frac{1}{5}$ i $5\frac{3}{8}\times3\frac{3}{7}\times5\frac{3}{5}$
   j $2\frac{7}{9}\times2\frac{2}{5}\times1\frac{3}{10}$ k $3\frac{1}{2}\times2\frac{11}{14}\times1\frac{1}{3}$ l $5\frac{2}{3}\times1\frac{1}{34}\times\frac{2}{35}$
   m $2\frac{3}{10}\times3\frac{3}{8}\times1\frac{1}{18}$ n $4\frac{2}{3}\times5\frac{2}{7}\times\frac{1}{2}$ o $\frac{5}{8}\times6\frac{2}{5}\times\frac{1}{4}$

3. Use the distributive property to evaluate the following:
   a $(2\frac{1}{3}\times\frac{2}{3})+(1\frac{2}{3}\times\frac{2}{3})$ b $(3\frac{1}{2}\times1\frac{1}{4})+(3\frac{1}{2}\times\frac{3}{4})$
   c $(\frac{3}{4}$ of $1\frac{1}{5})+(\frac{1}{4}$ of $1\frac{1}{5})$ d $(1\frac{5}{6}\times2\frac{9}{10})+(1\frac{5}{6}\times3\frac{1}{10})$
   e $(\frac{1}{2}$ of $2\frac{3}{4})+(1\frac{1}{2}\times2\frac{3}{4})$ f $(3\frac{4}{5}\times1\frac{1}{2})+(3\frac{1}{5}\times1\frac{1}{2})$
   g $(2\frac{1}{2}\times3\frac{5}{8})+(2\frac{1}{2}\times2\frac{3}{8})$ h $(4\frac{2}{3}\times2\frac{5}{9})+(5\frac{1}{3}\times2\frac{5}{9})$
   i $(5\frac{1}{4}\times1\frac{3}{8})+(2\frac{3}{4}\times1\frac{3}{8})$ j $(\frac{2}{3}$ of $\frac{9}{10})+(1\frac{1}{3}\times1\frac{9}{10})$

4. A rectangular prism is $6\frac{2}{3}$ ft. long, $2\frac{1}{5}$ ft. wide, and $2\frac{1}{4}$ ft. high. What is its volume?

5. Find the following products and graph each of them on a number line that shows $\frac{1}{2}$'s, $\frac{1}{4}$'s, and $\frac{1}{8}$'s.
   a $2\times1\frac{3}{4}$ b $\frac{1}{2}\times\frac{3}{4}$ c $\frac{2}{3}\times\frac{3}{8}$ d $1\frac{1}{2}\times\frac{5}{6}$ e $2\frac{1}{2}\times1\frac{1}{10}$
   f $\frac{5}{6}\times\frac{3}{5}$ g $1\frac{1}{8}\times\frac{5}{9}$ h $2\frac{2}{7}\times3\frac{1}{2}$ i $2\frac{5}{9}\times2\frac{1}{4}$ j $1\frac{2}{7}\times3\frac{1}{2}$

6. If x can represent any whole number, graph $\frac{1}{2}$ of x on a number line.

Exercise 17

1. Work the following:

a $2 \div \frac{1}{2}$ b $3\frac{1}{3} \div \frac{1}{6}$ c $4\frac{2}{3} \div 1\frac{5}{18}$ d $2\frac{5}{6} \div 1\frac{5}{12}$ e $\frac{5}{8} \div \frac{7}{12}$
f $\frac{9}{10} \div \frac{21}{25}$ g $\frac{5}{8} \div 25$ h $\frac{2}{3} \div 6$ i $\frac{5}{12} \div 1\frac{2}{3}$ j $4\frac{1}{2} \div 12$
k $2\frac{5}{8} \div 5\frac{1}{4}$ l $5\frac{1}{3} \div \frac{2}{9}$ m $12 \div \frac{7}{10}$ n $6 \div 1\frac{1}{11}$ o $\frac{5}{13} \div 1\frac{11}{39}$
p $\frac{2}{3} \div 5$ q $\frac{7}{8} \div \frac{21}{22}$ r $3\frac{5}{9} \div 8$ s $6\frac{2}{3} \div 2\frac{1}{12}$ t $5\frac{2}{9} \div \frac{7}{15}$

2. Divide and express the answers in their simplest forms:

a $\frac{1}{4} \div \frac{1}{8}$ b $\frac{4}{5} \div \frac{8}{15}$ c $9 \div \frac{2}{3}$ d $\$4.50 \div \frac{5}{16}$
e $\frac{3}{16} \div 9$ f $\frac{4}{25} \div 16$ g $3\frac{1}{2} \div 7$ h $8\frac{1}{3} \div 25$
i $2\frac{1}{2} \div 1\frac{1}{4}$ j $8\frac{1}{8} \div 16\frac{1}{4}$ k $27\frac{9}{10} \div 6\frac{1}{5}$ l $6\frac{2}{5} \div 5\frac{3}{5}$
m $\frac{9}{10} \div \frac{3}{4}$ n $\frac{5}{6} \div \frac{15}{16}$ o $\frac{4}{5} \div \frac{2}{3}$ p $\frac{4}{5} \div 10$
q $\frac{3}{8} \div 6$ r $9\frac{1}{6} \div 11$ s $12\frac{1}{2} \div 15$ t $11\frac{1}{9} \div 100$

3. Copy the following,replacing each ? by the symbol <, > or = and so make number sentences that are true.

a $\frac{7}{8} \div \frac{5}{6}$? 1 b $\frac{2}{5} \div \frac{4}{9}$? 1 c $\frac{5}{8} \div \frac{10}{16}$? 1 d $1\frac{2}{3} \div 1\frac{1}{4}$? 1
e $\frac{9}{10} \div \frac{3}{4}$? 1 f $\frac{5}{8} \div 3\frac{3}{4}$? 1 g $2\frac{3}{5} \div 2\frac{7}{10}$? 1 h $3\frac{5}{8} \div \frac{58}{16}$? 1

4. Name three fractions which lie between the two numbers in each of the sets of pairs of numbers below:

a $\{\frac{1}{4}, \frac{1}{2}\}$ b $\{\frac{2}{9}, \frac{5}{18}\}$ c $\{1\frac{1}{3}, 1\frac{5}{6}\}$ d $\{\frac{2}{5}, 1\frac{1}{3}\}$
e $\{1\frac{1}{2}, 1\frac{2}{3}\}$ f $\{\frac{5}{8}, \frac{5}{9}\}$ g $\{\frac{9}{10}, 1\}$ h $\{\frac{7}{12}, \frac{7}{24}\}$

5. How many pieces of metal can be cut from a metal bar that is $9\frac{3}{8}$ ft. long if each piece measures $\frac{5}{16}$ ft. long? (Assume that no metal is lost in cutting.)

6. A storekeeper cuts a roll of ribbon that was 36 yd. long into strips for favours. Each favour was $\frac{5}{12}$ yd. long. If he sold all his favours at 11¢ each, how much money did the storekeeper receive for his roll of ribbon?

Exercise 18

1. What is the place value of each '7' in the numerals below?

a 17.04 b 15.76 c 30729.8 d .07 e 39.107
f 0.00007 g 5.32174 h 69.5627 i 7,000,000 j 1362297

2. Write the following as expanded numerals and then write them as mixed numbers.

a 5.28	**b** 14.3	**c** 2.06	**d** 13.95
e 2.047	**f** 16.008	**g** 7.296	**h** 15.824
i 3.00101	**j** 17.5672	**k** 9.887	**l** 28.843
m 8.987	**n** 35.9014	**o** 11.00019	**p** 41.0808

3. Write the following in the form $\frac{a}{b}$, the form for fractions:

a 4.7	**b** 5.9	**c** 11.3	**d** 16.2
e 7.04	**f** 9.32	**g** 15.80	**h** 2.09
i .069	**j** 3.947	**k** 2.909	**l** 8.354

4. Write decimal numerals for each of the following:

a $\frac{3}{10}$ **b** $\frac{2}{10}+\frac{9}{100}$

c $\frac{4}{10}+\frac{7}{100}$ **d** $\frac{3}{1000}+\frac{5}{100}+\frac{4}{10}$

e $(3\times10)+(4\times1)+\frac{1}{10}+\frac{4}{100}$ **f** $(9\times100)+\frac{9}{100}+\frac{9}{1000}$

g $\frac{8}{10000}+\frac{6}{1000}+\frac{1}{10}$ **h** $(5\times10)+(4\times1)+\frac{8}{1000}+\frac{2}{10000}$

5. Change the following to decimal numerals:

a $\frac{17}{50}$	**b** $\frac{29}{40}$	**c** $\frac{13}{16}$	**d** $\frac{18}{25}$
e $\frac{33}{64}$	**f** $\frac{11}{64}$	**g** $\frac{3}{125}$	**h** $\frac{119}{200}$

Exercise 19

1. Add:

a .7 .3	**b** 3.49 2.8	**c** 4.26 5.09	**d** 13.29 9.8
e 2.75 4.869	**f** 9.832 46.8576	**g** 4.4132 3.798	**h** .0496 .0753
i 3.8 4.7 6.9 2.4	**j** 5.07 6.8 14.34 8.2	**k** 17.68 9.24 33.065 2.98	**l** 13.0329 9.6048 17.2335 5.0128

2. Subtract:

a 8.4 3.9	**b** 9.3 2.6	**c** 11.4 2.7	**d** 4.1 1.5

e 4	f 17.28	g 49.07	h 11.
3.6	9.64	15.88	2.08

i 12.769	j 50	k 9.5	l 11.23
3.889	5.706	4.516	2.741

m 5.3241	n 1.031	o 5	p 7.42
1.8779	.8695	4.7182	3.0695

3. Find the following products:

a 17.3×19.4	**b** 6.08×7.23	**c** 5.09×2.064	**d** .0429×.046
e 15.8×.09809	**f** .111×.111	**g** 1.203×4.86	**h** 38.94×9.684
i 74.5×.3228	**j** 5.048×.6957	**k** 2.018×.309	**l** 19.86×7.298

4. Work the following:

a 39.788÷19.6	**b** .0288÷.072	**c** 132.55÷27.5
d 170.448÷318	**e** $9.24\overline{)809.424}$	**f** $.257\overline{)28.013}$
g $43.2\overline{)25.056}$	**h** $8.76\overline{).809424}$	**i** $\frac{9.0592}{1.9}$
j $\frac{.0132678}{.00567}$	**k** $\frac{.0909}{.909}$	

Exercise 20

1. Add:

a	**b**	**c**	**d**
906.98	4.0098	979.89	27.0965
17.857	.969	86.73	6.0324
2.088	3.25	214.85	49.9877
36.394	7.2785	72.13	6.8514
8.5	0.869	308.12	25.3724
217.69	36.77	965.87	68.9799

e 1.069+3.527+8.76+9.8153+.08827

f 147.0892+673.75418+9.984+3.76793

g 206.9859+2.04321+86.09976+2.68194

2. Subtract:

a	**b**	**c**	**d**
1.00000	572.1623	9.09	5.88
0.90943	489.1729	8.009	5.879

e	**f**	**g**	**h**
14.203	589.645	49.08	51.273
7.5996	79.739	39.099	29.3866

i $3-1.985$ j $2.09-1.995$ k $3.247-1.28$

3. Multiply:

a $.915\times.067$ b 2.08×3.95 c $16.7\times.0987$

d $23.43\times.094$ e 1.5×3.216 f 4.084×79.8

g $\begin{array}{r}109.8\\ \underline{79.6}\end{array}$ h $\begin{array}{r}53.78\\ \underline{4.85}\end{array}$ i $\begin{array}{r}2.079\\ \underline{.987}\end{array}$ j $\begin{array}{r}7.165\\ \underline{48.3}\end{array}$

k $\begin{array}{r}401.9\\ \underline{53.9}\end{array}$ l $\begin{array}{r}.00867\\ \underline{.374}\end{array}$ m $\begin{array}{r}.1394\\ \underline{.876}\end{array}$ n $\begin{array}{r}.2126\\ \underline{3.94}\end{array}$

4. If $96\times114=10944$ write answers to the following:

a $.96\times11.4$ b 9.6×1.14 c $.96\times.114$

d 96×1.14 e $.096\times114$ f $.096\times.114$

g 9.60×1.140 h $.0096\times1140$ i $.096\times.0114$

j 960×1140 k 9.6×11400 l $.0096\times.114$

5. Divide:

a $88.57\div56$ b $108.48\div4.8$ c $40.32\div15.4$

d $82.75\div1.93$ e $7.514\div.074$ f $40.94\div2.58$

g $1.8052\div5.07$ h $14.04\div.38$ i $89.74\div42.3$

j $58.44\div306.6$ k $280.48\div84.8$ l $235.85\div.725$

m $12.0346\div2.424$ n $78.075\div2.45$ o $207.25\div6.42$

p $.7632\div1.23$ q $28.076\div4.85$ r $7.9125\div.685$

Exercise 21

1. Change the following to decimal numerals and circle the repeating decimal numerals.

$\frac{2}{9}$ $\frac{17}{200}$ $\frac{6}{7}$ $\frac{243}{250}$ $\frac{5}{21}$ $\frac{11}{12}$ $\frac{33}{64}$

$\frac{19}{60}$ $\frac{111}{160}$ $\frac{7}{15}$ $\frac{8}{11}$ $\frac{239}{400}$ $\frac{15}{16}$ $\frac{29}{30}$

2. A room is 12 feet long. Its length is 4.5 feet more than its width. What is the perimeter of the room?

3. A metal plate that is 3.708 inches thick is to be reduced to a thickness of 3.685 inches. How many inches of metal need to be removed?

4. A car travels 15.8 miles on a gallon of gasoline. How far will it travel on 5.25 gallons of gasoline?

5. The product of two numbers is 10.1556. If one of the numbers is 3.64, what is the other number?

Exercise 22

1. A kilometer is about .62 miles. About how many miles is 23.8 kilometers?

2. The four sides of a field measure 68.7 rods, 33.5 rods, 70.5 rods, and 34.3 rods respectively. What would be the cost of putting a wire fence around this field if 1 foot of wire fence costs 9¢?

3. A woman bought 3 baskets of peaches for canning at \$1.89 a basket. She bought 25 pounds of sugar at 13¢ a pound and 24 glass jars at \$2.75 a dozen. What change should she receive from a \$20 bill?

4. A truck is loaded with 5 crates weighing as follows: 237.8 lb., 617.45 lb., 482.9 lb., 385.38 lb., and 250.6 lb. What is the total weight of the load?

5. Multiply the sum of 3.09 and 5.83 by their difference.

Exercise 23

1. Four cans of maple syrup each weighing $7\frac{5}{8}$ lb. were packed for shipping. The total weight when packed was $35\frac{1}{4}$ lb. What was the weight of the packing?

2. What is the cost of $14\frac{2}{3}$ square yards of carpet at \$10.50 per square yard?

3. A pile of 60 pieces of sheet copper is $11\frac{1}{4}$ inches high. What is the thickness of one sheet of copper?

4. A boy walks $8\frac{1}{4}$ miles in $2\frac{3}{4}$ hours. What is his rate of walking in miles per hour?

5. A boy receives 20¢ a bushel for picking apples. In the first three hours he picked $15\frac{3}{4}$ bushels. At this rate, how much can he earn in an 8-hour day?

Exercise 24

1. Use (a) fractions
   (b) ordered number pairs

   to express the following rates
   (i) 9 cans for \$1.50
   (ii) 88 feet in 1 second
   (iii) 75¢ a score
   (iv) 69¢ for 5 lb.
   (v) 1 gallon for 700 square feet

2. Which of the following ordered number pairs are equivalent ordered number pairs?

a (3, 4) ; (27, 36)	**b** $(1\frac{1}{2}, 2)$; $(\frac{3}{4}, 1)$	**c** (1, 1) ; (1, 2)
d (5, 8) ; (8, 13)	**e** (90:15) ; (18:3)	**f** (0, 1) ; (1, 2)
g (1, 19) ; (2, 20)	**h** (7, 11 ; $(1\frac{3}{4}, 2\frac{3}{4})$	**i** (5, 10) ; (65, 130)
j (4, 8) ; (8, 4)	**k** (8, 7) ; $(2, 1\frac{3}{4})$	**l** (8, 12) ; (26, 39)

3. Below are equivalent ordered number pairs. What number is represented by x in each example?

a $(x, 2)$; (2, 4)	**b** $(3, x)$; (15, 25)	**c** (24, 15) ; $(x, 5)$
d (7, 13) ; $(21, x)$	**e** $(x, 6)$; (6, 24)	**f** $(11, x)$; (44, 5)
g (6, 8) ; $(x, 1\frac{1}{3})$	**h** (20, 9) ; $(5, x)$	**i** $(1, x)$; (11, 11)
j $(2\frac{1}{4}, 6\frac{3}{4})$; $(\frac{2}{3}, x)$	**k** $(x, 6)$; (10, 12)	**l** (4, 10) ; $(x, 15)$

4. A car travels 315 miles in $8\frac{3}{4}$ hours. How long will it take to travel 630 miles at this rate?

5. $3\frac{1}{2}$ gallons of paint cover 2,275 square feet of a barn. How many gallons would be needed to cover 1,300 square feet of the barn?

Exercise 25

1. Solve:

a $\frac{4}{5}=\frac{x}{15}$	**b** $\frac{7}{9}=\frac{28}{z}$	**c** $\frac{m}{24}=\frac{6}{12}$
d $\frac{p}{49}=\frac{1}{7}$	**e** $\frac{9}{13}=\frac{27}{p}$	**f** $\frac{15}{24}=\frac{x}{16}$
g $\frac{4}{1}=\frac{m}{6}$	**h** $\frac{3}{18}=\frac{x}{45}$	**i** $\frac{1}{2}=\frac{p}{50}$
j $\frac{30}{z}=\frac{60}{28}$	**k** $\frac{w}{10}=\frac{18}{30}$	**l** $\frac{4}{6}=\frac{6}{m}$
m $\frac{p}{24}=\frac{33}{36}$	**n** $\frac{10}{18}=\frac{x}{45}$	**o** $\frac{y}{26}=\frac{24}{39}$
p $\frac{16}{24}=\frac{18}{z}$	**q** $\frac{2}{m}=\frac{24}{12}$	**r** $\frac{x}{12}=\frac{6}{18}$
s $\frac{x}{25}=\frac{1}{5}$	**t** $\frac{6}{16}=\frac{a}{64}$	**u** $\frac{10}{b}=\frac{35}{42}$
v $\frac{21}{8}=\frac{42}{m}$	**w** $\frac{15}{z}=\frac{10}{20}$	**x** $\frac{2}{4}=\frac{3}{z}$

y $\frac{5}{25} = \frac{4}{b}$ z $\frac{17}{18} = \frac{c}{54}$ aa $\frac{2}{22} = \frac{s}{55}$

bb $\frac{3}{21} = \frac{7}{x}$ cc $\frac{4}{b} = \frac{6}{45}$ dd $\frac{x}{18} = \frac{3}{27}$

2. **a** Find the ratio of the measure in column **A** to the measure in column **B**.

   b Find the ratio of the measure in column **B** to the measure in column **A**.

   Write each answer in three different ways. Put your answers in their lowest terms.

A	**B**
18 inches	$1\frac{1}{2}$ yards
3 pints	6 gallons
750 pounds	2 tons
15¢	75¢
1,320 yards	2 miles
12 ounces	3 pounds
2 nickels	half-a-dollar
3 hours	1 day
$16\frac{1}{2}$ feet	1 rod
$1\frac{1}{2}$ inches	2 feet

3. Two partners shared their profits in the ratio of 2:3. If the partner with the larger share received $9,150.00, how much did the other partner receive?

4. An outboard motor used a mixture of $\frac{1}{2}$ pint of oil with each gallon of gasoline. How much oil should be used with $2\frac{1}{2}$ gallons of gasoline?

5. The ratio of the top speeds of two racing cars was 8:7. If the slower car had a top speed of 174.58 miles an hour what was the top speed of the other car?

6. A metal alloy contained copper and tin in the ratio 9:2. If an article made from this alloy contained 1 lb. 11 oz. of copper what weight of tin did it contain?

7. A headache tablet contains $4\frac{1}{2}$ grains of acetyl-salicylic acid and $\frac{1}{2}$ grain of starch. What is the ratio used here?

Exercise 26

1. Write the following as per cents:

$\frac{33}{100}$	$\frac{19}{100}$	$\frac{206}{100}$	$\frac{1000}{100}$	$\frac{1\frac{3}{4}}{100}$	$\frac{\frac{7}{8}}{100}$
$\frac{17}{50}$	$\frac{9}{10}$	$\frac{1}{4}$	$\frac{3}{5}$	$\frac{1}{10}$	$\frac{1}{50}$
$1\frac{11}{50}$	$2\frac{1}{4}$	$1\frac{4}{5}$	$3\frac{7}{10}$	$6\frac{11}{20}$	$3\frac{1}{2}$

2. Write the following as fraction numerals in their lowest terms:

24%	25%	18%	44%	75%	64%
115%	300%	110%	550%	200%	216%

3. Express each of the following as per cents:

.07	.14	.96	.5	.75	.8
1.2	3.58	7.92	5.6	2.84	3.21
.004	.058	.725	1.055	3.218	5.644

4. Write the following as decimal numerals:

15%	75%	60%	78%	95%	2%
13.6%	4.85%	3.2%	5.615%	209%	100%

5. Write the following as per cents:

$\frac{2}{3}$	$\frac{3}{5}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{11}{12}$	$\frac{9}{16}$
$1\frac{4}{5}$	$3\frac{3}{8}$	$4\frac{11}{20}$	$5\frac{2}{3}$	$4\frac{11}{15}$	$1\frac{4}{7}$
$\frac{12}{5}$	$\frac{13}{8}$	$1\frac{2}{15}$	$3\frac{5}{14}$	$\frac{29}{25}$	$6\frac{1}{8}$

Exercise 27

1. Find x:

a 30% of 65 is x **b** x is 20% of 75 **c** x is 32.5% of 80
d 180% of 125 is x **e** 65% of 80 is x **f** 190% of 240 is x
g 40% of 75 is x **h** x is 135% of 82 **i** x is 2.5% of 64
j x is 277% of 420 **k** x is $112\frac{1}{2}\%$ of 164 **l** x is $33\frac{1}{3}\%$ of 270

2. Find y (to the nearest tenth):

a 36 is $y\%$ of 40 **b** 35 is $y\%$ of 20 **c** 93 is $y\%$ of 124
d 64 is $y\%$ of 48 **e** $y\%$ of 96 is 72 **f** $y\%$ of 15 is 25
g $y\%$ of 432 is 36 **h** $y\%$ of 120 is 80 **i** 56 is $y\%$ of 64
j 66 is $y\%$ of 33 **k** $y\%$ of 88 is 99 **l** 27 is $y\%$ of 51

3. Solve for n:

a 15% of n is 36	**b** $66\frac{2}{3}\%$ of n is 112	**c** 75% of n is 336
d 63 is 45% of n	**e** 24 is 40% of n	**f** 35 is 62.5% of n
g 13 is 65% of n	**h** $16\frac{2}{3}\%$ of n is 144	**i** 56% of n is 133
j 95% of n is 57	**k** 15% of n is 33	**l** $22\frac{1}{2}\%$ of n is 90

4. Solve for z:

a 10% of z is 11	**b** 328 is z% of 492	**c** 125% of 84 is z
d 3.5% of 1000 is z	**e** 54 is z% of 27	**f** 100 is 55% of z
g 15 is 15% of z	**h** z% of 140 is 175	**i** z is 63% of 36
j 215% of 80 is z	**k** 3% of 54 is x	**l** z is $6\frac{2}{3}\%$ of 21

5. What number is represented by x in the statements below?

a $\frac{5}{6}=\frac{x}{100}$	**b** $\frac{15}{1000}=\frac{x}{100}$	**c** $\frac{x}{15}=\frac{20}{100}$	**d** $\frac{x}{40}=\frac{7\frac{1}{2}}{100}$
e $\frac{4}{x}=\frac{133\frac{1}{3}}{100}$	**f** $\frac{46}{x}=\frac{230}{100}$	**g** $\frac{5}{4}=\frac{125}{x}$	**h** $\frac{11}{8}=\frac{137.5}{x}$
i $\frac{15}{90}=\frac{x}{100}$	**j** $\frac{x}{16}=\frac{5.5}{100}$	**k** $\frac{7}{8}=\frac{x}{144}$	**l** $\frac{9}{10}=\frac{x}{5}$

Exercise 28

1. A boy earned $64.00 in his summer vacation. He saved $24.00 of his earnings. What per cent of his earnings did he save?

2. One year a girl attended school 98% of the days the school was opened. If she made 196 attendances that year, how many days was the school open?

3. A supermarket bought 250 bushels of potatoes and sold 36% of them in one day. How many bushels of potatoes did the store sell on that day?

4. In 1951 a town had a population of 24,000. Since then the population has increased by $12\frac{1}{2}\%$. By how many has the town's population increased? What is the present population of the town?

5. A class has 35 pupils. If 60% of the pupils are girls, how many boys are there in the class?

Exercise 29

1. The enrollment of a school last year was 640. This year it is 5% more. What is the present enrollment?

2. At a sale, rugs were reduced in price by $12\frac{1}{2}\%$. What was the original price of a rug that now sells for \$278.60?

3. The employees at a factory received a 5% increase in pay. What will be the weekly earnings of a man who works a 40 hour week and previously earned \$2.40 an hour?

4. A girl paid \$3.20 for a purse that previously sold for \$4.00. What was the percent reduction in price?

5. A commission agent received \$562.50 for selling a house. If his rate of commission was $3\frac{3}{4}\%$, what was the selling price of the house?

Exercise 30

1. A man received a raise in pay of $8\frac{1}{3}\%$ of what he had been earning. If his raise amounted to 20¢ an hour, what was the hourly pay of the man before he received a raise?

2. At a sale all goods were reduced in price by 30%. What was the original price of a television set that had a sale price of \$189.00?

3. An agent sold some goods for \$4,800. His rate of commission was $3\frac{3}{4}\%$. What was the amount of commission he received?

4. A man bought an automobile the price of which was \$3,900. He made a down payment of \$650. What per cent of the price of the car was his down payment?

5. In one province, sales tax was paid at a rate of 3%. What was the price of a pair of slacks if the sales tax on them was \$ .85?

Exercise 31

1. A boy deposited \$25.00 in the bank and left it there for $2\frac{1}{2}$ years. If the bank paid interest at the rate of $3\frac{1}{2}\%$ per annum what sum of money should the boy receive in interest?

2. A bank paid interest at the rate of $2\frac{3}{4}\%$ per annum. How much interest should be paid on a deposit of \$720 if the deposit was left in the bank for 9 months?

3. After 1 year 8 months the interest paid on a deposit of \$450 was \$30. At what rate per cent was the interest paid?

4. After leaving a sum of money in the bank for 2 years 4 months at a rate of interest of $3\frac{3}{4}\%$ a boy received \$5.20 interest. What sum of money did he deposit originally?

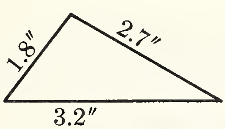
5. A bank pays interest at the rate of $4\frac{1}{2}\%$ per annum. If a boy deposits \$36.00 in this bank, how many months must he leave it there before he receives \$2.16 in interest?

Exercise 32

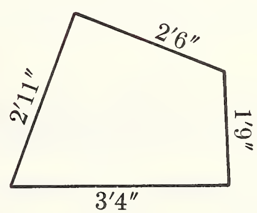
1. Draw a model of
   a a point
   b a ray
   c an angle
   d a simple polygon
   e a curve
   f an octagon
   g a line segment
   h two intersecting lines
   i a line segment perpendicular to a line
   j a pair of parallel lines

2. What is the perimeter of each of the figures drawn below?

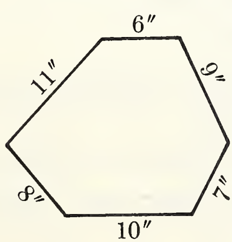
a



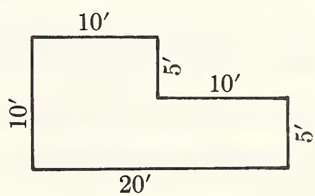
b



c



d

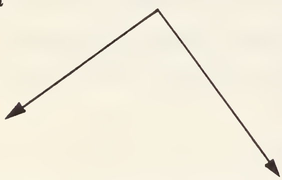


3. What other name may we use for (a) an angle of 90°
(b) an angle of 180°

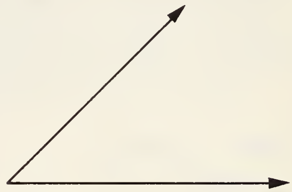
4. Draw angles of 35°, 10°, 95°, 140°, 75°. Which of these angles are acute angles?

5. Measure the angles below. Give their size and state the type of angle each one is.

a



b

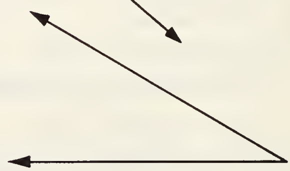


c

d

e

f



Exercise 33

1. Construct triangle ABC so that $\overline{AB}=1\frac{1}{2}''$, $\overline{BC}=2''$, $\overline{CA}=1\frac{3}{4}''$. What kind of triangle is it?

2. The perimeter of a triangle is $7\frac{1}{2}''$. The lengths of two of the sides are $2''$ and $3''$ respectively. Draw the triangle.

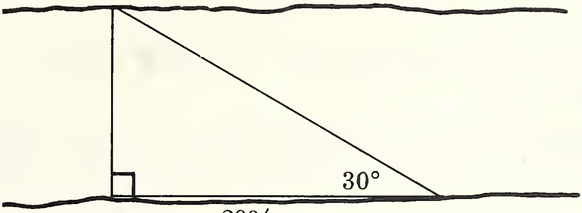
3. Draw triangle XYZ so that $\overline{XY}=4''$, $\overline{XZ}=4''$ and the perimeter is $12''$. Measure the angles. What do you notice?

4. Draw triangle LMN so that $\overline{LM}=3''$, $\overline{LN}=2\frac{1}{2}''$ and $\angle NML$ is 45°.

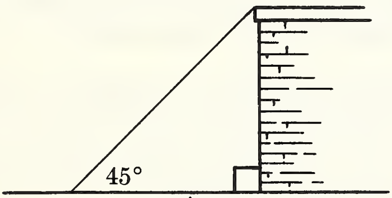
5. In triangle DEF, $\overline{EF}=3''$, $\angle EDF=40°$, $\angle EFD=50°$. Draw the triangle.

Exercise 34

1. Make a scale drawing using a scale of 1″ to represent 50′ and hence find the width of the river in the diagram below.



2. In the diagram below are shown the measurements that were made by a Grade 7 class when they were measuring the height of their school. What is the height of the school in yards?



3. The radius of a circle is 2.5″. What is its diameter? What is its circumference? (Take π as 3.1416)

4. When the pedals of a bicycle make one complete revolution, the back wheel makes $2\frac{1}{2}$ revolutions. How far will this bicycle travel when the rider turns the pedals through 4 revolutions, if the diameter of the back wheel is 28″?

5. The circumference of a circle is 6.58 inches. What is its radius? (Give your answer to the nearest $\frac{1}{10}$ of an inch).

Exercise 35

1. Construct triangles ABC with the following dimensions:

a	$\overline{AB}=3''$	$\overline{BC}=3\frac{1}{2}''$	$\overline{CA}=2\frac{3}{4}''$
b	$\overline{AB}=2''$	$\overline{AC}=2''$	$\angle CAB=60°$
c	$\overline{BC}=4''$	$\angle ACB=40°$	$\angle ABC=90°$
d	$\overline{AB}=1\frac{3}{4}''$	$\overline{BC}=1\frac{3}{4}''$	$\overline{CA}=1\frac{3}{4}''$

2. Draw triangle ABC so that $\overline{AB}=2''$, $\overline{BC}=1\frac{1}{2}''$, $\angle ABC=90°$. What is the area of this triangle?

3. The perimeter of a rectangle is 20.68″. If the length of the rectangle is 5.73″ what is its width?

4. A circle has a circumference of 4.71″. Find its radius if π is given as 3.14 (approx.)

5. A rectangular prism has a length of 1.5″, a width of 1″ and a volume of 1.125 cu. in. What is the height of this rectangular prism?

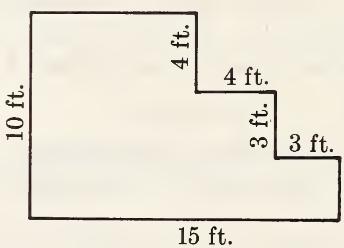
Exercise 36

1. A rectangular shaped room is 7 yards long and 14 feet wide. Find the cost of covering the floor of this room with wall-to-wall carpeting if the price of the carpet is $11.72 a square yard.

2. A farm is in the shape of a rectangle. Its length is 1,320 yards and its width is 880 yards. What is the area of the farm in acres?

3. A rectangular region has a length of $20\frac{1}{2}$ yards. If the area of this region is $191\frac{1}{3}$ square yards, what is its width?

4. Find the area of the region contained in the polygon below:

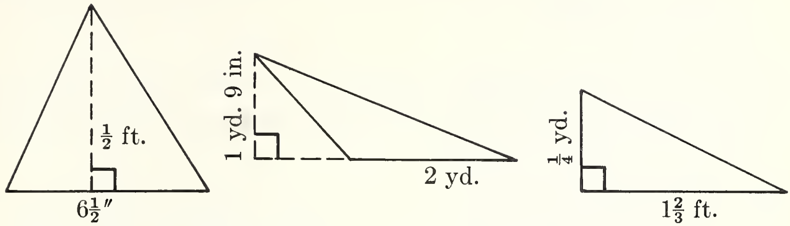


5. The perimeter of a square is $53\frac{1}{3}$ feet. What is its area?

Exercise 37

1. The measure of the base of a right triangle is 4 inches. The altitude of the triangle is $3\frac{1}{2}$ inches. What is its area?

2. Find the areas of the triangular regions drawn below:



Give each answer in:

(a) square inches

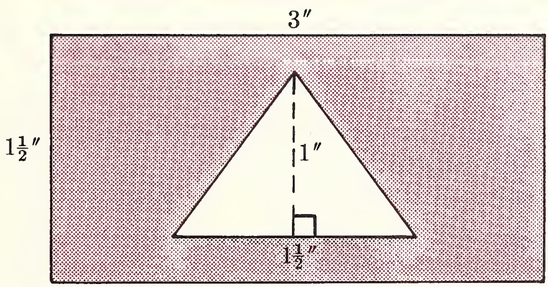
(b) square feet

(c) square yards

3. A field is in the shape of a right triangle. The lengths of the sides of this field are 90 yards, 120 yards, and 150 yards. What is its area in acres?

4. A section of lawn is triangular in shape with a base of 40 feet and an altitude of 30 feet. What will it cost to have this lawn transplanted with sod if sod costs 11¢ a square foot?

5. Find the area of the shaded region in the diagram below:



Exercise 38

1. A grain bin is 12 ft. long, 6 ft. wide, and 5 ft. high. Find its volume in (a) cubic feet (b) cubic yards.

2. A water tank is 6 ft. long, 3 ft. wide, and 3 ft. high. A cubic foot of water weighs about 62.5 lb. How many pounds will the water in the tank weigh when the tank is full?

3. A patio is 25 ft. long and 20 feet wide. This patio is to be covered with cement to a depth of 9 in. What will the cement cost if 1 cubic yard of cement costs $5.25?

4. A contractor charges $3.50 a cubic yard for excavations. What will he charge for excavating a basement that is 36 feet long, 30 feet wide, and 9 feet deep?

5. Find the missing measures of the rectangular prisms below.

Rectangular Prism	Length	Width	Height	Volume
A	$2\frac{1}{2}''$	$3\frac{1}{4}''$	$1\frac{3}{4}''$	▩
B	4 ft.	2 yd.	5 ft.	▩
C	6 yd.	▩	$2\frac{1}{2}$ yd.	30 cu. yd.
D	$16''$	$12''$	▩	1536 cu. in.
E	▩	1 ft.	1 ft.	1 cu. ft.
F	$3\frac{1}{3}$ ft.	$2\frac{1}{2}$ ft.	$1\frac{1}{4}$ ft.	▩
G	$2\frac{2}{3}$ yd.	$1\frac{1}{8}$ yd.	$\frac{5}{9}$ yd.	▩
H	$11\frac{1}{4}''$	$4\frac{1}{5}''$	▩	126 cu. in.
I	$4\frac{1}{2}$ yd.	▩	$2\frac{2}{3}$ yd.	32 cu. yd.
J	3.5 ft.	1.75 ft.	1.25 ft.	▩

Exercise 39

1. Use the rules you have learned for the order of performing operations to compute the following:

a $16+24+34-19-18+30$

b $56+9\times3+13$

c $315\div5\times2$

d $40\div8\times2+6$

e $17-36\div9+8$

f $27+14\div7-7$

g $303-300+2\times15$

h $4\times8\div2-12$

i $3\times9-6\div3+5$

j $14\times2-3\times6$

k $27+13-15-8+9-6$

l $5\times6+18\div9+54\div9$

m $7\times8-8+2\times3\div2$

n $4\times2-16\div2+3\times4-48\div4$

o $14+9\div3\times5$

p $4\times8-9\div3\times2$

q $15\times0+15\div5$

r $27-9+0\div9+3\times9$

s $36\div9+9\div3-12\div6$

t $15+15\times2+15\div5-15$

2. Simplify:

a $(5+4)\times(12-4)$

b $(17+9)\div(20-7)$

c $(4+3)\times(6+2)\div(12-8)$

d $(3+2+3)\times(12-2-6)$

e $16\div(2\times4)$

f $(16\div2)\times4$

g $5\times(7-3-2)\div(9-3-1)$

h $(3+1)\times(4+1)\times(5+1)$

i $(20+5)\times(20-5)$

j $20\times(5-4)-(3\times2-5)$

k $(7+2-4)\times(3\times2-1)$

l $(33-2\times11)\div(2\times44\div8)$

m $(6+2\times9)\div(27\div3\div3)$

n $(3+2)\times(2+3)\div(3+2)$

o $(60-6)\div(18-12)\div(13-4)$

p $(4+6\times2)\div(3\times2+2)$

q $(9-3\div3)\times(3+2\times2)$

r $(5-3)\times(6-3)\div(5+9-8)$

s $(4+2-6)\times(5+7-3)\times(16\div8+2)$

t $(14-3)\times(16+9\div3-9)$

3. Evaluate:

a $23\times(16-9)\div14$

b $27+189-36\times4$

c $27+(189-36)\times4$

d $36\div9\times3\div6$

e $706+189-198\times2$

f $(14-7)\times(288\div24)$

g $315-189-76+306$

h $4016\times(3956-3069)$

i $144\div12\times3\div9$

j $37-18-9+17+15$

4. Simplify:

a $\dfrac{6+15}{4+3}$

b $\dfrac{9+16-5}{100\div10}$

c $\dfrac{5\times3-7}{24\div3}$

d $\dfrac{3+6\times9}{9+2\times5}$

e $\dfrac{5\times4-2}{2\times2+2}$

f $\dfrac{4+8\times5}{2\times7+8}$

g $\dfrac{(8+2)\times(6-4)}{(2+8)\div(16-6)}$

h $\dfrac{2+(4\times5)-4}{(5-3)\times(7-5)}$

i $\dfrac{(18-9)\times(15+3)}{(15+3)\times(18-9)}$

j $\dfrac{29+5}{4+2}+\dfrac{6\times5}{2\times3}$

k $\dfrac{5\times7-2+7}{2\times5\times2}$

l $\dfrac{(19-15)\times(15-11)}{(11-7)\times(7-3)}$

m $\dfrac{16+9}{10-5}+\dfrac{2\times15}{3\times5}+\dfrac{16-8}{4\times2}$

n $\dfrac{8-16\div4}{9-21\div3}+\dfrac{14-5\times2}{13-4\times3}$

o $\dfrac{15+12}{19-10}-\dfrac{2\times15}{7+8}$

p $\dfrac{49\div7}{21\div3}+\dfrac{54\div9}{18\div3}-\dfrac{63\div7}{36\div4}$

q $\dfrac{31\times(2+2)-24}{(2+3)\times(1+4)}$

r $\dfrac{26-(19+5)-2}{7\times(3+4)-8}$

Exercise 40

1. Evaluate the following:

a	$2x+5$	if $x=3$	**b**	$m+2m-3$	if $m=2$
c	q^2+q	if $q=2$	**d**	$2p^2+3p$	if $p=1$
e	$5y^2+2y$	if $y=3$	**f**	$2z^2+3z+5$	if $z=2$
g	l^2+5l-8	if $l=4$	**h**	$3w^2+2w-9$	if $w=2$
i	y^2-3y	if $y=4$	**j**	$2x^2-4+x^2-x$	if $x=2$
k	$2y^2-3y+2$	if $y=2$	**l**	$8z-2z^2$	if $z=3$
m	$3x^2-3x+5$	if $x=0$	**n**	x^2-x+x^2-x	if $x=1$
o	$3p^2-(p+4)$	if $p=2$	**p**	$(z+4)\times(z-4)$	if $z=6$
q	$m^2-(2+m)$	if $m=9$	**r**	$(w^2-3)\times(w^2+3)$	if $w=2$
s	$(x-4)\times(x^2+x)$	if $x=5$	**t**	$(2z-4)-(z-1)$	if $z=3$

2. Evaluate the following:

a $3x+2x-x$ if $x=12$
b $2y^2-2y+9$ if $y=15$
c n^3-n^2-n if $n=9$
d $2z^2+3z^2+z-8$ if $z=4$
e p^4-p^2 if $p=2$

3. Evaluate the following:

a	$2a+3b$	if $a=2$ and $b=1$
b	x^2+y^2	if $x=1$ and $y=3$
c	$2mn+3m+2n$	if $m=3$ and $n=2$
d	$3p^2+2pq+q^2$	if $p=2$ and $q=4$
e	$2xy-2x^2$	if $x=3$ and $y=4$
f	$3p^2-2q^2+pq$	if $p=2$ and $q=1$
g	u^2+64s	if $u=0$ and $s=2$
h	$\frac{x^2}{3}+\frac{y^2}{4}$	if $x=3$ and $y=2$
i	$\frac{2xy-y}{x+y}$	if $x=2$ and $y=1$
j	$\frac{x^2-xy+y^2}{2xy-x+y}$	if $x=0$ and $y=3$

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